

**ROGERS–RAMANUJAN TYPE IDENTITIES RELATED TO
MODULI $4k$ AND $4k + 2$**

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Abstract: In this paper, we provide the combinatorial interpretations of Rogers–Ramanujan type identities involving double sums series related to moduli $4k$ and $4k + 2$ using $(n + t)$ –color partitions.

Keywords and Phrases: $(n + t)$ –color partitions, combinatorial interpretations, Rogers–Ramanujan type identities.

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1. Introduction

The following two celebrated identities, known as Rogers–Ramanujan Identities (RRIs), were discovered by Rogers in [9] and later rediscovered by Ramanujan:

$$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q, q)_n} = \frac{(q^2, q^3; q^5)_{\infty}}{(q; q)_{\infty}},$$
$$\sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(q, q)_n} = \frac{(q, q^4; q^5)_{\infty}}{(q; q)_{\infty}},$$

where we have used the standard q –series notation

$$(x_1, x_2, \dots, x_r; q)_{\infty} = \prod_{j=0}^{\infty} (1 - x_1 q^j) \cdots (1 - x_r q^j).$$

Bailey derived a number of Rogers–Ramanujan type identities (RRTIs) in [6, 7]. His idea was extensively used by Slater to find 130 such identities, which were cataloged in [13]. Several RRTIs are studied in [6, 7, 8, 9, 14] using various approaches. In the last few decades, some q -series have been extended to multiple series identities, and results of infinite families of RRTIs can be found in [4, 16].

Using Bailey’s idea and the q -hypergeometric transformations of Verma and Jain in [16], Sills discovered some new double sum identities in [11, 12]. However, we are unable to find any work in the literature that provides insight into the combinatorial interpretations of these double sum identities. In this paper, we provide the combinatorial interpretations of double sum identities of Sills given in [12] using $(n + t)$ -color partitions. Before giving the definition of $(n + t)$ -color partitions, we first recall ordinary partitions in Definition 1.1.

Definition 1.1. *A partition λ of a positive integer n into m parts is a non-increasing sequence of positive integers $\lambda_1, \lambda_2, \dots, \lambda_m$ such that*

$$n = \sum_{j=1}^m \lambda_j$$

where λ_j ’s are parts of the partition.

In [5], Andrews et al. exhibited a partition identity including hook difference that generalized the successive ranks theorem. Agarwal and Andrews [3], considered a special case of the general identity proved in [5] and restated the conditions on hook differences in respect of the conditions on the corresponding Frobenius symbol. Then they encoded these partitions as another class of restricted partitions arising from a new category of partitions introduced by them, now commonly known as $(n + t)$ -color partitions.

Definition 1.2. *In [3], the $(n + t)$ -color partition is a partition in which a part of size $n, (n \geq 0)$ can come in $(n + t)$ -different colors denoted by the subscripts, $n_1, n_2, n_3, \dots, n_{(n+t)}$. For $t = 0$ the $(n + t)$ -color partitions are simply called n -color partitions. And if $t \geq 1$ then 0_t is allowed as a part. Also, zeros are not allowed to repeat. The weighted difference between any two parts $(\lambda_i)_{\alpha_i}, (\lambda_j)_{\alpha_j}$ of the $(n + t)$ -color partition $(\lambda_1)_{\alpha_1} + (\lambda_2)_{\alpha_2} + \dots + (\lambda_m)_{\alpha_m}$ with $(\lambda_1)_{\alpha_1} \geq (\lambda_2)_{\alpha_2} \geq \dots \geq (\lambda_m)_{\alpha_m}$ is defined as $\delta_{i,j} = \lambda_i - \lambda_j - \alpha_i - \alpha_j$; $\lambda_i \geq \lambda_j$, where the parts follow order $1_1 > 1_2 > 2_1 > 2_2 > 2_3 \dots$.*

Remark 1.1. *We use $(\lambda_m)_{\alpha_m}$ as the least n -color partition throughout the paper.*

In this paper, we present the combinatorial interpretations of the following

double sum RRTIs appeared in [12] with Identity no. A.7, A.6, A.4 respectively:

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{q^{n^2+2k^2}}{(q; q^2)_n (q^2; q^2)_k (q; q)_{n-2k}} = \frac{(q^8, q^{10}, q^{18}; q^{18})_{\infty}}{(q; q)_{\infty}}, \quad (1.1)$$

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{q^{n^2+2k^2+2k}}{(q; q^2)_n (q^2; q^2)_k (q; q)_{n-2k}} = \frac{(q^6, q^{12}, q^{18}; q^{18})_{\infty}}{(q; q)_{\infty}}, \quad (1.2)$$

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{q^{n^2+2n+2k^2+2k}}{(q; q^2)_{n+1} (q^2; q^2)_k (q; q)_{n-2k}} = \frac{(q^2, q^{16}, q^{18}; q^{18})_{\infty}}{(q; q)_{\infty}}. \quad (1.3)$$

Verma and Jain in [16] gave the analytic generalization of (1.1) and (1.3) respectively, in the following form

$$\sum_{n=0}^{\infty} \sum_{r_1, \dots, r_{p-3}}^{\infty} \frac{q^{2(M_1^2 + \dots + M_{p-3}^2) + 4M_{p-3}^3 + n^2 + 4nM_{p-3}}}{(q; q)_n (q^2; q^2)_{r_1} \cdots (q^2; q^2)_{r_{p-3}} (q; q^2)_{n+2M_{p-3}}} = \prod_{n=0, \pm 2p(\text{mod } 4p+2)} (1 - q^n)^{-1} \quad (1.4)$$

$$\sum_{n=0}^{\infty} \sum_{r_1, \dots, r_{p-3}}^{\infty} \frac{q^{2(M_1^2 + \dots + M_{p-3}^2) + 4M_{p-3}^3 + n^2 + 4nM_{p-3}}}{(q; q)_n (q^2; q^2)_{r_1} \cdots (q^2; q^2)_{r_{p-3}}} \cdot \frac{q^{2(n+3M_{p-3} + M_1 + \dots + M_{p-4})}}{(q; q^2)_{n+2M_{p-3}}} = \prod_{n \neq 0, \pm 2(\text{mod } 4p+2)} (1 - q^n)^{-1}. \quad (1.5)$$

Remark 1.2. We note $M_i = r_1 + r_2 + \dots + r_i$ and $M_{-1} = M_0 = 0$. The identities (1.1) and (1.3) are the special case of (1.4) and (1.5) respectively for $p = 4$ and $n + 2r = n$.

In [3], Agarwal and Andrews also discussed the product side of (1.4). In the sequel, we give combinatorial interpretations of the identities given in (1.1)-(1.3). We present our main results in Section 2 and our concluding remarks in Section 3.

2. Main Results

In this section, to provide the combinatorial interpretations of (1.1)-(1.3) RRTIs, we employ the tool $(n+t)$ -color partitions in Theorem 2.1-Theorem 2.3 respectively. The summation sides of (1.1)-(1.3) are enumerated by $X_i(n)$, and the product sides are enumerated by $Y_i(n)$, where $n \geq 0$ and $1 \leq i \leq 3$.

Theorem 2.1. Let $P_1(k, n)$ count n -color partitions of n into k parts satisfying

1. λ_i is even and $\alpha_i = 2, \forall i$
2. $\delta_{i,j} \geq 0, \forall i, j$

and $Q_1(k, m, n)$, $k \geq 0$ count n -color partitions of n into m parts satisfying

1. $\lambda_m = \alpha_m$,
2. $\delta_{i,i+1} \geq 0$, if $1 \leq i \leq m - 2k - 1$, and
 $\delta_{i,i+1} = 0$, if $m - 2k - 1 \leq i \leq m$.

And, let $X_1(n) = \sum_{k=0}^{\infty} \sum_{j=0}^n P_1(k, j) Q_1(k, n - j)$. Then

$$\sum_{n=0}^{\infty} X_1(n) q^n = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{q^{n^2+2k^2}}{(q; q^2)_n (q^2; q^2)_k (q; q)_{n-2k}}. \quad (2.1)$$

Let $Y_1(n)$ count the partitions of n into parts $\not\equiv 0, \pm 8 \pmod{18}$. Then,

$$X_1(n) = Y_1(n) \quad \forall n \geq 0$$

where

$$\sum_{n=0}^{\infty} Y_1(n) q^n = \frac{(q^8, q^{10}, q^{18}; q^{18})_{\infty}}{(q; q)_{\infty}}.$$

Theorem 2.2. Let $P_2(k, n)$ count n -color partitions of n into k parts satisfying all the conditions of $P_1(k, n)$ with λ_i greater than 2. $Q_2(k, m, n)$ counted same as $Q_1(k, m, n)$. And, let $X_2(n) = \sum_{k=0}^{\infty} \sum_{j=0}^n P_2(k, j) Q_2(k, n - j)$. Then

$$\sum_{n=0}^{\infty} X_2(n) q^n = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{q^{n^2+2k^2+2k}}{(q; q^2)_n (q^2; q^2)_k (q; q)_{n-2k}}. \quad (2.2)$$

Let $Y_2(n)$ count the partitions of n into parts $\not\equiv 0, \pm 6 \pmod{18}$. Then,

$$X_2(n) = Y_2(n) \quad \forall n \geq 0$$

where

$$\sum_{n=0}^{\infty} Y_2(n) q^n = \frac{(q^6, q^{12}, q^{18}; q^{18})_{\infty}}{(q; q)_{\infty}}.$$

Theorem 2.3. $P_3(k, n)$ count the same as $P_2(k, n)$ and $Q_3(k, m, n)$ count the $(n+2)$ -color partitions of n into m parts with subscript $\alpha_m = \lambda_m + 2$ and remaining conditions of $Q_3(k, m, n)$ are the same as $Q_2(k, m, n)$.

And, let $X_3(n) = \sum_{k=0}^{\infty} \sum_{j=0}^n P_3(k, j) Q_3(k, n - j)$. Then

$$\sum_{n=0}^{\infty} X_3(n) q^n = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{q^{n^2+2n+2k^2+2k}}{(q; q^2)_{n+1} (q^2; q^2)_k (q; q)_{n-2k}}. \quad (2.3)$$

Let $Y_3(n)$ count the partitions of n into parts $\not\equiv 0, \pm 2 \pmod{18}$. Then,

$$X_3(n) = Y_3(n) \quad \forall n \geq 0$$

where

$$\sum_{n=0}^{\infty} Y_3(n) q^n = \frac{(q^2, q^{16}, q^{18}; q^{18})_{\infty}}{(q; q)_{\infty}}.$$

Example 2.1. For $n = 8$, $P_1(k, n)$ and $Q_1(k, m, n)$ as counted in Theorem 2.1 are given in Table 1 and 2 respectively.

Table 1: $P_1(k, 8)$

$k \backslash n$	0	2	4	6	8
0	1	0	0	0	0
1	0	1	1	1	1
2	0	0	0	0	1
≥ 3	0	0	0	0	0

Table 2: $Q_1(k, m, 8)$

$m \backslash k$	0	1	≥ 2
0	0	0	0
1	8	0	0
2	11	2	0
≥ 3	0	0	0

$P(1, 2) = 1$, the relevant partition is 2_2 ; $P(1, 4) = 1$, the relevant partition is 4_2 ; $P(1, 6) = 1$, the relevant partition is 6_2 ; $P(1, 8) = 1$, the relevant partition is 8_2 and $P(2, 8) = 1$ the relevant partition is $6_2 2_2$.

Also $Q_1(0, 1, 8) = 8$, where the relevant partitions are $8_1, 8_2, 8_3, 8_4, 8_5, 8_6, 8_7, 8_8$.

$Q_1(0, 2, 8) = 11$, the relevant partitions are $7_5 1_1, 7_4 1_1, 7_3 1_1, 7_2 1_1, 7_1 1_1, 6_3 2_1, 6_2 2_1, 6_1 2_1, 5_2 2_2, 5_1 2_2, 5_1 3_1$. $Q_1(1, 2, 8) = 2$, the relevant partitions are $7_5 1_1, 6_2 2_2$. Hence,

$$Q_1(0, 8) = \sum_{m=0}^8 Q_1(0, m, 8) = 19,$$

and so $Q_1(k, n), 1 \leq k, n \leq 8$, is given in the Table 3.

Table 3: $Q_1(k, n)$

$m \backslash k$	0	1	≥ 2
0	0	0	0
1	8	0	0
2	11	2	0
≥ 3	0	0	0

Therefore,

$$\begin{aligned}
 X_1(8) &= \sum_{k=0}^8 \sum_{n=0}^8 P_1(k, n) Q_1(k, 8-n) \\
 &= (19 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0) + (0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0) \\
 &+ (0 + 1 + 0 + 0 + 0 + 0 + 0 + 0 + 0) + (0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0) \\
 &+ (0 + 1 + 0 + 0 + 0 + 0 + 0 + 0 + 0) + (0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0) \\
 &+ (0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0) + (0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0) \\
 &= 21
 \end{aligned}$$

and $Y_1(8)$ is

$$\begin{array}{cccc}
 7+1 & 3+3+2 & 3+3+1+1 & 2+2+2+1+1 \\
 6+2 & 6+1+1 & 3+2+2+1 & 4+1+1+1+1 \\
 5+3 & 5+2+1 & 2+2+2+2 & 3+2+1+1+1 \\
 4+4 & 4+2+2 & 4+2+1+1 & 2+2+1+1+1+1 \\
 & 4+3+1 & 5+1+1+1 & 3+1+1+1+1+1 \\
 & & & 2+1+1+1+1+1+1 \\
 & & & 1+1+1+1+1+1+1+1
 \end{array}$$

Proofs.**Proof of Theorem 2.1.**

We have

$$\sum_{n=0}^{\infty} P_1(k, n) q^n = \frac{q^{2k^2}}{(q^2; q^2)_k}. \quad (2.4)$$

In $\frac{q^{2k^2}}{(q^2; q^2)_k}$, the factor $q^{2k^2} = q^{2((2k-1)+(2k-3)+\dots+3+1)}$ generates an n -color partition into k parts of the form

$$(4k - 2)_2 + (4k - 6)_2 + \dots + 2_2. \quad (2.5)$$

The factor $\frac{1}{(q^2; q^2)_k}$ generates k nonnegative parts which are $\equiv 0 \pmod{2}$, say, $2 \times a_1, 4 \times a_2, \dots, 2k \times a_k$ such that (2.5) transforms to

$$(4k - 2 + a_1 + a_2 + \dots + a_k)_2 + (4k - 6 + a_1 + a_2 + \dots + a_{k-1})_2 + \dots + (2 + a_1)_2. \quad (2.6)$$

Hence the parts are even with subscript 2, $\delta_{i,i+1} \geq 0 \quad \forall i$.

Next we prove,

$$\sum_{n=0}^{\infty} Q_1(k, m, n) q^n = \frac{q^{m^2}}{(q; q^2)_m (q; q)_{m-2k}}.$$

The factor $q^{m^2} = q^{(2m-1)+(2m-3)+\dots+3+1}$ generates an n -color partition into m parts of the form

$$(2m - 1)_1 + (2m - 3)_1 + \dots + 3_1 + 1_1. \quad (2.7)$$

The factor $\frac{1}{(q; q^2)_m}$ generates m nonnegative odd parts, say, $1 \times l_1, 3 \times l_2, \dots, (2m - 1) \times l_m$. It transforms the partition (2.7) into

$$\begin{aligned} & \left((2m - 1) + 2l_m + 2l_{m-1} + \dots + 2l_2 + l_1 \right)_{(1+l_1)} \\ & + \left((2m - 3) + 2l_m + 2l_{m-1} + \dots + 2l_3 + l_2 \right)_{(1+l_2)} + \dots \\ & + (3 + 2l_m + l_{m-1})_{(1+l_{m-1})} + (1 + l_m)_{(1+l_m)}. \end{aligned} \quad (2.8)$$

The factor $\frac{1}{(q; q)_{m-2k}}$ depends on the value of k . Here, we discuss the cases separately for $k = 0$, $0 < k \leq \lfloor \frac{m}{2} \rfloor$ and $k > \lfloor \frac{m}{2} \rfloor$.

For the last case, $k > [\frac{m}{2}]$, the factor $(q; q)_{m-2k} = 0$ and hence $\frac{1}{(q; q)_{m-2k}}$ will not make any addition in counting of the relevant partitions.

For $k = 0$, $\frac{1}{(q; q)_{m-2k}}$ reduces to $\frac{1}{(q; q)_m}$ which generates m nonnegative parts, $1 \times c_1, 2 \times c_2, \dots, m \times c_m$. So, (2.8) becomes

$$\begin{aligned} & \left((2m-1) + 2l_m + \dots + 2l_2 + l_1 + c_m + \dots + c_1 \right)_{(1+l_1)} \\ & + \left((2m-3) + 2l_m + \dots + 2l_3 + l_2 + c_m + \dots + c_2 \right)_{(1+l_2)} + \dots \\ & + (3 + 2l_m + l_{m-1} + c_m + c_{m-1})_{(1+l_{m-1})} \\ & + (1 + l_m + c_m)_{(1+l_m)}. \end{aligned} \quad (2.9)$$

Clearly the weighted difference between any two consecutive parts is $\delta_{i,i+1} = c_i \geq 0 \forall i$.

For $0 < k \leq [\frac{m}{2}]$ $\frac{1}{(q; q)_{m-2k}}$ generates $(m-2k)$ nonnegative parts, say, $1 \times c_1, 2 \times c_2, \dots, (m-2k) \times c_{m-2k}$. For this case, (2.8) translates to

$$\begin{aligned} & \left((2m-1) + 2l_m + \dots + 2l_2 + l_1 + c_{m-2k} + \dots + c_1 \right)_{(1+l_1)} + \dots \\ & + \left((2m-2(m-2k+1)+1) + 2l_m + \dots + 2l_{m-2k} + l_{m-2k+1} + c_{m-2k} + \right. \\ & \left. c_{m-2k+1} \right)_{(1+l_{m-2k+1})} \\ & + \left((2m-2(m-2k)+1) + 2l_m + \dots + 2l_{m-2k-1} + l_{m-2k} + c_{m-2k} \right)_{(1+l_{m-2k})} \\ & + \left((2m-2(m-2k-1)+1) + 2l_m + \dots + 2l_{m-2k-2} + l_{m-2k-1} \right)_{(1+l_{m-2k-1})} \\ & + \dots + (3 + 2l_m + l_{m-1})_{(1+l_{m-1})} + (1 + l_m)_{(1+l_m)}. \end{aligned} \quad (2.10)$$

Using (2.10),

$$\delta_{i,i+1} = c_i \geq 0, \text{ if } 1 \leq i \leq m-2k-1,$$

and

$$\delta_{i,i+1} = 0, \text{ if } m-2k-1 \leq i \leq m.$$

Now,

$$\sum_{n=0}^{\infty} Q_1(k, m, n) q^n = \frac{q^{m^2}}{(q; q^2)_m (q; q)_{m-2k}}, \quad (2.11)$$

and

$$\begin{aligned} \sum_{n=0}^{\infty} Q_1(k, n)q^n &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} Q_1(k, m, n)q^n \\ &= \sum_{m=0}^{\infty} \frac{q^{m^2}}{(q; q^2)_m (q; q)_{m-2k}}. \end{aligned} \quad (2.12)$$

Also,

$$\begin{aligned} X_1(n) &= \sum_{k,j} P_1(k, j) Q_1(k, n - j), \\ \sum_{n=0}^{\infty} X_1(n)q^n &= \sum_{k,j,n} P_1(k, j) Q_1(k, n - j)q^n \\ &= \sum_{k,j,n} P_1(k, j) Q_1(k, n)q^{n+j} \\ &= \sum_{k=0}^{\infty} \left(\sum_{j=0}^{\infty} P_1(k, j)q^j \right) \left(\sum_{n=0}^{\infty} Q_1(k, n)q^n \right) \\ &= \sum_{k=0}^{\infty} \frac{q^{2k^2}}{(q^2; q^2)_k} \sum_{m=0}^{\infty} \frac{q^{m^2}}{(q; q^2)_m (q; q)_{m-2k}} \\ &= \sum_{k,m=0}^{\infty} \frac{q^{m^2+2k^2}}{(q; q^2)_m (q; q)_{m-2k} (q^2; q^2)_k}. \end{aligned}$$

Remark 2.1. *The proofs of Theorems 2.2 – 2.3 follow on similar argument, hence omitted.*

The results presented in this paper are sufficient to prove the following two identities (A.14)–(A.15) of [12].

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{q^{n^2+4k^2+4k}}{(q; q^2)_n (q^4; q^4)_k (q; q)_{n-2k}} = \frac{(q^8, q^{20}, q^{28}; q^{28})_{\infty} (-q; q^2)_{\infty}}{(q^2; q^2)_{\infty}}, \quad (2.13)$$

$$\sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{q^{n^2+4k^2}}{(q; q^2)_n (q^4; q^4)_k (q; q)_{n-2k}} = \frac{(q^{12}, q^6, q^{28}; q^{28})_{\infty} (-q; q^2)_{\infty}}{(q^2; q^2)_{\infty}}. \quad (2.14)$$

In this paper, we have explored double sum RRTIs using colored partitions. An obvious question arises: Is there a possibility of establishing bijection between

the $(n + t)$ -color partitions and some other combinatorial tools for double sum RRTIs, as analogues to bijections available for single sum RRTIs between different combinatorial objects, such as lattice paths and F-partitions? For related work, refer to [2, 10, 15].

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