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COINCIDENCE AND COMMON FIXED-POINT THEOREM USING COMPATIBLE MAPPING OF TYPE (P) ON INTUITIONISTIC FUZZY b-METRIC SPACES

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Abstract: In this paper we have defined compatible type (P) mapping in the structure of intuitionistic fuzzy b-metric space and have proved a coincidence point theorem in intuitionistic fuzzy b-metric space.

Keywords and Phrases: Fuzzy b-metric space, Intuitionistic fuzzy b-metric space, Compatible mapping, Compatible type (P) mapping.

2020 Mathematics Subject Classification: 47H10, 54H25.

1. Introduction

The thought of b-metric space was introduced by Bakhtin [2] in 1989. The class of b-metric spaces is larger than that of metric spaces. In 2016, Nadaban [7] introduced the concept of fuzzy b-metric space and approved that the study in fuzzy b-metric spaces will obtain a lot of applications of as well as in mathematical engineering than in computer science. With the idea of intuitionistic fuzzy sets, Park [8] in 2004 defined the concept of intuitionistic fuzzy metric spaces with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space. In 2020, Konwar [5] extended fixed point results and studied the existence of uniqueness of self-mapping on the intuitionistic fuzzy b-metric space. In 2022, Azam and Kanwal [1] have established some conventional fixed-point theorem in the setting of complete intuitionistic fuzzy b-metric spaces. On the other hand, in 2014, Tripathi et al. [9] defined compatible type (P) mapping in fuzzy metric space. In this paper we have extend Azam and Kanwal [1] fixed-point theorems in the setting of compatible mapping of type (P) in intuitionistic fuzzy b-metric spaces with other contraction.

2. Preliminaries

For the reader convenience some definitions and results are recalled. The perception of b-metric space was announced by Bakhtin [2] and extensively used by Czerwik [3].

Definition 2.1. [8] *A binary operation* $* : [0,1]\times[0,1] \to [0,1]$ *is called continuous triangular norm (t-norm) if it satisfies the following conditions:*
(1) $*$ *is associative and commutative;*
(2) $*$ *is continuous;*
(3) $a * 1 = a, \forall a \in [0,1]$;
(4) if $a \le c$ *and* $b \le d$ *with* $a, b, c, d \in [0,1]$, *then* $a * b \le c * d$.

Example 2.1.1. [6] Three basic t-norms are defined as follows:
(1) The minimum t-norm, $a *_1 b = \min(a, b)$,
(2) The product t-norm, $a *_2 b = a.b$,
(3) The Lukasiewicz t-norm $a *_3 b = \max(a + b - 1, 0)$.

Definition 2.2. [8] *A binary operation* $\Diamond : [0,1] \times [0,1] \to [0,1]$ *is called a continuous triangular conorm (t-conorm) if it satisfies the following conditions:*
(1) $\Diamond$ *is associative and commutative;*
(2) $\Diamond$ *is continuous;*
(3) $a\Diamond 0 = a, \forall a \in [0,1]$;
(4) $a\Diamond b \le c\Diamond d$, *whenever* $a \le c$ *and* $b \le d$ $\forall a, b, c, d \in [0,1]$.

Example 2.2.1. [6] Three basic t-conorms are given below:
(1) $a\Diamond_1 b = \min(a + b, 1)$,
(2) $a\Diamond_2 b = (a + b - ab)$,
(3) $a\Diamond_3 b = \max(a, b)$.

Definition 2.3. [1] *A 6-tuple* $(X, M, N, *, \Diamond, s)$ *is said to be an intuitionistic fuzzy b-metric space (IFb-MS), if* X *is an arbitrary set,* $s \ge 1$ *is a given real number,* $*$ *is a continuous* t*-norm,* $\Diamond$ *is a continuous* t*-conorm.* M *and* N *are fuzzy sets on* $X^2 \times [0,\infty)$ *satisfying the following conditions: for all* $x, y, z \in X$,

(a) $M(x,y,t) + N(x,y,t) \leq 1$;
(b) $M(x,y,0) = 0$;
(c) $M(x,y,t) = 1, \forall t > 0$ *iff* $x{=}y$;
(d) $M(x,y,t) = M(y,x,t) \forall t > 0$;
(e) $M(x,z,s(t+u)) \geq M(x,y,t) * M(y,z,u), \forall t,u > 0$;
(f) $M(x,y) : [0,\infty) \to [0,1]$ *is left continuous and* $\lim_{t\to\infty} M(x,y,t) = 1$,
(g) $N(x,y,0) = 1$;
(h) $N(x,y,t) = 0 \forall t > 0$ *iff* $x = y$;
(i) $N(x,y,t) = N(y,x,t) \forall t > 0$;
(j) $N(x,z,s(t+u)) \leq N(x,y,t) \Diamond N(y,z,u), \forall t,u > 0$;
(k) $N(x,y,\centerdot) : [0,\infty) \to [0,1]$ *is right continuous and* $\lim_{t\to\infty} N(x,y,t) = 0$.
Here, $M(x,y,t)$ *and* $N(x,y,t)$ *represent the nearness degree and the non-nearness degree with respect to* t *between* x *and* y *respectively.*

Definition 2.4. [1] *Let* $s \geq 1$ *be a given real number. A function* $f : R \to R$ *will be called s-nondecreasing if* $t < u$ *implies that* $f(t) \leq f(su)$ *and* f *is called s-nonincreasing if* $t < u$ *implies that* $f(t) \geq f(su)$.

Proposition 2.5. [1] *Let* $(X, M, N, *, \Diamond, s)$ *is an intuitionistic fuzzy b-metric space, then for all* $x, y \in X$*, the fuzzy set* M *and* N *are defined with respect to product such that* $M(x,y,\centerdot) : [0,\infty) \to [0,1]$ *is s-nondecreasing and* $N(x,y,\centerdot) : [0,\infty) \to [0,1]$ *is s-nonincreasing.*

Definition 2.6. [1] *Let* $(X, M, N, *, \Diamond, s)$ *be an intuitionistic fuzzy b-metric space.*

(a) *A sequence* $\{x_n\}$ *in* X *is said to be convergent if there exists* $x \in X$ *such that* $\lim_{n\to\infty} M(x_n,x,t) = 1$ *and* $\lim_{n\to\infty} N(x_n,x,t) = 0$ $\forall t > 0$. *In this case* x *is called the limit of the sequence* $\{x_n\}$ *and we write* $\lim_{n\to\infty} x_n = x$, *or* $x_n \to x$.

(b) *A sequence* $\{x_n\}$ *in* $(X, M, *, \Diamond, s)$ *is said to be a Cauchy sequence if for every* $\epsilon \in (0,1)$, *there exists* $n_0 \in N$ *such that* $M(x_n,x_m,t) > 1-\epsilon$ *and* $N(x_n,x_m,t) < \epsilon, \forall m,n \geq n_0$ *and* $t > 0$.

(c) *The space* X *is said to be complete if every Cauchy sequence is convergent and it is called compact if every sequence has a convergent subsequence.*

The following result of Shazia Kanwal [4] gives common fixed point of Π and σ with the assumption of weakly compatibility:

Theorem 2.7. [4] *Let* $(\zeta, \Phi, \varphi, \bigodot, *, s)$ *be a compete IFb-MS and* $\Pi, \sigma : \zeta \to \zeta$ *be mappings satisfying the following conditions:*
(1) $\sigma(\zeta) \subseteq \Pi(\zeta)$,

(2) Π *and* σ *are weakly compatible.*
(3) The is $k, 0 \leq k < 1$, *such that, for all* $\omega, \nu \in \zeta$,
$\Phi(\sigma(\omega), \sigma(\nu), kt) \geq \Phi(\Pi(\omega), \Pi(\nu), t)$ *and* $\varphi(\sigma(\omega), \sigma(\nu), kt) \geq \varphi(\Pi(\omega), \Pi(\nu), t)$.
Then, Π *and* σ *have a unique common fixed point in* ζ.

3. Main Result

We define compatible and compatible type-P mappings in intuitionistic fuzzy b-metric spaces.

Definition 3.1. *Two self-mappings* A *and* S *of an intuitionistic fuzzy b-metric space* $(X, M, N, *, \Diamond)$ *are called compatible if* $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ *and* $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$ *whenever* $\{x_n\}$ *is a sequence in* X *such that* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ *for some* $x \in X$.

Definition 3.2. *Two self-mappings* A *and* S *of an intuitionistic fuzzy b-metric space* $(X, M, N, *, \Diamond)$ *are called compatible of type (P) if* $\lim_{n\to\infty} M(AAx_n, SSx_n, t) = 1$ *and* $\lim_{n\to\infty} N(AAx_n, SSx_n, t) = 0$ *whenever* $\{x_n\}$ *is a sequence in* X *such that* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ *for some* $x \in X$.

Example 3.2.1. Let $X = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$ with $*$ continuous t-norm and $\Diamond$ continuous t-conorm defined by $a * b = ab$ and $a \Diamond b = \min\{1, a+b\}$ respectively, for $a, b \in [0,1]$. For each $t \in [0, \infty)$ and $x, y \in X$, define (M, N) by

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|^2}, & \text{if} \quad t > 0, \\ 0 & \text{if} \quad t = 0, \end{cases} \qquad \text{and} \quad N(x,y,t) = \begin{cases} \frac{|x-y|^2}{t+|x-y|^2}, & \text{if} \quad t > 0, \\ 1 & \text{if} \quad t = 0, \end{cases}$$

Clearly $(X, M, N, *, \Diamond)$ is an intuitionistic fuzzy metric space.
Define $Sx = \frac{x}{6}$ and $Tx = \frac{x}{2}$ on X and $x_n = \frac{1}{n}$.
Clearly, it can be easily observed that S and T are compatible type (P) mapping.
Our main result is to extend Theorem 2.7 of Kanwal et.al, using other contractive mapping in intuitionistic fuzzy b-metric space with compatible type-(P) mapping.

Theorem 3.3. *Let* $(X, M, N, *, \Diamond, s)$ *be a complete intuitionistic fuzzy b-metric space with* $*$ *t-norm and* $\Diamond$ *t-conorm defined as:*

(I) $a * b = \min\{a, b\}, a \Diamond b = \max\{a, b\}$,

(II) $M(x, y, .)$ *and* $N(x, y, .)$ *are strictly increasing and strictly deceasing functions respectively.*

Let $S, T : X \to X$ *be two self-mapping on* X *satisfy following conditions:*
(i) $T(X) \subseteq S(X)$,

(ii) One of S or T is continuous,
(iii) (S, T) is compatible of type (P)
(iv) If for all $x, y \in X, k \in \left(0, \frac{1}{2s}\right)$, $t > 0$,

$$M(Tx, Ty, kt) \geq \min\{M(Tx, Sy, t), M(Ty, Sy, t), M(Ty, Sx, t)\},$$

$$N(Tx, Ty, kt) \leq \max\{N(Tx, Sy, t), N(Ty, Sy, t), N(Ty, Sx, t)\}.$$

Then x is common fixed point of S and T.

Proof. Let $x_0 \in X$. Since $T(X) \subseteq S(X)$ there exist x_{2n+1} and x_{2n} in X such that

$$Tx_{2n} = Sx_{2n+1} = y_{2n+1} \qquad \text{for,} \quad n = 1, 2, 3, ... \tag{3.3.1}$$

Case I. Putting $x = x_{2n}$ and $y = x_{2n+1}$ in (iv) we get

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, kt) &= M(Sx_{2n+1}, Sx_{2n+2}, kt) = M(Tx_{2n}, Tx_{2n+1}, kt) \\ &\geq \min\{M(Tx_{2n}, Sx_{2n+1}, t), M(Tx_{2n+1}, Sx_{2n+1}, t), M(Tx_{2n+1}, Sx_{2n}, t)\}, \\ &= \min\{M(Sx_{2n+1}, Sx_{2n+1}, t), M(Sx_{2n+2}, Sx_{2n+1}, t), M(Sx_{2n+2}, Sx_{2n}, t)\}, \\ &= \min\{M(y_{2n+1}, y_{2n+1}, t), M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+2}, y_{2n}, t)\}, \quad \text{(By 3.3.1)} \end{aligned}$$

Since $M(y_{2n+1}, y_{2n+1}, t) = 1$.

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, kt) &\geq \min\{(1, M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+2}, y_{2n}, t))\}, \\ &\geq \min\{(M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+2}, y_{2n}, t))\}, \end{aligned}$$

Since $kt < \frac{t}{2s}$ and by (II) of theorem (3.3), $M(x, y, .)$ is a strictly increasing function.
If $\min\{(M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+2}, y_{2n}, t))\} = M(y_{2n+2}, y_{2n+1}, t)$
Then we will reach to a contradiction $M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n+2}, y_{2n+1}, t)$.
Therefore,

$$\begin{aligned} &M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n+2}, y_{2n}, t) \\ &\geq M\left(y_{2n+2}, y_{2n+1}, \frac{t}{2s}\right) * M\left(y_{2n+1}, y_{2n}, \frac{t}{2s}\right) \quad \text{(By using (e) of definition 2.3)} \\ &= \min\left\{M\left(y_{2n+2}, y_{2n+1}, \frac{t}{2s}\right), M\left(y_{2n+1}, y_{2n}, \frac{t}{2s}\right)\right\} \quad \text{(By (I) of theorem 3.3)} \end{aligned}$$

Since $kt < \frac{t}{2s}$ and by (II) of theorem 3.3. $M(x, y, .)$ is a strictly increasing function.
If $\min\left\{M\left(y_{2n+2}, y_{2n+1}, \frac{t}{2s}\right), M\left(y_{2n+1}, y_{2n}, \frac{t}{2s}\right)\right\} = M\left(y_{2n+2}, y_{2n+1}, \frac{t}{2s}\right)$,
then we will again reach to contradiction, $M(y_{2n+1}, y_{2n+2}, kt) \geq M\left(y_{2n+1}, y_{2n+2}, \frac{t}{2s}\right)$.

which is not possible. Therefore, $M(y_{2n+2}, y_{2n+1}, kt) \geq M\left(y_{2n+1}, y_{2n}, \frac{t}{2s}\right)$
In the similar manner, $M(y_{2n+3}, y_{2n+2}, kt) \geq M\left(y_{2n+2}, y_{2n+1}, \frac{t}{2s}\right)$.
In general, $M(y_{n+1}, y_{n+2}, kt) \geq M\left(y_n, y_{n+1}, \frac{t}{2s}\right)$ for $n = 1, 2, 3, ...$
And, $M(y_{n+2}, y_{n+3}, kt) \geq M\left(y_{n+1}, y_{n+2}, \frac{t}{2s}\right)$ for $n = 1, 2, 3, ...$
Also, it follows that, $M(y_{n+1}, y_{n+2}, kt) \geq M\left(y_n, y_{n+1}, \frac{t}{2s}\right) \geq M\left(y_{n-1}, y_n, \frac{t}{(2s)^2 k}\right)$.
Continuing this, we get, $M(y_{n+1}, y_{n+2}, kt) \geq M\left(y_0, y_1, \frac{t}{(2s)^{n+1}k^n}\right) \to 0$ as $n \to \infty$.
Thus, in general, when $n \to \infty$, Clearly,
$1 \geq M(y_n, y_{n+1}, kt) \geq M\left(y_0, y_1, \frac{t}{(2s)^n k^{n-1}}\right) \to 1$
Thus, $\lim_{n\to\infty} M(y_n, y_{n+1}, kt) = 1$.
Furthermore,

$$
\begin{aligned}
N(y_{2n+1}, y_{2n+2}, kt) = N(Sx_{2n+1}, Sx_{2n+2}, kt) &= N(Tx_{2n}, Tx_{2n+1}, kt) \\
&\leq \max\{(N(Tx_{2n}, Sx_{2n+1}, t), N(Tx_{2n+1}, Sx_{2n+1}, t), N(Tx_{2n+1}, Sx_{2n}, t))\}, \\
&= \max\{(N(Sx_{2n+1}, Sx_{2n+1}, t), N(Sx_{2n+2}, Sx_{2n+1}, t), N(Sx_{2n+2}, Sx_{2n}, t)\}, \\
&= \max\{(N(y_{2n+1}, y_{2n+1}, t), N(y_{2n+2}, y_{2n+1}, t), N(y_{2n+2}, y_{2n}, t))\}, \\
&\Rightarrow N(y_{2n+1}, y_{2n+2}, kt) \leq \max\{N(y_{2n+2}, y_{2n+1}, t), N(y_{2n+2}, y_{2n}, t)\}, \\
&[\text{Since } N(y_{2n+1}, y_{2n+1}, t) = 0]
\end{aligned}
$$

Since $kt < \frac{t}{2s}$ and by (II) of theorem (3.3) $N(x, y, .)$ is a strictly decreasing function.
If $\max\{(N(y_{2n+2}, y_{2n+1}, t), N(y_{2n+2}, y_{2n}, t))\} = N(y_{2n+2}, y_{2n+1}, t)$
Then we reach to a contradiction, $N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n+2}, y_{2n+1}, t)$ is not possible.
Therefore,

$$
\begin{aligned}
&N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n+2}, y_{2n}, t) \\
&\leq N\left(y_{2n+2}, y_{2n+1}, \frac{t}{2s}\right) \Diamond N\left(y_{2n+1}, y_{2n}, \frac{t}{2s}\right) \quad \text{(By using (j) of definition 2.3)} \\
&= \max\left\{N\left(y_{2n+2}, y_{2n+1}, \frac{t}{2s}\right), N\left(y_{2n+1}, y_{2n}, \frac{t}{2s}\right)\right\} \quad \text{(By (I) of theorem 3.3)}
\end{aligned}
$$

Since $kt < \frac{t}{2s}$ and by (II) of theorem (3.3)$N(x, y, .)$ is a strictly decreasing function
If, $\max\left\{N\left(y_{2n+2}, y_{2n+1}, \frac{t}{2s}\right), N\left(y_{2n+1}, y_{2n}, \frac{t}{2s}\right)\right\} = N\left(y_{2n+1}, y_{2n+2}, \frac{t}{2s}\right)$
Then we reach to a contradiction, $N(y_{2n+1}, y_{2n+2}, kt) \leq N\left(y_{2n+1}, y_{2n+2}, \frac{t}{2s}\right)$ which is not possible.
Therefore, $N(y_{2n+1}, y_{2n+2}, kt) \leq N\left(y_{2n+1}, y_{2n}, \frac{t}{2s}\right)$

By similar pattern $N(y_{2n+3}, y_{2n+2}, kt) \leq N\left(y_{2n+2}, y_{2n+1}, \frac{t}{2s}\right)$
Thus, we have $N(y_{2n+1}, y_{2n+2}, kt) \leq N\left(y_{2n}, y_{2n+1}, \frac{t}{2s}\right)$
And $N(y_{2n+2}, y_{2n+3}, kt) \leq N\left(y_{2n+1}, y_{2n+2}, \frac{t}{2s}\right)$
In general, $N(y_{n+1}, y_{n+2}, kt) \leq N\left(y_n, y_{n+1}, \frac{t}{2s}\right)$ for $n = 1, 2, 3, ...$
And $N(y_{n+2}, y_{n+3}, kt) \leq N\left(y_{n+1}, y_{n+2}, \frac{t}{2s}\right)$ for $n = 1, 2, 3, ...$
Also, it follows that, $N(y_{n+1}, y_{n+2}, kt) \leq N\left(y_n, y_{n+1}, \frac{t}{2s}\right) \leq N\left(y_{n-1}, y_n, \frac{t}{(2s)^2 k}\right)$
Continuing this, we have, $N(y_{n+1}, y_{n+2}, kt) \leq N\left(y_0, y_1, \frac{t}{(2s)^{n+1} k^n}\right) \to 0$ as $n \to \infty$,
Thus, in general, when $n \to \infty$, $0 \leq N(y_n, y_{n+1}, kt) \leq N\left(y_0, y_1, \frac{t}{(2s)^n k^{n-1}}\right) \to 0$
Therefore, $\lim_{n\to\infty} N(y_n, y_{n+1}, kt) = 0$
Hence, $M(y_n, y_{n+1}, kt) \to 1$ and $N(y_n, y_{n+1}, kt) \to 0$ as $n \to \infty$ for any $t > 0$,
Next, we show that the sequence $\{y_n\}$ is a Cauchy sequence.
For each $\varepsilon > 0$ and $t > 0$, we may be chosen $n_0 \in N$ such that
$M(y_n, y_{n+1}t) > 1 - \varepsilon$ for all $n > n_0$ and $N(y_n, y_{n+1}t) < \varepsilon$ for all $n > n_0$
For $m, n \in N$, we suppose $m \geq n$. Then we have

$$\begin{aligned}
M(y_n, y_m, t) &\geq M\left(y_n, y_{n+1}, \frac{t}{2s}\right) * M\left(y_{n+1}, y_m, \frac{t}{2s}\right) \\
&\geq M\left(y_n, y_{n+1}, \frac{t}{2s}\right) * M\left(y_{n+1}, y_{n+2}, \frac{t}{(2s)^2}\right) * M\left(y_{n+2}, y_m, \frac{t}{(2s)^3}\right) \\
&\geq M\left(y_n, y_{n+1}, \frac{t}{2s}\right) * M\left(y_{n+1}, y_{n+2}, \frac{t}{(2s)^2}\right) * M\left(y_{n+2}, y_m, \frac{t}{(2s)^3}\right) ... \\
&\Rightarrow M(y_n, y_m, t) \geq (1-\varepsilon) * (1-\varepsilon) * (1-\varepsilon) ... (1-\varepsilon)
\end{aligned}$$

$= \min\{(1-\varepsilon), (1-\varepsilon), (1-\varepsilon), ...(1-\varepsilon)\} = (1-\varepsilon)$ (by (I) of Theorem 3.3)

And

$$\begin{aligned}
N(y_n, y_m, t) &\leq N\left(y_n, y_{n+1}, \frac{t}{2s}\right) \Diamond N\left(y_{n+1}, y_m, \frac{t}{2s}\right) \\
&\leq N\left(y_n, y_{n+1}, \frac{t}{2s}\right) \Diamond N\left(y_{n+1}, y_{n+2}, \frac{t}{(2s)^2}\right) \Diamond N\left(y_{n+2}, y_m, \frac{t}{(2s)^3}\right) \\
&\leq N\left(y_n, y_{n+1}, \frac{t}{2s}\right) \Diamond N\left(y_{n+1}, y_{n+2}, \frac{t}{(2s)^2}\right) \Diamond N\left(y_{n+2}, y_m, \frac{t}{(2s)^3}\right) ... \\
&\leq \varepsilon \Diamond \varepsilon \Diamond \varepsilon ... \Diamond \varepsilon = \max\{\varepsilon, \varepsilon, \varepsilon, ..., \varepsilon\} = \varepsilon \quad \text{(by (I) of Theorem 3.3)}
\end{aligned}$$

Hence, $\{y_n\}$ is a Cauchy sequence in X.
Since $(X, M, N, *, \Diamond)$ is complete. In view of completeness of the space, sequence

$\{y_n\}$ converges to some point $u \in X$. Also its subsequence converges to the same point i.e., $Sx_{2n} = Tx_{2n} \to u$. Now, we shall prove $Su = u$ then
$M(u, Su, kt) \geq M\left(u, Tx_{2n}, \frac{kt}{2s}\right) * M\left(Tx_{2n}, Su, \frac{kt}{2s}\right)$, S is continuous and S,T are compatible type P such that $n \to \infty$. $TTx_{2n} \to Su, SSx_{2n} \to Su$,

$$\begin{aligned}M(u,Su,kt) &\geq M\left(u, Tx_{2n}, \frac{kt}{2s}\right) * M\left(Tx_{2n}, TTx_{2n}, \frac{kt}{2s}\right),\\ &\geq M\left(u, Tx_{2n}, \frac{kt}{2s}\right) * \min\left\{M\left(Tx_{2n}, STx_{2n}, \frac{t}{2s}\right), M\left(TTx_{2n}, STx_{2n}, \frac{t}{2s}\right),\right.\\ &\left. M\left(TTx_{2n}, Sx_{2n}, \frac{t}{2s}\right)\right\} \quad \text{(by(iv) of Theorem 3.3)}\end{aligned}$$

Since $Sx_{2n} = Tx_{2n} \to u$ and S and T are compatible type (P) Mapping.
Therefore, as $n \to \infty$, we get, $TTx_{2n} \to Su, SSx_{2n} \to Su$.

$$\begin{aligned}&\leq M\left(u, u, \frac{kt}{2s}\right) * \min\left\{M\left(u, Su, \frac{t}{2s}\right), M\left(Su, Su, \frac{t}{2s}\right), M\left(Su, u, \frac{t}{2s}\right)\right\}\\ &\leq M\left(u, u, \frac{kt}{2s}\right) * \min\left\{M\left(u, Su, \frac{t}{2s}\right), M\left(Su, Su, \frac{t}{2s}\right), M\left(u, Su, \frac{t}{2s}\right)\right\}\\ &\Rightarrow M(u, Su, kt) \geq M\left(u, Su, \frac{t}{2s}\right)\\ &\text{(Since, } M\left(u, u, \frac{kt}{2s}\right) = 1 \text{ and } M\left(Su, Su, \frac{t}{2s}\right) = 1 \text{ for all } t > 0)\end{aligned}$$

Therefore, $Su = u$. Now we will show that $Tu = u$.
For that let $x = u$ and $y = Tx_{2n}$ then, (iv) of Theorem (3.3) becomes
$M(Tu, TTx_{2n}, kt) \geq \min\{M(Tu, STx_{2n}, t), M(TTx_{2n}, STx_{2n}, t), M(TTx_{2n}, Su, t)\}$
Since $Sx_{2n} = Tx_{2n} \to u$, S is continuous and S, T are compatible of type P such that

$$\begin{aligned}&TTx_{2n} = SSx_{2n} = Su = u\\ &\quad M(Tu, u, kt) \geq \min\{M(Tu, Su, t), M(u, Su, t), M(u, Su, t)\}\\ &\quad M(Tu, u, kt) \geq \min\{M(Tu, u, t), M(u, u, t), M(u, u, t)\},\end{aligned}$$

Since, $M(u, u, t) = 1$ for all $t > 0$. Therefore, $M(Tu, u, kt) \geq M(Tu, u, t)$
Thus, $Tu = u$. Hence, u is a fixed point of S and T.
Now, we prove $Su = u$ for N, $N(u, Su, kt) \leq N\left(u, Tx_{2n}, \frac{kt}{2s}\right) \Diamond N\left(Tx_{2n}, Su, \frac{kt}{2s}\right)$,

S is continuous and S, T are compatible type P such that $n \to \infty$.

$$TTx_{2n} \to Su, SSx_{2n} \to Su,$$

$$N(u,Su,kt) \leq N\left(u, Tx_{2n}, \frac{kt}{2s}\right) \Diamond N\left(Tx_{2n}, TTx_{2n}, \frac{kt}{2s}\right),$$
$$\leq N\left(u, Tx_{2n}, \frac{kt}{2s}\right) \Diamond \max\left\{N\left(Tx_{2n}, STx_{2n}, \frac{t}{2s}\right), N\left(TTx_{2n}, STx_{2n}, \frac{t}{2s}\right), N\left(TTx_{2n}, Sx_{2n}, \frac{t}{2s}\right)\right\}$$

Since $Sx_{2n} = Tx_{2n} \to u$ and S and T are compatible type (P) mapping.
Therefore, as $n \to \infty$, we get, $TTx_{2n} \to Su, SSx_{2n} \to Su$.

$$\leq N\left(u, u, \frac{kt}{2s}\right) \Diamond \max\left\{N\left(u, Su, \frac{t}{2s}\right), N\left(Su, Su, \frac{t}{2s}\right), N\left(Su, u, \frac{t}{2s}\right)\right\}$$
$$\leq N\left(u, u, \frac{kt}{2s}\right) \Diamond \max\left\{N\left(u, Su, \frac{t}{2s}\right), N\left(Su, Su, \frac{t}{2s}\right), N\left(u, Su, \frac{t}{2s}\right)\right\}$$
$$\Rightarrow N(u, Su, kt) \leq N\left(u, Su, \frac{t}{2s}\right)$$
$$\text{(Since, } N\left(u, u, \frac{kt}{2s}\right) = 0 \text{ and } N\left(Su, Su, \frac{t}{2s}\right) = 0 \text{ for all } t > 0)$$

Therefore, $Su = u$. Now we will show that $Tu = u$.
For that let $x = u$ and $y = Tx_{2n}$ then, (iv) of Theorem (3.3) becomes
$N(Tu, TTx_{2n}, kt) \leq \max\{N(Tu, STx_{2n}, t), N(TTx_{2n}, STx_{2n}, t), N(TTx_{2n}, Su, t)\}$
Since $Sx_{2n} = Tx_{2n} \to u$, S is continuous and S, T are compatible of type P such that

$$\begin{aligned} TTx_{2n} = SSx_{2n} &= Su = u \\ N(Tu, u, kt) &\leq \max\{N(Tu, Su, t), N(u, Su, t), N(u, Su, t)\} \\ &\leq \max\{N(Tu, u, t), N(u, u, t), N(u, u, t)\}, \end{aligned}$$

(Since, $N(u, u, t) = 0$ for all $t > 0$).
$\Rightarrow N(Tu, u, kt) \geq N(Tu, u, t)$
Thus, $Tu = u$. Hence, u is a fixed point of S and T.
Uniqueness. Let u' be another common fixed point of S and T.
Then $Su' = Tu' = u'$. we get
$M(Tu, Tu', kt) \geq \min\{M(Tu, Su', t), M(Tu', Su', t), M(Tu', Su, t)\}$,

$M(u,u',kt) \geq \min\{M(u,u',t), M(u',u',t), M(u',u,t)\}$,
(Since $M(u',u',t) = 1$ for all $t > 0$)
Therefore,
$M(u,u',kt) \geq M(u,u',t) \geq M\left(u,u',\frac{t}{k}\right) \geq M\left(u,u',\frac{t}{k^2}\right) ... \geq M\left(u,u',\frac{t}{k^{n-1}}\right) \to 1$,
as $n \to \infty$.
And, $N(Tu,Tu',kt) \leq \max\{N(Tu,Su',t), N(Tu',Su',t), N(Tu',Su,t)\}$,
$N(u,u',kt) \leq \max\{N(u,u',t), N(u',u',t), N(u',u,t)\}$,
(Since $N(u',u',t) = 0$ for all $t > 0$)
Therefore,
$N(u,u',kt) \leq N(u,u',t) \leq N\left(u,u',\frac{t}{k}\right) \leq N\left(u,u',\frac{t}{k^2}\right) ... \leq N\left(u,u',\frac{t}{k^{n-1}}\right) \to 0$,
as $n \to \infty$. By (c) and (h) of definition 2.3, we get $u = u'$. Therefore, u is the common fixed point of self-mappings S and T.

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