J. of Ramanujan Society of Mathematics and Mathematical Sciences
Vol. 11, No. 2 (2024), pp. 153-162

DOI: 10.56827/JRSMMS.2024.1102.12 **ISSN (Online): 2582-5461**

ISSN (Print): 2319-1023

MULTIPLY DIVISOR CORDIAL LABELING IN CONTEXT OF JOINT SUM OF GRAPHS

J. T. Gondalia

Department of Mathematics,
The Rajkumar College,
Rajkot - 360001, Gujarat, INDIA

E-mail : jatingondalia98@gmail.com

(**Received:** *Jan. 04, 2024* **Accepted:** *Jun. 16, 2024* **Published:** *Jun. 30, 2024*)

Abstract: Multiply divisor cordial labeling of a graph G^* having set of node V^* is a bijective h from $V(G^*)$ to $\{1, 2, ..., |V(G^*)|\}$ such that an edge xy is assigned the label 1 if 2 divides $(h(x) \cdot h(y))$ and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph having multiply divisor cordial labeling is said to be a multiply divisor cordial graph.

Keywords and Phrases: Multiply Divisor Cordial Labeling, Multiply Divisor Cordial Graph, Joint sum of graphs.

2020 Mathematics Subject Classification: 05C76, 05C78.

1. Introduction

Graph theory is a useful tool for quantifying and simplifying the various moving aspects of dynamic systems given a collection of nodes and connections that can abstract anything from city plans to computer data. Many arrangements, networking, optimization, matching, and operational problems can indeed be solved by studying graphs using a framework. Many people in Artificial Intelligence and Data Science use graph theory for presentation, and there are some good libraries for it. To store graph data directly in native graph form and support graph-oriented queries, many people will also adopt graph databases like Neo4j. But there are still certain specialised applications for graph databases.

All graphs included here are without loops and parallel edges, having no orientation, finite and connected. We follow the basic notations and terminologies of graph theory as in [9]. A graph labeling is a mapping that carries the graph components to the set of numbers, usually to the set of natural numbers. If the domain is the set of nodes the labeling is called node labeling. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both nodes and edges then the labeling is called total labeling. For a dynamic survey of various graph labeling, we refer to [2].

The concept of multiply divisor cordial labeling was introduced by J. T. Gondalia and A. H. Rokad [5]. J. T. Gondalia and A. H. Rokad et al. [5], [6] proved that cycle, cycle with one chord, cycle with twin chord, cycle with triangle, path graph, star graph, jellyfish and coconut tree are multiply divisor cordial graphs. Further, they proved that ring sum of a star with cycle, ring sum of a star with cycle having one chord, ring sum of a star with cycle having twin chords, ring sum of a star with cycle having triangle, ring sum of a star with double fan, ring sum of a star with double wheel and ring sum of a star with helm graph are multiply divisor cordial labeling.

A. H. Rokad [11] proved that the graph obtained by the joint sum of two copies of Globe $Gl(n)$ is Fibonacci cordial. A. H. Rokad and G. V. Ghodasara [12] proved that the graph obtained by the joint sum of two copies of wheel W_n is Fibonacci cordial. They also proved that the graph obtained by the joint sum of two copies of the Petersen graph is Fibonacci cordial.

A. H. Rokad [13] proved that the graph obtained by the joint sum of two copies of wheel W_n, shell S_n, Double wheel DW_n, Petersen graph, closed helm CH_n, coconut tree $CT_{n,n}$ are different cordial graphs.

2. Preliminaries

Definition 2.1. *The joint sum of two graphs G and H is the graph obtained by the joining a node of G with a node of H by an edge.*

Definition 2.2. *A helm $H_n, n \geq 3$ is the graph obtained from the wheel W_n by adding a pendant edge at each node on the rim of the wheel W_n.*

Definition 2.3. *A closed helm CH_n is the graph obtained by taking a helm H_n and adding edges between the pendant nodes.*

Definition 2.4. *A gear graph is obtained from the wheel W_n by adding a node between every pair of adjacent nodes of rim of the wheel W_n.*

Definition 2.5. *A globe is a graph obtained from two isolated node are joined by n paths of length two. It is denoted by $Gl(n)$.*

Definition 2.6. *A double-wheel graph DW_n of size n can be composed of $2C_n + K_1$, i.e. it consists of two cycles of size n, where the nodes of the two cycles are all connected to a common hub.*

3. Main Results

Theorem 3.1. *The graph obtained by joint sum of two copies of cycle C_n is multiply divisor cordial graph.*

Proof. Let G be the joint sum of two copies of C_n. Let $\{u_1, u_2, ..., u_n\}$ and $\{v_1, v_2, ..., v_n\}$ be the nodes of first and second copy of C_n respectively.
We define labeling function $f : V(G) \to \{1, 2, ..., V(G)\}$, as follows.
$f(u_i) = 2i–1; 1 \leq i \leq n$.
$f(v_i) = 2i; 1 \leq i \leq n$.
In view of above defined labeling pattern we have $e_f(0) = n$ and $e_f(1) = n + 1$.
Therefore $|e_f(0)–e_f(1)| = 1$.
Thus, the graph obtained by joint sum of two copies of cycle C_n is multiply divisor cordial graph.

Illustration 3.1. The multiply divisor cordial labeling of joint sum of two copies of C_5 is shown in Graph 1.

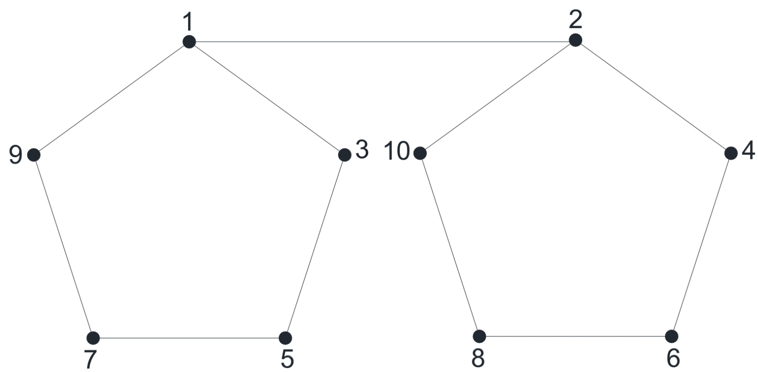

Figure 1: Graph 1

Theorem 3.2. *The graph obtained by joint sum of two copies of Globe $Gl(n)$ is multiply divisor cordial graph.*

Proof. Let G be the joint sum of two copies of $Gl(n)$. Let $\{u, u', u_1, u_2, ..., u_n\}$ and $\{v, v', v_1, v_2, ..., v_n\}$ be the nodes of first and second copy of $Gl(n)$ respectively.
We define labeling function $f : V(G) \to \{1, 2, ..., V(G)\}$, as follows.
$f(u) = 1$,
$f(u') = 3$,

$f(u_i) = 2i + 3; 1 \leq i \leq n.$
$f(v) = 2,$
$f(v') = 4,$
$f(v_i) = 2i + 4; 1 \leq i \leq n.$
In view of above defined labeling pattern we have $e_f(0) = 2n$ and $e_f(1) = 2n + 1$. Therefore $|e_f(0)–e_f(1)| \leq 1$. Thus, the graph obtained by joint sum of two copies of Globe $Gl(n)$ is multiply divisor cordial graph.

Illustration 3.2. The Multiply divisor cordial labeling of joint sum of two copies of Gl_5 is shown in Graph 2.

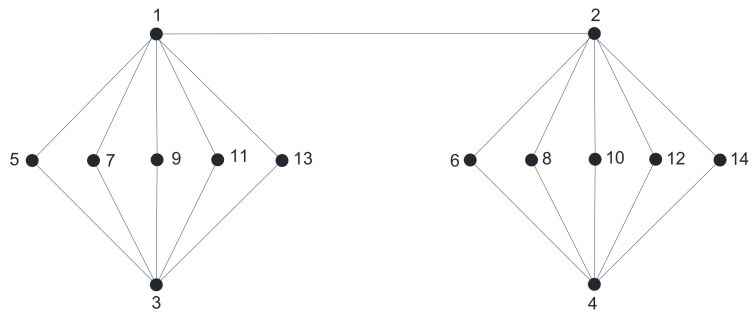

Figure 2: Graph 2

Theorem 3.3. *The graph obtained by joint sum of two copies of wheel W_n is multiply divisor cordial.*
Proof. Let G be the joint sum of two copies of W_n. Let $\{u_0, u_1, u_2, ..., u_n\}$ and $\{v_0, v_1, v_2, ..., v_n\}$ be the nodes of first and second copy of W_n respectively, where u_0 and v_0 be the apex nodes of first and second copy of W_n respectively.
Here we define labeling function $f : V(G) \rightarrow \{1, 2, ..., V\}$ as follows.
$f(u_0) = 1,$
$f(u_i) = 2i + 1; 1 \leq i \leq n.$
$f(v_0) = 2,$
$f(v_i) = 2i; 1 \leq i \leq n.$
We have $e_f(0) = 2n$ and $e_f(1) = 2n + 1$.
Therefore $|e_f(0) - e_f(1)| = 1$ in each case.
Hence joint sum of two copies of W_n is multiply divisor cordial.

Illustration 3.3. The Multiply divisor cordial labeling of joint sum of two copies of W_6 is shown in Graph 3.

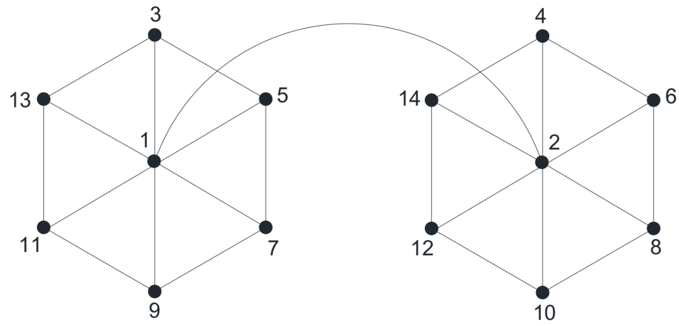

Figure 3: Graph 3

Theorem 3.4. *The graph obtained by joint sum of two copies of double wheel DW_n is multiply divisor cordial.*

Proof. Let G be the joint sum of two copies of DW_n. Let $\{u_0, u_1, u_2, ..., u_n, u'_1, u'_2, ..., u'_n\}$ and $\{v_0, v_1, v_2, ..., v_n, v'_1, v'_2, ..., v'_n\}$ be the nodes of first and second copy of DW_n respectively, where u_0 and v_0 be the apex nodes of first and second copy of DW_n respectively. Here we define labeling function $f : V(G) \rightarrow \{1, 2, ..., V\}$ as follows.

$f(u_0) = 1,$

$f(u_i) = 2i + 1; 1 \leq i \leq n.$

$f(u'_i) = 2n + 2i + 1; 1 \leq i \leq n.$

$f(v_0) = 2,$

$f(v_i) = 2i + 2; 1 \leq i \leq n.$

$f(v'_i) = 2n + 2i + 2; 1 \leq i \leq n.$

We have $e_f(0) = 4n$ and $e_f(1) = 4n + 1$.

Therefore $|e_f(0) - e_f(1)| = 1$ in each case.

Hence joint sum of two copies of DW_n is multiply divisor cordial.

Illustration 4. The Multiply divisor cordial labeling of joint sum of two copies of DW_5 is shown in Graph 4.

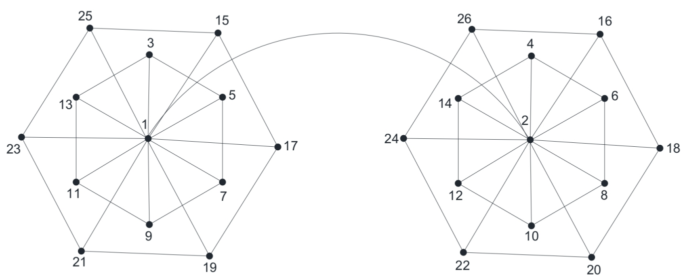

Figure 4: Graph 4

Theorem 3.5. *The graph obtained by joint sum of two copies of double fan DF_n is multiply divisor cordial.*

Proof. Let G be the joint sum of two copies of DF_n. Let $\{u, u', u_1, u_2, ..., u_n\}$ and $\{v, v', v_1, v_2, ..., v_n\}$ be the nodes of first and second copy of DF_n respectively, where u, u' and v, v' be two apex nodes of first and second copy of DF_n respectively.
Here we define labeling function $f : V(G) \to \{1, 2, ..., V\}$ as follows.
$f(u) = 1$,
$f(u') = 3$,
$f(u_i) = 2i + 3; 1 \leq i \leq n$.
$f(v) = 2$,
$f(v') = 4$,
$f(v'_i) = 2i + 4; 1 \leq i \leq n$.
We have $e_f(0) = 3n - 1$ and $e_f(1) = 3n$.
Therefore $|e_f(0) - e_f(1)| = 1$ in each case.
Hence joint sum of two copies of DF_n is multiply divisor cordial.

Illustration 3.5. The Multiply divisor cordial labeling of joint sum of two copies of DF_7 is shown in Graph 5.

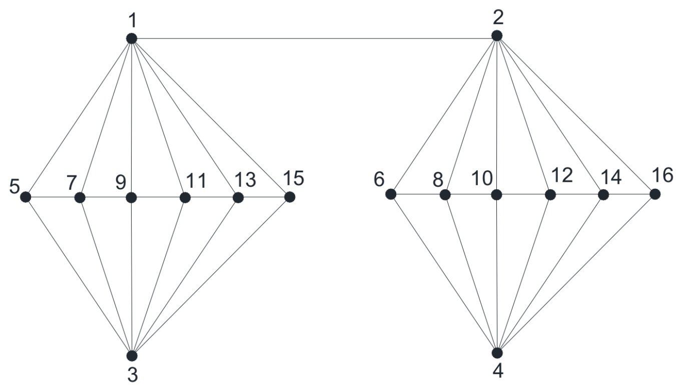

Figure 5: Graph 5

Theorem 3.6. *The graph obtained by joint sum of two copies of helm H_n is multiply divisor cordial.*

Proof. Let G be the joint sum of two copies of H_n. Let $\{u_0, u_1, u_2, ..., u_n, u'_1, u'_2, ..., u'_n\}$ and $\{v_0, v_1, v_2, ..., v_n, v'_1, v'_2, ..., v'_n\}$ be the nodes of first and second copy of H_n respectively, where u_0 and v_0 be the apex nodes of first and second copy of H_n respectively.
The labeling pattern is the same as Theorem 3.4.

We have $e_f(0) = 3n$ and $e_f(1) = 3n + 1$.
Therefore $|e_f(0) - e_f(1)| = 1$ in each case.
Hence joint sum of two copies of H_n is multiply divisor cordial.

Illustration 3.6. The Multiply divisor cordial labeling of joint sum of two copies of H_5 is shown in Graph 6.

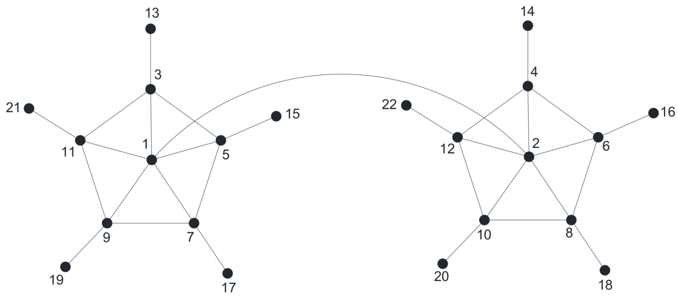

Figure 6: Graph 6

Theorem 3.7. *The graph obtained by joint sum of two copies of helm CH_n is multiply divisor cordial.*
Proof. Let G be the joint sum of two copies of CH_n. Let $\{u_0, u_1, u_2, ..., u_n, u'_1, u'_2, ..., u'_n\}$ and $\{v_0, v_1, v_2, ..., v_n, v'_1, v'_2, ..., v'_n\}$ be the nodes of first and second copy of CH_n respectively, where u_0 and v_0 be the apex nodes of first and second copy of CH_n respectively.
The labeling pattern is the same as Theorem 3.4.
We have $ef(0) = 4n$ and $e_f(1) = 4n + 1$.
Therefore $|e_f(0) - e_f(1)| = 1$ in each case.
Hence joint sum of two copies of CH_n is multiply divisor cordial.

Illustration 3.7. The Multiply divisor cordial labeling of joint sum of two copies of CH_5 is shown in Graph 7.

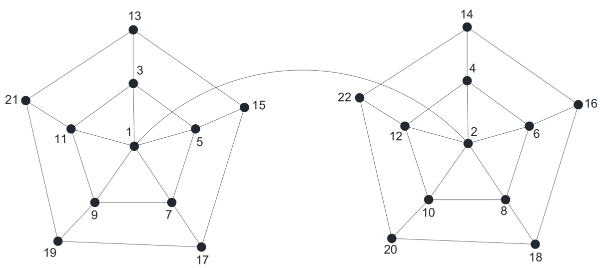

Figure 7: Graph 7

Theorem 3.8. *The graph obtained by joint sum of two copies of gear G_n is multiply divisor cordial.*

Proof. Let G be the joint sum of two copies of G_n. Let $\{u_0, u_1, u_2, ..., u_{2n}\}$ and $\{v_0, v_1, v_2, ..., v_{2n}\}$ be the nodes of first and second copy of G_n respectively, where u_0 and v_0 be the apex nodes of first and second copy of G_n respectively.
Here we define labeling function $f : V(G) \to \{1, 2, ..., V\}$ as follows.
$f(u) = 1$,
$f(u_i) = 2i + 1; 1 \leq i \leq 2n$.
$f(v) = 2$,
$f(v'_i) = 2i + 2; 1 \leq i \leq 2n$.
We have $e_f(0) = 3n$ and $e_f(1) = 3n + 1$.
Therefore $|e_f(0) - e_f(1)| = 1$.
Hence joint sum of two copies of G_n is multiply divisor cordial.

Illustration 3.8. The Multiply divisor cordial labeling of joint sum of two copies of G_6 is shown in Graph 8.

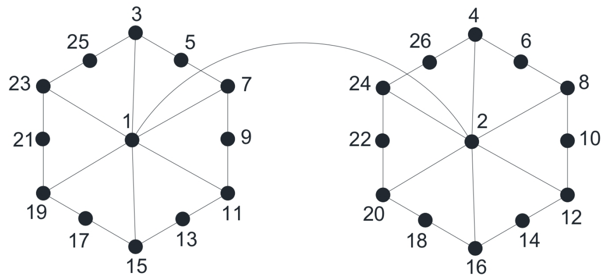

Figure 8: Graph 8

4. Conclusion

In this paper, I investigated eight new multiply divisor cordial graphs. The results proved in this paper are novel. It will create a bridge between graph theory and number theory for researchers. Illustrations are provided at the end of each theorem for a better understanding of the labeling pattern defined in each theorem.

References

[1] Cahit I., Cordial graphs: A weaker version of graceful and harmonious graphs, Ars Combinatoria, 23 (1987), 201-207.

[2] Gallian J. A., A dynamic survey of graph labeling, The Electronics Journal of Combinatorics, 19 (2012), #DS6, 1-260.

[3] Ghodasara G. V., Rokad A. H. and Jadav I. I., Cordial Labeling of Grid Related Graphs, International Journal of Combinatorial Graph Theory and Applications, Vol. 6, No. 2 (2013), 55-62.

[4] Ghodasara G. V., Rokad A. H., Cordial Labeling of $K_{n,n}$ related graphs, International Journal of Science and Research, Vol. 6, No. 2 (2013), 55-62.

[5] Gondalia J. T. and Rokad A. H., Multiply Divisor Cordial Labeling, International Journal of Engineering and Advanced Technology, Vol. 9 No. 2 (2019), 1901-1904.

[6] Gondalia J. T. and Rokad A. H., Multiply Divisor Cordial Labeling in context of Ring sum of Graphs, Advances in Mathematics: Scientific Journal, 9(11) (2020), 9037–9044.

[7] Gondalia J. T. and Rokad A. H., Integer Cordial Labeling of Some Star and Bistar Related Graphs, South East Asian Journal of Mathematics and Mathematical Sciences, Vol. 17, No. 2 (2021), 277-286.

[8] Gross J. and Yellen J., Graph Theory and its Applications, CRC Press, 1999.

[9] Harary F., Graph theory, Addision-Wesley, Reading, MA, 1969.

[10] Rokad A. H. and Ghodasara G. V., Fibonacci Cordial Labeling of Some Special Graphs, Annals of Pure and Applied Mathematics, Vol. 11, No. 1 (2016), 133-144.

[11] Rokad A. H., Fibonacci Cordial labeling of Some Star & Bistar Related Graphs, Multilogic in Science, Vol. vii, Issue xxv (2018).

[12] Rokad A. H. and Ghodasara G. V., Fibonacci Cordial Labeling of Some Special Graphs, Annals of Pure and Applied Mathematics, Vol. 11, No. 1 (2016), 133-144.

[13] Rokad A. H., Difference cordial labeling in context of joint sum of some graphs, Research & Reviews: Discrete Mathematical Structures, Vol. 5, No. 1 (2018), 21-25.

[14] Rokad A. H. and Ghodasara G. V., Cordial Labeling in Context of Ring Sum of Graphs, International Journal on Recent and Innovation Trends in Computing and Communication, Vol. 3, No. 11 (2015), 6272–6275.

[15] Sridevi R., Navanaeethakrishnan S. and Nagarajun K., Super Fibonacci Graceful Labeling, International Journal Mathematical Combinatorics, Vol. 3(2010), 22-40.