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NEW SOFT GENERALIZED ISOMETRIES WITH SOFT POINTS

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Abstract: Guzide Senel first time introduced the concept of soft distance functions between two soft points in soft metric spaces. Motivated by the study of Senel on soft metric spaces, we have define the new definition of soft isometries with soft points and to investigate some important prepositions on soft isometries. Moreover, we also introduce the examples depend on soft isometries. We hope that this result will provide a good and fruitful results for beginners and also help for some new results for researcher in the field of soft isometric spaces.

Keywords and Phrases: Soft set, soft point, soft isometric, soft metric spaces soft distance injective function, surjective function, bijective function and so on.

2020 Mathematics Subject Classification: 28C15, 03E75, 03E72.

1. Introduction and Definitions

Soft set theory is given a general framework for solving problems of unpredictable data. And in many field the complexity of unpredictable data is available like environment, engineering, economics, science, social science etc. To solve all these type of problem in 1999, Molodtsov [6] introduced the most useful tools for dealing with uncertainties. They gave the basic notations of theory of soft set to present the first result of soft set theory and to discuss some problems of the future and find some new results on fuzzy sets and soft set. Soft Sets represent a powerful tool for decision making about information systems, data mining and drawing conclusions from data, especially in those cases where some uncertainty exists in the data. In 2003 Maji et al., [5] studied the theory of soft sets initiated by Molodtsov, and gave the equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set. In 2011 Shabir M. and Naz M. [14] introduced the notion of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms and their basic properties. It is also show that a soft topological space gives a parameterized family of topological spaces with the help of an examples. In 2010 K. V. Babitha and J. J. Sunil initiated [2] the concepts of relations and functions in soft set theory. They have also introduce the equivalent version of some theories on relations and functions in the background of soft sets. All these concepts are basic supporting structures for research and development on soft set theory. In 2017, Muhammad Riaz and Zain Fatima [7] introduced some properties of soft metric spaces by using the idea of soft points in the soft set theory. They also studied soft first category, soft second category, and given the Baire's category theorem for soft metric spaces and also suggested the notion of soft isometric spaces and the completion of a soft metric space. First time in 2017, Güzide Şenel [9] introduced soft distance function between two soft points such as soft distance that cannot be defined in soft topological spaces. They generated the soft metric spaces whose structure is represented by soft distance function shown by $\tilde{d}$. By using this method, they studied new classes of soft mappings and also present different definitions of $\tilde{d}$. In 2018, Guzide senel [10] introduced soft spheres with soft real numbers and soft points in soft metric spaces of curvature bounded below and a survey of applicable results and also gave differences among soft real numbers and soft points are used for the first time to revive the soft sphere mathematically thus the locus of the soft sphere can be analyzed. In 2018, Cigdem gunduz aras et. al., [1] introduced Metric spaces wide area provides a powerful tool to the study of optimization and approximation theory, variational inequalities and so many. After Molodtsov initiated a novel concept of soft set theory as a new mathematical tool for dealing with uncertainties, applications of soft set theory in other disciplines and real life problems was progressing rapidly, the study of soft metric space which is based on soft point of soft sets was initiated by Das and Samanta [3]. The purpose of this paper is to contribute for investigating on new generalized soft isometries on soft metric space which is based on soft point of soft sets and give some of their preposition. We briefly next section give some basic definitions of concepts which serve a background to this work.

2. Preliminaries

Definitions 2.1. [5] *A soft set (F, A) is called a soft set over U, where F is a* *mapping given by $F : A \to P(U)$ In other words, a soft set over U is a parametrized family of subsets of the universal $U, e \in A$. $F(e)$ may be considered as the set of ϵ-approximate elements of the set (F, A). Clearly a soft set is not a set. Inspired by Molodtsov we have given one new example of soft set.*

Definition 2.2. [2] *A function f from (F, A) to (G, B) is called injective (one–one) if $F(a) \neq F(b)$ implies $f(F(a)) \neq f(F(b))$. i. e. f is called injective if each element of the range f appears exactly once in the function.*

Definition 2.3. [2] *A function f from (F, A) to (G, B) is called surjective (onto) if rang $f = (G, B)$.*

Definition 2.4. [2] *A function which is both injective and surjective is called a bijective function. In the above example if we take $A = \{a_1, a_3\}$. Then the function f from (F, A) to (G, B) defined as $f = \{F(a_1) \times G(b_1), F(a_3) \times G(b_2)\}$ is a bijective function.*

Definition 2.5. [8] *A soft mapping $f = (\tilde{f}, \hat{f})$ is said to be injective if $\tilde{f}, \hat{f}$ are both injective. A soft mapping $f = (\tilde{f}, \hat{f})$ is said to be surjective if $\bar{f}, \hat{f}$ are both surjective, A soft mapping $f = (\tilde{f}, \hat{f})$ is said to be bijective if $\tilde{f}, \hat{f}$ are both bijective.*

Definition 2.6. [7] *Let $(\tilde{X}, d, E)$ and $(\tilde{Y}, d^*, E')$ are soft metric spaces and $\varphi_{fu} : (\tilde{X}, d, E) \to (\tilde{Y}, d^*, E')$ is a soft mapping, where $f : X \to Y$ and $u : E \to E'$. Then φ_{fu} is called soft isometry or soft isometric mapping if and only if, $d^*(\varphi_{fu}(T_w^a), \varphi_{fu}(T_\mu^b)) = d(T_w^a, T_\mu^b)$ for all $T_w^a, T_\mu^b \tilde{\in} \tilde{X}$. If φ_{fu} is a bijective soft isometry, then $(\tilde{X}, d, E)$ and $(\tilde{Y}, d^*, E')$ are said to be soft isometric spaces.*

Definitions 2.7. [9] *Soft point - The soft set f is called a soft point in S, if for the parameter $e_i \in E$ such that $f(e_i) \neq \varnothing$ and $f(e_k) = \varnothing$, for all $e_k \in E/\{e_i\}$ is denoted by $(e_{i_f})j$ for all $ijk \neq N^+$. (Note that the set of all soft points of f will be denoted by $SP(f)$)*

Definitions 2.8. [9] *Soft metric spaces - Let $\varnothing \neq X \subseteq E$, $f \in Sx(U)$ and $f : X \to P(U)$ be one to one function $fi, fj, fs \in Sx(U)$ and $(e_{i_f})i, (e_{j_f})j, (e_s)s \in f$. A mapping*

$$\tilde{d} : SP(f) \times SP(f) \to \tilde{R}(E)$$

is said to be a soft metric on the soft set f if $\tilde{d}$ satisfies the following condition:

(i) $\tilde{d}((e_{i_f})i, (e_{j_f})j) \geq 0$.

(ii) $\tilde{d}((e_{i_f})i, (e_{j_f})j) = 0 \Leftrightarrow \tilde{d}(e_{i_f})i = (e_{j_f})j$

(iii) $\tilde{d}((e_{i_f})i, (e_{j_f})j) = \tilde{d}((e_{j_f})j, (e_{i_f})i)$.

(iv) $\tilde{d}((e_{i_f})i, (e_{j_f})j) \leq \tilde{d}((e_{i_f})i, (e_s)s) + \tilde{d}((e_{s_f})s, (e_{j_f})j)$

If $\tilde{d}$ is a soft metric on the soft set then, f is called soft metric space and denoted by $(f, \tilde{d})$

Definitions 2.9. [9] *Soft distance - Let $(e_{i_f})i, (e_{j_f})j$ be soft points of a soft metric space. The value of $\tilde{d}((e_{i_f})i, (e_{j_f})j)$ is called as the soft distance between the soft points $(e_{i_f})i$ and $(e_{j_f})j$.*

3. Main Result

Soft Isometries and their prepositions:

In this section we present the concept of soft isometry of soft metric spaces and their prepositions with example.

Suppose $(g, \tilde{d})$ and $(h, \tilde{e})$ are soft metric spaces and $\tilde{f} : g \to h$ be any soft function. Then $\tilde{f}$ is said to be a soft isometry or a soft isometric map if

$$\tilde{e}(f(e_{a_f})a), f(e_{b_f})b)) = \tilde{d}(f(e_{a_g})a), f(e_{b_g})b) \;\; \forall (e_{a_g})a, (e_{b_g})b \in (g, \tilde{d})$$

In view of the definition, we observe that an soft isometry is a distance preserving function. We observe that an isometry is an injective function.

$$\begin{aligned} f(e_{a_g})a) = f(e_{b_h})b) &\Leftrightarrow e(f(e_{a_g})a), f(e_{b_h})b) = 0 \\ &\Leftrightarrow d((e_{a_g})a), (e_{b_h})b) = 0 \\ &\Leftrightarrow (e_{a_g})a = (e_{b_h})b \end{aligned}$$

If $\tilde{f}$ is an soft isometry, then we say that the soft metric subspace $(\tilde{f}(g, \tilde{d}), \tilde{e})$ of $((h, \tilde{e}), \tilde{e}))$ is an soft isometric copy of the space $(h, \tilde{e})$. Thus metric space $(h, \tilde{e})$ is isometric copy of $(g, \tilde{d})$ if and only if $\tilde{f}$ is a soft surjective isometry.

Preposition 3.1. *Inverse of a surjective isometry is an isometry.*

Proof. Suppose $(g, \tilde{d})$ and $(h, \tilde{e})$ are metric spaces and $\tilde{f} : g \to h$ be any soft isometry.

Claim: $\tilde{f}^{-1} : h \to g$ is a soft isometry.

Consider any $((e_{a_h})a), (e_{b_h})b) \in (h, \tilde{e})$, then there exists, $((e_{x_g})x), (e_{y_g})y) \in (g, \tilde{d})$ Such that $\tilde{f}(e_{x_g})x = (e_{a_h})a$ and $\tilde{f}(e_{y_g})y = (e_{b_h})b$. Now

$$\begin{aligned} \tilde{d}(f^{-1}(e_{a_h})a, f^{-1}(e_{b_h})b) &= \tilde{d}(f^{-1}(f(e_{x_g})x), f^{-1}(e_{y_g})y) \\ &= \tilde{d}(e_{x_g})x, (e_{y_g})y \\ &= \tilde{e}(f(e_{x_g})y, f(e_{y_g})y) \qquad [\because \tilde{f} \text{ is an soft isometry}] \\ &= \tilde{e}((e_{a_h})a, f(e_{b_h})b) \end{aligned}$$

Thus $\tilde{d}(f^{-1}((e_{a_h})a, f^{-1}(e_{b_h})b) = \tilde{e}((e_{a_h})a, f(e_{b_h})b)\ \forall((e_{a_h})a, f(e_{b_h})b) \in (h, \tilde{e})$. Hence it Follows that f^{-1} is a soft isometry.
From above theorem it follows that if $(h, \tilde{e})$ is an isometric copy of $(g, \tilde{d})$, then $(g, \tilde{d})$ is also An isometric copy of $(h, \tilde{e})$. Thus we can simply say that $(g, \tilde{d})$ is isometric to $(h, \tilde{e})$.

4. Examples of Soft Isometries

Example 4.1. $\mathbb{R}^2$ is soft isometric to $\mathbb{C}$
Let $(\tilde{f}, d)$ be the usual soft metric on $\mathbb{R}^2$ and $(h, \tilde{e})$ be the usual soft metric on $\mathbb{C}$. We shall show that the function $\tilde{f} : \mathbb{R}^2 \to \mathbb{C}$ defined as
$\tilde{f}((e_{a_f})a, f(e_{b_f})b)) = (e_{a_f})a + i(e_{b_f})b\ \ \forall((e_{a_f})a, (e_{b_f})b) \in \mathbb{R}^2$ is an soft isometry.
For any $((e_{a_f})a, (e_{b_f})b))((e_{c_f})c, (e_{d_f})d) \in \mathbb{R}^2$,

$$\begin{aligned}
\tilde{e}(f(e_{a_f})a, (e_{b_f})b), f(e_{c_f})c, (e_{d_f})d) &= |f(e_{a_f})a, (e_{b_f})b) – f((e_{c_f})c, (e_{d_f})d)| \\
&= |(e_{a_f})a + i(e_{b_f})b) - ((e_{c_f})c + i(e_{d_f})d))| \\
&= |(e_{a_f})a) – (e_{c_f})c + i(e_{b_f})b) – (e_{d_f})d)| \\
&= \sqrt{((e_{a_f})a) – (e_{c_f})c)^2 - ((e_{b_f})b) – (e_{d_f})d)^2} \\
&= \tilde{d}((e_{a_f})a, (e_{b_f})b))((e_{c_f})c, (e_{d_f})d)
\end{aligned}$$

Hence it follows that $\mathbb{R}^2$ is soft isometric to $\mathbb{C}$.

Example 4.2. Consider the real line $(\mathcal{R}, \tilde{u})$ with $\tilde{u}$ as usual soft metric and the Euclidean soft space $(\mathcal{R}^3, \tilde{u})$, then the inclusion map $\tilde{\mu}_1 : \mathcal{R} \to \mathcal{R}^3$ given by given by $\tilde{\mu}_1(e_{x_f})x = ((e_{x_f})x, 0, 0)\ \ \forall(e_{x_f})x \in \mathcal{R}$ is an soft isometry. for any $(e_{x_f})x, (e_{y_f})y \in \mathcal{R}$

$$\begin{aligned}
\tilde{d}(\tilde{\mu}_1(e_{x_f})x, \tilde{\mu}_1(e_{x_f})x) &= \sqrt{((e_{x_f})x - (e_{y_f})y))^2 + (\tilde{0} - \tilde{0})^2 + (\tilde{0} - \tilde{0})^2} \\
&= \sqrt{((e_{x_f})x - (e_{y_f})y))^2} \\
&= |(e_{x_f})x - (e_{y_f})y)| \\
&= \tilde{u}((e_{x_f})x, (e_{y_f})y)
\end{aligned}$$

Thus, $\tilde{\mu}_1$ is an soft isometry and $\{((e_{x_f})x), 0, 0) : (e_{x_f})x \in \mathcal{R}\}$ is soft isometry of $\mathcal{R}$ in $\mathcal{R}^3$. Hence under soft isometry, $\mathcal{R}$ can considered as a soft metric subspace of $\mathcal{R}^3$, however as a set $\mathcal{R}$ is not even a subset of $\mathcal{R}^3$.
Similarly, it can be shown that the other inclusion map $\tilde{\mu}_2 : \mathcal{R} \to \mathcal{R}^3$ given by $\tilde{\mu}_2(e_{x_f})x = (0, (e_{y_f})y, 0)\ \ \forall(e_{y_f})y \in \mathcal{R}$ is an soft isometry, Now $\{(0, (e_{y_f})y, 0)$ $\forall(e_{x_f})x \in \mathcal{R}\}$ (y-axis) is isometry copy of $\mathcal{R}$ in $\mathcal{R}^3$.
Also, xy- plane $\{((e_{x_f})x), (e_{y_f})y), 0) : (e_{x_f})x, (e_{y_f})y) \in \mathcal{R}\}$ in $\mathcal{R}^3$ is an soft isometry

copy of $\mathcal{R}^2$ under the isometry $\tilde{\mu} : \mathcal{R}^2 \to \mathcal{R}^3$ given by
$\tilde{\mu}((e_{x_f})x, (e_{y_f})y) = ((e_{x_f})x, (e_{y_f})y, 0) \ \ \forall ((e_{x_f})x, (e_{y_f})y) \in \mathcal{R}^2$.
Obviously, all other lines and planes in $\mathcal{R}^2$ are soft isometric copy of $\mathcal{R}$ and $\mathcal{R}^2$, respectively.

5. Conclusion

In this paper, we introduce the new concept of soft isometries with the help of soft points depend on soft metric spaces and also investigate the prepositions of soft isometries moreover we also introduce the example of soft isometries. We hope that the results investigated in this paper make a significant and very helpful to researcher in the field of soft metric spaces and soft isometric space. Also We believe that this study will help researchers to upgrade and support the further studies on soft isometries, soft metric space and it's useful prepositions to carry out a general framework for their applications in real life.

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