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NEUTROSOPHIC Λ_P-HOMEOMORPHISM IN NEUTROSOPHIC TOPOLOGICAL SPACES

C. Reena and M. Karthika

Department of Mathematics,
St. Mary's College (Autonomous),
Thoothukudi - 628001, Tamil Nadu, INDIA

E-mail : reenastephany@gmail.com, karthikamarimuthu97@gmail.com

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Abstract: In this article, we have defined neutrosophicΛ_P-open, neutrosophicΛ_P-closed mappings and neutrosophicΛ_P-homeomorphism in neutrosophic topological spaces.Finally, we have extended our study to neutrosophicΛ_P-$i$homeomorphism which is a stronger form of neutrosophic Λ_P-homeomorphism.

Keywords and Phrases: Neutrosophic Λ_P-open, neutrosophic Λ_P-continuous, neutrosophic Λ_P-open map, neutrosophic Λ_P-closed map, neutrosophic Λ_P- homeomorphism, neutrosophic Λ_P-i homeomorphism.

2020 Mathematics Subject Classification: 54C10.

1. Introduction

Since its introduction by Zadeh [12], fuzzy sets have been prevalent in nearly every field of mathematics. Florentine Smarandache [10] created the concept of neutrosophy and neutorsophic sets at the beginning of 20^{th} century. Later Salama [8] and Alblowi initiated the neutrosophic sets in a topology entitled as neutrosophic topological space.Recently, the authors [6] of this paper defined a new notion of neutrosophic sets namely neutrosophic Λ_P-open and neutrosophic Λ_P-closed sets. Also,we have studied about novel concept of neutrosophic Λ_P-neighbourhood with quasi coincident. Also extended the neutrosophic continuous functions to neutrosophic Λ_P-continuous [7] and neutrosophic Λ_P-irresolute functions in neutrosophic topological space. The topological isomorphism commonly called homeomorphism plays a vital role in the properties of topological spaces. Parimala et.al [4] introduced the concept of neutrosophic homeomorphism and neutrosophic $\alpha\psi-$homeomorphism in neutrosophic topological spaces. This paper aspires to overly erunicate the thought of neutrosophic Λ_P-homeomorphism which is extended to neutrosophic Λ_P-$i$homeomorphism in neutrosophic topological spaces. Initially the article gives brief explanation on neutrosophic Λ_P-open and neutrosophic Λ_P-closed mapping.

2. Preliminaries

Definition 2.1. [8] *Let U be a non-empty fixed set. A **Neutrosophic set** K is an object having the form $K = \{\langle u, \ \mu_K(u), \ \sigma_K(u), \gamma_K(u)\rangle : u \in U\}$ where $\mu_K(u)$, $\sigma_K(u)$ and $\gamma_K(u)$ represents the degree of membership, the degree of indeterminacy and the degree of non-membership respectively of each element $u \in U$ to the set$U$. A neutrosophic set $K = \{\langle u, \mu_K(u), \ \sigma_K(u), \gamma_K(u)\rangle : u \in U\}$ can be identified to an ordered triple $\langle \mu_K(u), \ \sigma_K(u), \gamma_K(u)\rangle$ in on U.*

Definition 2.2. [8] *Let U be a non-empty set and $K = \{\langle u, \ \mu_K(u), \ \sigma_K(u), \gamma_K(u)\rangle : u \in U\}$ and $M = \{\langle u, \ \mu_M(u), \ \sigma_M(u), \gamma_M(u)\rangle : u \in U\}$ are neutrosophic sets, then*

i. $K \subseteq M \Leftrightarrow \ \mu_K(u) \leq \mu_M(u), \ \sigma_K(u) \leq \sigma_M(u)$ *and* $\gamma_K(u) \geq \gamma_M(u) \forall u \in U$

ii. $$K \bigcup M = \{\langle u, \ \max(\mu_K(u), \mu_M(u)), \ \max(\sigma_K(u), \sigma_M(u)), \\ \min(\gamma_K(u), \gamma_M(u)) : u \in U\rangle\}$$

iii. $$K \bigcap M = \{\langle u, \ \min(\mu_K(u), \mu_M(u)), \ \min(\sigma_K(u), \sigma_M(u)), \\ max(\gamma_K(u), \gamma_M(u)) : u \in U\rangle\}$$

iv. $K^C = \{\langle u, \ (\gamma_K(u), \ 1 - \sigma_K(u), \ \mu_K(u))\rangle : u \in U\}$

v. $0_{N_{tr}} = \{\langle u, \ 0, \ 0, \ 1\rangle : u \in U\}$ *and* $1_{N_{tr}} = \{\langle u, \ 1, \ 1, \ 0\rangle : u \in U\}$

Definition 2.3. [8] *A **Neutrosophic topology** on a non-empty set U is a family $\tau_{N_{tr}}$ of neutrosophic sets in U satisfying the following axioms:*

i. $0_{N_{tr}}, \ 1_{N_{tr}} \in \tau_{N_{tr}}$.

ii. $K_1 \bigcap K_2 \in \tau_{N_{tr}}$ *for any* $K_1, K_2 \in \tau_{N_{tr}}$.

iii. $\bigcup K_i \in \tau_{N_{tr}}$ *for every* $\{K_i : i \in I\} \subseteq \tau_{N_{tr}}$.

In this case the ordered pair $(U, \tau_{N_{tr}})$ is called a neutrosophic topological space. The members of $\tau_{N_{tr}}$ are neutrosophic open set and its complements are neutrosophic closed.

Definition 2.4. [3] *Let $(U, \tau_{N_{tr}})$ be a neutrosophic topological space and S be a non-empty subset of U. Then, a neutrosophic relative topology on S is defined by*

$$\tau^{S}_{N_{tr}} = \{K \cap 1^{S}_{N_{tr}} \ : K \in \tau_{N_{tr}}\}$$

where

$$1^{S}_{N_{tr}} = \begin{cases} <1,1,0>, & \text{if } s \in S \\ <0,0,1>, & \text{otherwise} \end{cases}$$

*Thus, $(S, \tau^{S}_{N_{tr}})$ is called a **neutrosophic subspace** of $(U, \tau_{N_{tr}})$.*

Definition 2.5. [9] *$(U, \tau_{N_{tr}})$ and $(V, \rho_{N_{tr}})$ be neutrosophic topological spaces. Then the function $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be **neutrosophicopen** if $f_{N_{tr}}(K)$ is N_{tr}-open in $(V, \rho_{N_{tr}})$ for every N_{tr}-open set K in $(U, \tau_{N_{tr}})$.*

Definition 2.6. [4] *A bijective function $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be a **neutrosophic homeomorphism** if $f_{N_{tr}}$ and $f^{-1}_{N_{tr}}$ are N_{tr}-continuous.*

Definition 2.7. [6] *A neutrosophic set K of a neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be **neutrosophic$\Lambda_{\mathbf{P}}$-open** if there exist a neutrosophicpre-open set $E \neq 0_{N_{tr}}, 1_{N_{tr}}$ such that $K \subseteq N_{tr}cl(K \bigcap E)$. The complement of neutrosophic Λ_P-open set is neutrosophic Λ_P-closed. The class of neutrosophic Λ_P-open sets is denoted by $N_{tr}\Lambda_P O(U, \tau_{N_{tr}})$.*

Theorem 2.8. [6] *A neutrosophic set K in a neutrosophic topological space $(U, \tau_{N_{tr}})$ is $N_{tr}\Lambda_P$-open if and only if for every neutrosophic point $u_{a,b,c} \in K$, there exists a $N_{tr}\Lambda_P$-open set $M_{u_{a,b,c}}$ such that $u_{a,b,c} \in M_{u_{a,b,c}} \subseteq K$.*

Theorem 2.9. [6] *Every N_{tr}-open set is $N_{tr}\Lambda_P$-open.*

Definition 2.10. [7] *Let $u_{a,b,c}$ be a neutrosophic point in a neutrosophic topological space $(U, \tau_{N_{tr}})$. Then a neutrosophic set N in U is said to be **neutrosophic$\Lambda_{\mathbf{P}}$-neighbourhood**($N_{tr}\Lambda_P$-nbhd) of $u_{a,b,c}$ if there exists a $N_{tr}\Lambda_P$-open set M such that $u_{a,b,c} \in M \subseteq N$.*

Definition 2.11. [7] *A neutrosophic topological space $(U, \tau_{N_{tr}})$ is said to be $\mathbf{N}_{tr}\mathbf{T}_{\Lambda_{\mathbf{P}}}$-**space** if every $N_{tr}\Lambda_P$-open set in $(U, \tau_{N_{tr}})$ is N_{tr}-open.*

Definition 2.12. [7] *Let $(U, \tau_{N_{tr}})$ and $(V, \rho_{N_{tr}})$ be neutrosophic topological spaces. Then the function $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be **neutrosophic$\Lambda_{\mathbf{P}}$-continuous** if $f^{-1}_{N_{tr}}(M)$ is $N_{tr}\Lambda_P$-open in $(U, \tau_{N_{tr}})$ for every N_{tr}-open set M in*

$(V, \rho_{N_{tr}})$.

Theorem 2.13. [7] *Every N_{tr}-continuous function is $N_{tr}\Lambda_P$-continuous.*

Definition 2.14. [7] *Let $(U, \tau_{N_{tr}})$ and $(V, \rho_{N_{tr}})$ be neutrosophic topological spaces. Then the function $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be **neutrosophicΛ_P-irresolute** if $f_{N_{tr}}^{-1}(M)$ is $N_{tr}\Lambda_P$-open in $(U, \tau_{N_{tr}})$ for every $N_{tr}\Lambda_P$-open set M in $(V, \rho_{N_{tr}})$.*

Theorem 2.15. [7] *Every $N_{tr}\Lambda_P$-irresolute function is $N_{tr}\Lambda_P$-continuous.*

Theorem 2.16. [7] *Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a function between two neutrosophic topological spaces. Then the following statements are equivalent:*

i. $f_{N_{tr}}$ *is* $N_{tr}\Lambda_P$*-irresolute.*

ii. $f_{N_{tr}}(N_{tr}\Lambda_P cl(K)) \subseteq N_{tr}\Lambda_P cl(f_{N_{tr}}(K))$ *for every neutrosophic set* K *in* U*.*

iii. $N_{tr}\Lambda_P cl\left(f_{N_{tr}}^{-1}(M)\right) \subseteq f_{N_{tr}}^{-1}(N_{tr}\Lambda_P cl(M))$ *for every neutrosophic set* M *in* V*.*

Theorem 2.17. [7] *If $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ and $g_{N_{tr}} : (V, \rho_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ are $N_{tr}\Lambda_P$-irresolute functions, then their composition $g_{N_{tr}} \circ f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ is also $N_{tr}\Lambda_P$-irresolute.*

3. NeutrosophicΛ_P-open and Λ_P-closed Maps

Definition 3.1. *Let $(U, \tau_{N_{tr}})$and$(V, \rho_{N_{tr}})$ be neutrosophic topological spaces. Then the mapping$f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be a **neutrosophicΛ_P-open** if $f_{N_{tr}}(K)$ is $N_{tr}\Lambda_P$-open in $(V, \rho_{N_{tr}})$ for every N_{tr}-open set K in $(U, \tau_{N_{tr}})$.*

Example 3.2. Let $U = \{a, b\}$, $V = \{x, y\}$, $\tau_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, K_1, K_2\}$, $\rho_{N_{tr}} = \{0_{N_{tr}}, 1_{N_{tr}}, M\}$ where $K_1 = \{< a, 0.1, 0.2, 0.5 >< b, 0.3, 0.3, 0.6 >\}$,
$K_2 = \{< a, 0.1, 0.1, 0.4 >< b, 0.2, 0.3, 0.7 >\}$ and $M = \{< x, 0.1, 0.1, 0.6 >$
$< y, 0.1, 0.2, 0.8 >\}$. Consider the collections $\mathcal{A} = \{A : 0_{N_{tr}} \subset A \subset K^C\}$, $\mathcal{B} = \{B : K^C \subset B \subset K\}$ and $\mathcal{C} = \{C : K \subset C \subset 1_{N_{tr}}\}$of neutrosophic sets in V. Then $N_{tr}\Lambda_P O(V, \rho_{N_{tr}}) = \{0_{N_{tr}}, M, M^c, \mathcal{A}, \mathcal{B}, \mathcal{C}, 1_{N_{tr}}\}$. Define $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ as $f_{N_{tr}}(a) = x$ and $f_{N_{tr}}(b) = y$. Then $f_{N_{tr}}(K_1) = \{< x, 0.1, 0.2, 0.5 >< y, 0.3, 0.3, 0.6 >\} \in \mathcal{B}$ and $f_{N_{tr}}(K_2) = \{< x, 0.1, 0.1, 0.4 >< y, 0.2, 0.3, 0.7 >\} \in \mathcal{C}$. Hence $f_{N_{tr}}$ is $N_{tr}\Lambda_P$-open.

Theorem 3.3. *Every N_{tr}-open map is $N_{tr}\Lambda_P$-open.*
Proof. Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a N_{tr}-open map and K be a N_{tr}-open set in U. Then, $f_{N_{tr}}(K)$ is N_{tr}-open in V. By theorem 2.9, $f_{N_{tr}}(K)$ is $N_{tr}\Lambda_P$-open in V. Hence $f_{N_{tr}}$ is a $N_{tr}\Lambda_P$-open map.

Remark 3.4. *The converse of theorem 3.3 does not hold in general.*

Example 3.5. Let $U = \{a, b\}$, $V = \{x, y\}$, $\tau_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, K\}and\rho_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, M\}$ where $K = \{< a, 0.7, 0.5, 0.3 >< b, 0.6, 0.3, 0.3 >\}$,
$M = \{< a, 0.3, 0.7, 0.6 >< b, 0.3, 0.5, 0.7\}$. Consider the collections $\mathcal{A} = \{A : K^C \subset A \subset K\}$ and $\mathcal{B} = \{B : K^C \subset B \subsetneq K\}$ of neutrosophic sets in V. Then $N_{\rm tr}\Lambda_P O\,(V, \rho_{N_{\rm tr}}) = \{0_{N_{\rm tr}}, M, M^c, \mathcal{A}, \mathcal{B}, 1_{N_{\rm tr}}\}$. Define $f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (V, \rho_{N_{\rm tr}})$ as $f_{N_{\rm tr}}(a) = y$ and $f_{N_{\rm tr}}(b) = x$. Then $f_{N_{\rm tr}}(K) = \{< x, 0.6, 0.3, 0.3 >< y, 0.7, 0.5, 0.3 >\}$ $= M^c$is $N_{\rm tr}\Lambda_P$-open but not $N_{\rm tr}$-open in V. Hence $f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-open but not $N_{\rm tr}$-open.

Theorem 3.6. *Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a mapping between neutrosophic topological spaces. Then, the following are equivalent:*

i. $f_{N_{tr}}$ is $N_{tr}\Lambda_P$-open.

ii. $f_{N_{tr}}\left(N_{tr}int(K)\right) \subseteq N_{tr}\Lambda_P int\left(f_{N_{tr}}(K)\right)$ for every neutrosophic set K in U.

iii. $N_{tr}int\left(f^{-1}_{N_{tr}}(M)\right) \subseteq f^{-1}_{N_{tr}}\left(N_{tr}\Lambda_P int(M)\right)$ for every neutrosophic set M in V.

(i)$\Longrightarrow$(ii) Let K be a neutrosophic set in U and $f_{N_{\rm tr}}$ be a $N_{\rm tr}\Lambda_P$-open function. Then $f_{N_{\rm tr}}\left(N_{\rm tr}{\rm int}(K)\right)$ is a $N_{\rm tr}\Lambda_P$-open set in V which implies$f_{N_{\rm tr}}\left(N_{\rm tr}{\rm int}(K)\right) \subseteq f_{N_{\rm tr}}(K)$. Also, $N_{\rm tr}\Lambda_P{\rm int}\left(f_{N_{\rm tr}}(K)\right)$ is the largest $N_{\rm tr}\Lambda_P$-open set contained in $f_{N_{\rm tr}}(K)$. Hence $f_{N_{\rm tr}}\left(N_{\rm tr}{\rm int}(K)\right) \subseteq N_{\rm tr}\Lambda_P{\rm int}\left(f_{N_{\rm tr}}(K)\right)$for every neutrosophic set K in U.

(ii)$\Longrightarrow$(iii) Let M be a neutrosophic set in V. Then, by assumption
$f_{N_{\rm tr}}\left(N_{\rm tr}{\rm int}\left(f^{-1}_{N_{\rm tr}}(M)\right)\right) \subseteq N_{\rm tr}\Lambda_P{\rm int}\left(f_{N_{\rm tr}}\left(f^{-1}_{N_{\rm tr}}(M)\right)\right) \subseteq N_{\rm tr}\Lambda_P{\rm int}(M)$.
Hence $N_{\rm tr}{\rm int}\left(f^{-1}_{N_{\rm tr}}(M)\right) \subseteq f^{-1}_{N_{\rm tr}}\left(N_{\rm tr}\Lambda_P{\rm int}(M)\right)$.

(iii)$\Longrightarrow$(i) Let K be a $N_{\rm tr}$-open set in U. By assumption,$N_{\rm tr}{\rm int}\left(f^{-1}_{N_{\rm tr}}\left(f_{N_{\rm tr}}(K)\right)\right) \subseteq f^{-1}_{N_{\rm tr}}\left(N_{\rm tr}\Lambda_P{\rm int}\left(f_{N_{\rm tr}}(K)\right)\right)$.Now, $K = N_{\rm tr}int(K) \subseteq N_{\rm tr}{\rm int}\left(f^{-1}_{N_{\rm tr}}\left(f_{N_{\rm tr}}(K)\right)\right)$ implies $f_{N_{\rm tr}}(K) \subseteq N_{\rm tr}\Lambda_P{\rm int}\left(f_{N_{\rm tr}}(K)\right)$. Also, $N_{\rm tr}\Lambda_P{\rm int}\left(f_{N_{\rm tr}}(K)\right) \subseteq f_{N_{\rm tr}}(K)$.
Consequently$f_{N_{\rm tr}}(K)$ is $N_{\rm tr}\Lambda_P$-open in V. Hence $f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-open.

Theorem 3.7. *A mapping$f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is $N_{tr}\Lambda_P$-open if and only if for each neutrosophic set K in $(V, \rho_{N_{tr}})$ and for each N_{tr}-closed set M in $(U, \tau_{N_{tr}})$ containing $f^{-1}_{N_{tr}}(K)$, there is a $N_{tr}\Lambda_P$-closed set N in $(V, \rho_{N_{tr}})$ such that $K \subseteq N$ and $f^{-1}_{N_{tr}}(N) \subseteq M$.*
Proof. Let$f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (V, \rho_{N_{\rm tr}})$ is a $N_{\rm tr}\Lambda_P$-open map. Let K be a neutrosophic set in V and M be a $N_{\rm tr}$-closed set in U such that $f^{-1}_{N_{\rm tr}}(K) \subseteq M$. Then,$N = \left(f_{N_{\rm tr}}\left(M^C\right)\right)^C$is $N_{\rm tr}\Lambda_P$-closed in V and$f^{-1}_{N_{\rm tr}}(N) \subseteq M$ since $N = \left(f_{N_{\rm tr}}\left(M^C\right)\right)^C \subseteq f_{N_{\rm tr}}(M^C)^C = f_{N_{\rm tr}}(M)$. Conversely, let O be a $N_{\rm tr}$-open set in U. Then, $O^C$is $N_{\rm tr}$-

closed in U and $f_{N_{\rm tr}}^{-1}(f_{N_{\rm tr}}(O))^C \subseteq O^C$. Now by assumption, there is a $N_{\rm tr}\Lambda_P$-closed set N in Vsuch that $(f_{N_{\rm tr}}(O))^C \subseteq N$and $f_{N_{\rm tr}}^{-1}(N) \subseteq O^C$. Hence $N^C \subseteq f_{N_{\rm tr}}(O) \subseteq f_{N_{\rm tr}}\left(\left(f_{N_{\rm tr}}^{-1}(N)\right)^C\right) = f_{N_{\rm tr}}\left(f_{N_{\rm tr}}^{-1}\left(N^C\right)\right) \subseteq N^C$ implies $f_{N_{\rm tr}}(O) = N^C$. Consequently$f_{N_{\rm tr}}(O)$ is $N_{\rm tr}\Lambda_P$-open in V.Hence $f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-open.

Theorem 3.8. *A mapping $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is $N_{tr}\Lambda_P$-open if and only if $f_{N_{tr}}^{-1}(N_{tr}\Lambda_P cl(K)) \subseteq N_{tr}cl\left(f_{N_{tr}}^{-1}(K)\right)$ for every neutrosophic set K in $(V, \rho_{N_{tr}})$.*

Proof. Suppose $f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (V, \rho_{N_{\rm tr}})$ is a $N_{\rm tr}\Lambda_P$-open function. For any neutrosophic set K in$V, f_{N_{\rm tr}}^{-1}(K) \subseteq N_{\rm tr}{\rm cl}\left(f_{N_{\rm tr}}^{-1}(K)\right)$. Then, by theorem 3.7, there exists a $N_{\rm tr}\Lambda_P$-closed set N in V such that $K \subseteq N$ and $f_{N_{\rm tr}}^{-1}(N) \subseteq N_{\rm tr}{\rm cl}\left(f_{N_{\rm tr}}^{-1}(K)\right)$. Now, since N is $N_{\rm tr}\Lambda_P$-closed and $K \subseteq N$, $f_{N_{\rm tr}}^{-1}\left(N_{\rm tr}\Lambda_P{\rm cl}(K)\right) \subseteq f_{N_{\rm tr}}^{-1}\left(N_{\rm tr}\Lambda_P{\rm cl}(N)\right) = f_{N_{\rm tr}}^{-1}(N) \subseteq N_{\rm tr}{\rm cl}\left(f_{N_{\rm tr}}^{-1}(K)\right)$. Hence $f_{N_{\rm tr}}^{-1}\left(N_{\rm tr}\Lambda_P{\rm cl}(K)\right) \subseteq N_{\rm tr}{\rm cl}\left(f_{N_{\rm tr}}^{-1}(K)\right)$ for every neutrosophic set K in V. Conversely, assume that K is a neutrosophic set in V and M is $N_{\rm tr}$-closed set in U containing $f_{N_{\rm tr}}^{-1}(K)$. Now, let $N = N_{\rm tr}\Lambda_P{\rm cl}(K)$. Then N is a $N_{\rm tr}\Lambda_P$-closed set in V such that $K \subseteq N$ and by assumption $f_{N_{\rm tr}}^{-1}(N) = f_{N_{\rm tr}}^{-1}\left(N_{\rm tr}\Lambda_P{\rm cl}(K)\right) \subseteq N_{\rm tr}{\rm cl}\left(f_{N_{\rm tr}}^{-1}(K)\right) \subseteq N_{\rm tr}{\rm cl}(M) = M$. Hence, by theorem 3.7, $f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-open.

Remark 3.9. *The composition of two $N_{tr}\Lambda_P$-open maps need not be $N_{tr}\Lambda_P$-open.*

Example 3.10. Let $U = \{a, b\}$, $V = \{x, y\}$ and $W = \{p, q\}$. Consider the neutrosophic topologies $\tau_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, K\}$, $\rho_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, M\}$ and $\omega_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, N\}$ where $K = \{< a, 0.6, \ 0.5, 0.2 >< b, 0.7, 0.6, 0.1 >\}$, $M = \{< x, 0.1, 0.4, 0.7 >< y, 0.2, 0.5, 0.6 >\}$ and $N = \{< p, 0.1, \ 0.3, 0.8 >< q, 0.2, 0.4, 0.7 > \}$ Consider the collections $\mathcal{A} = \{A \ : \ 0_{N_{\rm tr}} \subset A \subset K^c\}$,$\mathcal{B} = \{B \ : M^c \subset B \subset 1_{N_{\rm tr}}\}$ and $\mathcal{C} = \{C \ : M \subset C \subset M^c\}$ of neutrosophic sets in V and
$\mathcal{P} = \{P \ : \ 0_{N_{\rm tr}} \subset P \subset K^c\}$, $\mathcal{Q} = \{Q \ : M^c \subset Q \subset 1_{N_{\rm tr}}\}$, $\mathcal{R} = \{R \ : M \subset R \subset M^c\}$ the collection of neutrosophic sets in W.
Then, $N_{\rm tr}\Lambda_P O\left(V, \rho_{N_{\rm tr}}\right) = \{0_{N_{\rm tr}}, M, M^c, \mathcal{A}, \mathcal{B}, \mathcal{C}, 1_{N_{\rm tr}}\}$ and
$N_{\rm tr}\Lambda_P O\left(W, \omega_{N_{\rm tr}}\right) = \{0_{N_{\rm tr}}, N, N^c, \mathcal{P}, \mathcal{Q}, \mathcal{R}, 1_{N_{\rm tr}}\}$. Define $f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (V, \rho_{N_{\rm tr}})$ as $f_{N_{\rm tr}}(a) = y$ and $f_{N_{\rm tr}}(b) = x$.
Then $f_{N_{\rm tr}}(K) = \{< x, 0.7, 0.6, 0.1 >< y, 0.6, 0.5, 0.2 >\} = M^c$ is $N_{\rm tr}\Lambda_P$-open in $(V, \rho_{N_{\rm tr}})$. Also, define $g_{N_{\rm tr}} : (V, \rho_{N_{\rm tr}}) \longrightarrow (W, \omega_{N_{\rm tr}})$ as $g_{N_{\rm tr}}(x) = q$and$g_{N_{\rm tr}}(y) = p$. Then $g_{N_{\rm tr}}(M) = \{< p, 0.2, 0.5, 0.6 >< q, 0.1, 0.4, 0.7 >\} \in \mathcal{R}$ which implies $g_{N_{\rm tr}}(M)$ is $N_{\rm tr}\Lambda_P$-open in $(W, \omega_{N_{\rm tr}})$. This implies that both $f_{N_{\rm tr}}$ and $g_{N_{\rm tr}}$ are $N_{\rm tr}\Lambda_P$-open. Now, let $g_{N_{\rm tr}} \circ f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (W, \omega_{N_{\rm tr}})$ be the composition of two $N_{\rm tr}\Lambda_P$-open functions. Then, $g_{N_{\rm tr}} \circ f_{N_{\rm tr}}$ is not $N_{\rm tr}\Lambda_P$-open since $g_{N_{\rm tr}} \circ f_{N_{\rm tr}}(K) = g_{N_{\rm tr}}(f_{N_{\rm tr}}(K)) = \{< p, 0.6, 0.5, 0.2 >< q, 0.7, 0.6, 0.1 >\}$ is not $N_{\rm tr}\Lambda_P$-open in $(W, \omega_{N_{\rm tr}})$.

Theorem 3.11. *Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ and $g_{N_{tr}} : (V, \rho_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$*

be $N_{tr}\Lambda_P$-open and let $(V, \rho_{N_{tr}})$ be a $N_{tr}T_{\Lambda_P}$-space. Then $g_{N_{tr}} \circ f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ is also $N_{tr}\Lambda_P$-open.

Proof. Let K be any N_{tr}-open set in U. Since $f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open, $f_{N_{\text{tr}}}(K)$ is $N_{\text{tr}}\Lambda_P$-open in V. Then, by assumption $f_{N_{\text{tr}}}(K)$ is N_{tr}-open in V. Again, since $g_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open$g_{N_{\text{tr}}}\left(f_{N_{\text{tr}}}(K)\right) = g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}(K)$ is $N_{\text{tr}}\Lambda_P$-open in W. Hence $g \circ f$ is $N_{\text{tr}}\Lambda_P$-open.

Theorem 3.12. *Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a bijective map between two neutrosophic topological spaces. Then, the following are equivalent*

i. $f_{N_{tr}}$ is $N_{tr}\Lambda_P$-open.

ii. For each $(u_{a,b,c}) \in (U, \tau_{N_{tr}})$ and for every $N_{tr}nbhdK$ of $u_{a,b,c}$in $(U, \tau_{N_{tr}})$, there exists a $N_{tr}\Lambda_P$-nbhd$M$ of $f_{N_{tr}}(u_{a,b,c})$ in $(V, \rho_{N_{tr}})$ such that $M \subseteq f_{N_{tr}}(K)$.

Proof. **(i)⇒(ii)** Let $f_{N_{\text{tr}}}$ be a $N_{\text{tr}}\Lambda_P$-open map and K be an arbitrary N_{tr}nbhd of $u_{a,b,c}$ in U. Then, there exists a N_{tr}-open set$N$ in U such that $u_{a,b,c} \in N \subseteq K$. Since $f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open, $f_{N_{\text{tr}}}(N)$ is $N_{\text{tr}}\Lambda_P$-open in V. Now, let $f_{N_{\text{tr}}}(N) = M$. Then, $f_{N_{\text{tr}}}(u_{a,b,c}) \in M \subseteq f_{N_{\text{tr}}}(K)$. This implies M is a $N_{\text{tr}}\Lambda_P$-nbhd of $f_{N_{\text{tr}}}(u_{a,b,c})$ and $M \subseteq f_{N_{\text{tr}}}(K)$.

(ii) ⇒(i) Let K be a N_{tr}-open set in U, $u_{a,b,c} \in K$ and $f_{N_{\text{tr}}}(u_{a,b,c}) = v_{x,y,z} \in f_{N_{\text{tr}}}(K)$. By assumption, there exists a $N_{\text{tr}}\Lambda_P$-nbhd$M_{v_{x,y,z}}$ of $v_{x,y,z}$ such that $M_{v_{x,y,z}} \subseteq f_{N_{\text{tr}}}(K)$. Since $M_{v_{x,y,z}}$is a $N_{\text{tr}}\Lambda_P$-nbhd, there exists a $N_{\text{tr}}\Lambda_P$-open set $N_{v_{x,y,z}}$ in V such that $v_{x,y,z} \in N_{v_{x,y,z}} \subseteq M_{v_{x,y,z}}$. By theorem 2.8, $f_{N_{\text{tr}}}(K)$ is $N_{\text{tr}}\Lambda_P$-open. Hence $f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open.

Theorem 3.13. *Let $(U, \tau_{N_{tr}})$and $(V, \rho_{N_{tr}})$ be neutrosophic topological spaces and $(S, \tau^*_{N_{tr}})$ be a subspace of $(U, \tau_{N_{tr}})$. If $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is a $N_{tr}\Lambda_P$-open map and $1^S_{N_{tr}}$ is N_{tr}-open in $(U, \tau_{N_{tr}})$, then the restriction $f_{N_{tr}}|_S : (S, \tau^*_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is also $N_{tr}\Lambda_P$-open.*

Proof. Let K be N_{tr}-open in S. Then, $K = 1^S_{N_{\text{tr}}} \bigcap M$for some N_{tr}-open set M in U. Now, K is N_{tr}-open in U and since $f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open, $f_{N_{\text{tr}}}(K)$ is $N_{\text{tr}}\Lambda_P$-open in V. But $f_{N_{\text{tr}}}(K) = f_{N_{\text{tr}}}|_S(K)$. Therefore $f_{N_{\text{tr}}}|_S$ is $N_{\text{tr}}\Lambda_P$-open.

Definition 3.14. *A mapping $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ between two neutrosophic topological spaces is said to be **neutrosophicΛ_P-closed** if $f_{N_{tr}}(K)$ is $N_{tr}\Lambda_P$-closed in $(V, \rho_{N_{tr}})$ for every N_{tr}-closed set K in $(U, \tau_{N_{tr}})$.*

Example 3.15. Consider the neutrosophic topological spaces defined in example 3.2. Here $K_1^C = \{< a, 0.5, 0.8, 0.1 >< b, 0.6, 0.7, 0.3 >\}$,
$K_2^C = \{< a, 0.4, 0.9, 0.1 >< b, 0.7, 0.7, 0.2 >\}$ and

$N_{\rm tr}\Lambda_P C\,(V,\ \rho_{N_{\rm tr}}) = \left\{0_{N_{\rm tr}}, M, M^c, \mathcal{A}', \mathcal{B}, \mathcal{C}', 1_{N_{\rm tr}}\right\}$ where $\mathcal{A}' = \{A^c : A \in \mathcal{A}\}$ and $\mathcal{C}' = \left\{C^C : C \in \mathcal{C}\right\}$. Now, define $f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (V, \rho_{N_{\rm tr}})$ as $f_{N_{\rm tr}}(a) = y$ and $f_{N_{\rm tr}}(b) = x$. Then $f_{N_{\rm tr}}\left(K_1^C\right) = \{< x, 0.6, 0.7, 0.3 >< y,\ 0.5, 0.8, 0.1 >\} \in \mathcal{B}$ and $f_{N_{\rm tr}}\left(K_2^C\right) = \{< x, 0.7, 0.7, 0.2 >< y,\ 0.4, 0.9, 0.1 >\} \in \mathcal{C}'$. Hence $f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-closed.

Theorem 3.16. *A mapping $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ between two neutrosophic topological spaces is $N_{tr}\Lambda_P$-closed if and only if $N_{tr}\Lambda_P cl\,(f_{N_{tr}}(K)) \subseteq f_{N_{tr}}\,(N_{tr}cl(K))$ for every neutrosophic set K in U.*

Proof. Let K be a neutrosophic set in U and $f_{N_{\rm tr}}$ be a $N_{\rm tr}\Lambda_P$-closed map. Then, $f_{N_{\rm tr}}\,(N_{\rm tr}{\rm cl}(K))$ is a $N_{\rm tr}\Lambda_P$-closed set in $V$and $f_{N_{\rm tr}}(K) \subseteq f_{N_{\rm tr}}\,(N_{\rm tr}{\rm cl}(K))$ .Also, $N_{\rm tr}\Lambda_P{\rm cl}\,(f_{N_{\rm tr}}(K))$ is the smallest $N_{\rm tr}\Lambda_P$-closed set containing $f_{N_{\rm tr}}(K)$.
Hence $N_{\rm tr}\Lambda_P{\rm cl}\,(f_{N_{\rm tr}}(K)) \subseteq f_{N_{\rm tr}}\,(N_{\rm tr}{\rm cl}(K))$ for every neutrosophic set K in U. Conversely, let K be a $N_{\rm tr}$-closed set in U. Then $K = N_{\rm tr}{\rm cl}(K)$ implies $f_{N_{\rm tr}}(K) = f_{N_{\rm tr}}\,(N_{\rm tr}{\rm cl}(K))$. Hence, by assumption $N_{\rm tr}\Lambda_P{\rm cl}\,(f_{N_{\rm tr}}(K)) \subseteq f_{N_{\rm tr}}(K)$. Also, since $f_{N_{\rm tr}}(K) \subseteq N_{\rm tr}\Lambda_P{\rm cl}\,(f_{N_{\rm tr}}(K))\,, f_{N_{\rm tr}}(K) = N_{\rm tr}\Lambda_P{\rm cl}\,(f_{N_{\rm tr}}(K))$ which implies $f_{N_{\rm tr}}(K)$ is $N_{\rm tr}\Lambda_P$-closed in V. Hence $f_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$-closed map.

Theorem 3.17. *A bijective map $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ between two neutrosophic topological spaces is $N_{tr}\Lambda_P$-closed if and only if $f^{-1}_{N_{tr}}\,(N_{tr}\Lambda_P cl(K)) \subseteq N_{tr}cl\left(f^{-1}_{N_{tr}}(K)\right)$ for every neutrosophic set K in V.*

Proof. Let K be a neutrosophic set in V and $f_{N_{\rm tr}}$ be a $N_{\rm tr}\Lambda_P$-closed map. Then $f_{N_{\rm tr}}\left(N_{\rm tr}{\rm cl}\left(f^{-1}_{N_{\rm tr}}(K)\right)\right)$ is $N_{\rm tr}\Lambda_P$-closed in V which implies
$N_{\rm tr}\Lambda_P{\rm cl}\left(f_{N_{\rm tr}}\left(N_{\rm tr}{\rm cl}\left(f^{-1}_{N_{\rm tr}}(K)\right)\right)\right) = f_{N_{\rm tr}}\left(N_{\rm tr}{\rm cl}\left(f^{-1}_{N_{\rm tr}}(K)\right)\right)$. Also, since
$K = f_{N_{\rm tr}}\left(f^{-1}_{N_{\rm tr}}(K)\right)$ and $f^{-1}_{N_{\rm tr}}(K) \subseteq N_{tr}{\rm cl}\left(f^{-1}_{N_{\rm tr}}(K)\right)$ implies
$f_{N_{\rm tr}}\left(f^{-1}_{N_{\rm tr}}(K)\right) \subseteq f_{N_{\rm tr}}\left(N_{\rm tr}{\rm cl}\left(f^{-1}_{N_{\rm tr}}(K)\right)\right)$,
$N_{\rm tr}\Lambda_P{\rm cl}(K) \subseteq N_{\rm tr}\Lambda_P{\rm cl}\left(f_{N_{\rm tr}}\left(N_{\rm tr}{\rm cl}\left(f^{-1}_{N_{\rm tr}}(K)\right)\right)\right) = f_{N_{\rm tr}}\left(N_{\rm tr}{\rm cl}\left(f^{-1}_{N_{\rm tr}}(K)\right)\right)$. Hence $f^{-1}_{N_{\rm tr}}\,(N_{\rm tr}\Lambda_P{\rm cl}(K)) \subseteq N_{\rm tr}{\rm cl}\left(f^{-1}_{N_{\rm tr}}(K)\right)$ for every neutrosophic set K in V. Conversely, let K be a $N_{\rm tr}$-closed set in U. By assumption, $f^{-1}_{N_{\rm tr}}\,(N_{\rm tr}\Lambda_P{\rm cl}\,(f_{N_{\rm tr}}(K))) \subseteq N_{\rm tr}{\rm cl}\left(f^{-1}_{N_{\rm tr}}\,(f_{N_{\rm tr}}(K))\right) = N_{\rm tr}{\rm cl}(K) = K$.
Consequently,$N_{\rm tr}\Lambda_P{\rm cl}\,(f_{N_{\rm tr}}(K)) \subseteq f_{N_{\rm tr}}(K)$. Also, $f_{N_{\rm tr}}(K) \subseteq N_{\rm tr}{\rm cl}\,(f_{N_{\rm tr}}(K))$ .Hence $f_{N_{\rm tr}}(K)$ is $N_{\rm tr}\Lambda_P$-closed which implies $f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-closed.

Remark 3.18. *The composition of two $N_{tr}\Lambda_P$-closed maps need not be $N_{tr}\Lambda_P$-closed.*

Example 3.19. Let $U = \{a, b\}$, $V = \{x, y\}$and $W = \{p, q\}$. Consider the neutrosophic topologies $\tau_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, K_1, K_2\}, \rho_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, M\}and\omega_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, N\}$ where $K_1 = \{< a, 0.8, 0.7, 0.1 >< b, 0.8, 0.5, 0.2 >\}$, $K_2 = \{< a, 0.5, 0.7, 0.2 >< b, 0.6, 0.5, 0.5 >\} M = \{< x, 0.1, 0.2, 0.8 >< y, 0.1, 0.1, 0.7 >\}$

and $N = \{< p, 0.1, 0.1, 0.7 >< q, 0.1, 0.2, 0.8 >\}$ Consider the collections $\mathcal{A} = \{A : A \subset M^c,\ A \subset K\}$, $\mathcal{B} = \{B : B \subset M^c,\ M \subsetneq B, B \subsetneq K^c\}, \mathcal{C} = \{C : K \subset C \subsetneq K^c\}$ of neutrosophic sets in V and $\mathcal{P} = \{P\ :\ M^c \subset P \subset 1_{N_{\rm tr}}\}$, $\mathcal{Q} = \{Q\ : M \subset Q \subset M^c\}$, $\mathcal{R} = \{R\ : M \subset R \subsetneq M^c\}$ the collection of neutrosophic sets in W. Then, $N_{\rm tr}\Lambda_P C\,(V, \rho_{N_{\rm tr}}) = \{0_{N_{\rm tr}}, M, M^c, \mathcal{A}, \mathcal{B}, \mathcal{C}, 1_{N_{\rm tr}}\}$,$N_{\rm tr}\Lambda_P C\,(W, \omega_{N_{\rm tr}}) = \{0_{N_{\rm tr}}, N, N^c, \mathcal{P}, \mathcal{Q}, \mathcal{R}, 1_{N_{\rm tr}}\}$. Define $f_{N_{\rm tr}}\ :\ (U, \tau_{N_{\rm tr}}) \longrightarrow (V, \rho_{N_{\rm tr}})$ as $f_{N_{\rm tr}}(a) = y$ and $f_{N_{\rm tr}}(b) = x$. Then $f_{N_{\rm tr}}\left(K_1^C\right) = \{< x, 0.1, 0.3, 0.8 >< y, 0.2, 0.5, 0.8 >\}$ and $f_{N_{\rm tr}}\left(K_2^C\right) = \{< x, 0.5, 0.5, 0.6 >< y, 0.2, 0.3, 0.5 >\} \in \mathcal{B}$. Also, define $g_{N_{\rm tr}} : (V, \rho_{N_{\rm tr}}) \longrightarrow (W, \omega_{N_{\rm tr}})$ as $g_{N_{\rm tr}}(x) = p$ and $g_{N_{\rm tr}}(y) = q$. Then, $g_{N_{\rm tr}}(M^c) = \{< p, 0.8, 0.8, 0.1 >< q, 0.7, 0.9, 0.1 >\} \in$ R. This implies that both $f_{N_{\rm tr}}$ and $g_{N_{\rm tr}}$ are $N_{\rm tr}\Lambda_P$-closed. Now, let $g_{N_{\rm tr}} \circ f_{N_{\rm tr}}\ :\ (U, \tau_{N_{\rm tr}}) \longrightarrow (W, \omega_{N_{\rm tr}})$ be the composition of two $N_{\rm tr}\Lambda_P$-closed maps. Then, $g_{N_{\rm tr}} \circ f_{N_{\rm tr}}\left(K_1^C\right) = g_{N_{\rm tr}}(f_{N_{\rm tr}}\left(K_1^C\right)) = \{< p, 0.2, 0.5, 0.8 >< q, 0.1, 0.3, 0.8 >\}$ and $g_{N_{\rm tr}} \circ f_{N_{\rm tr}}\left(K_2^C\right) = g_{N_{\rm tr}}(f_{N_{\rm tr}}\left(K_2^C\right)) = \{< p, 0.5, 0.5, 0.6 >< q, 0.2, 0.3, 0.5 >\}$ are not $N_{\rm tr}\Lambda_P$-closed in $(W, \omega_{N_{\rm tr}})$. Hence $g_{N_{\rm tr}} \circ f_{N_{\rm tr}}$ is not $N_{\rm tr}\Lambda_P$-closed.

Theorem 3.20. *Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ and $g_{N_{tr}} : (V, \rho_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ be $N_{tr}\Lambda_P$-closed map and let $(V, \rho_{N_{tr}})$ be a $N_{tr}T_{\Lambda_P}$-space. Then $g_{N_{tr}} \circ f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ is also $N_{tr}\Lambda_P$-closed map.*

Proof. The proof is similar to theorem 3.11.

Theorem 3.21. *Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a bijective map between two neutrosophic topological spaces. Then, the following are equivalent*

- *i. $f_{N_{tr}}$ is $N_{tr}\Lambda_P$-open.*
- *ii. $f_{N_{tr}}$ is $N_{tr}\Lambda_P$-closed.*
- *iii. $f^{-1}_{N_{tr}}$ is $N_{tr}\Lambda_P$-continuous.*

Proof. (i)$\Longrightarrow$(ii) Let $f_{N_{\rm tr}}$ be a $N_{\rm tr}\Lambda_P$-open function and let K be a $N_{\rm tr}$-closed set in U. Then K^c is $N_{\rm tr}$-open in U and by assumption, $f_{N_{\rm tr}}(K^c)$ is $N_{\rm tr}\Lambda_P$-open in V. That is, $(f_{N_{\rm tr}}(K))^c$is $N_{\rm tr}\Lambda_P$-open in V. Hence $f_{N_{\rm tr}}(K)$ is $N_{\rm tr}\Lambda_P$-closed in V. Therefore $f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-closed.

(ii)$\Longrightarrow$(iii) Let K be a $N_{\rm tr}$-closed set in U. Then, by assumption $f_{N_{\rm tr}}(K)$ is $N_{\rm tr}\Lambda_P$-closed in V. Hence $\left(f^{-1}_{N_{\rm tr}}\right)^{-1}(K) = f_{N_{\rm tr}}(K)$ is $N_{\rm tr}\Lambda_P$-closed in V. Therefore $f^{-1}_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-continuous.

(iii)$\Longrightarrow$(i) Let K be a $N_{\rm tr}$-open set in U. Then, by assumption $\left(f^{-1}_{N_{\rm tr}}\right)^{-1}(K) = f_{N_{\rm tr}}(K)$ is $N_{\rm tr}\Lambda_P$-open in V. Hence $f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-open.

Theorem 3.22. *Let$f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ and $g_{N_{tr}} : (V, \rho_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$*

be mappings between neutrosophic topological spaces such that their composition $g_{N_{tr}} \circ f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ *is* $N_{tr}\Lambda_P$*-open. Then the following are true*

i. *If* $f_{N_{tr}}$ *is* N_{tr}*-continuous and surjective, then* $g_{N_{tr}}$ *is* $N_{tr}\Lambda_P$*-open.*

ii. *If* $f_{N_{tr}}$ *is a* $N_{tr}\Lambda_P$*-continuous, surjective and* $(V, \rho_{N_{tr}})$ *is a* $N_{tr}T_{\Lambda_P}$*-space, then* $g_{N_{tr}}$ *is* $N_{tr}\Lambda_P$*-open.*

iii. *If* $g_{N_{tr}}$ *is* $N_{tr}\Lambda_P$*-irresolute and injective, then* $f_{N_{tr}}$ *is* $N_{tr}\Lambda_P$*-open.*

Proof.

i. Let K be a N_{tr}-open set in V. Since $f_{N_{\text{tr}}}$ is N_{tr}-continuous, $f^{-1}_{N_{\text{tr}}}(K)$ is N_{tr}open in U. Now, since $g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open, $g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}} \left(f^{-1}_{N_{\text{tr}}}(K)\right) = g_{N_{\text{tr}}} \left(f_{N_{\text{tr}}} \left(f^{-1}_{N_{\text{tr}}}(K)\right)\right) = g_{N_{\text{tr}}}(K)$ is $N_{\text{tr}}\Lambda_P$-open in W. Hence $g_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open.

ii. Let K be a N_{tr}-open set in V. Since $f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-continuous, $f^{-1}_{N_{\text{tr}}}(K)$ is $N_{\text{tr}}\Lambda_P$-open in U. Now, since V is a $N_{\text{tr}}T_{\Lambda_P}$-space, $f^{-1}_{N_{\text{tr}}}(K)$ is N_{tr}-open in $U$and since $g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open, $g_{N_{\text{tr}}} \circ f_{N_{tr}} \left(f^{-1}_{N_{\text{tr}}}(K)\right) = g_{N_{\text{tr}}} \left(f_{N_{\text{tr}}} \left(f^{-1}_{N_{\text{tr}}}(K)\right)\right) = g_{N_{\text{tr}}}(K)$ is $N_{\text{tr}}\Lambda_P$-open in W. Hence $g_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open.

iii. Let K be a N_{tr}-open set in U. Since $g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open, $(g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}})(K)$ is $N_{\text{tr}}\Lambda_P$-open. Now, since $g_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-irresolute injective function, $g^{-1}_{N_{\text{tr}}} \left(g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}(K)\right) = g^{-1}_{N_{\text{tr}}} \left(g_{N_{\text{tr}}} \left(f_{N_{\text{tr}}}(K)\right)\right) = f_{N_{\text{tr}}}(K)$ is $N_{\text{tr}}\Lambda_P$-open in V. Hence $f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open.

Theorem 3.23. *If*$f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ *is a* $N_{tr}\Lambda_P$*-open function and* $g_{N_{tr}} : (V, \rho_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ *is a surjection such that their composition is* N_{tr}*-open, then* $g_{N_{tr}}$ *is* $N_{tr}\Lambda_P$*-continuous.*
Proof. Let K be a N_{tr}-open set in U. Since $g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}$ is N_{tr}-open, $g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}(K)$ is N_{tr}-open in W. Now, $g^{-1}_{N_{\text{tr}}} \left(g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}(K)\right) = g^{-1}_{N_{\text{tr}}} \left(g_{N_{\text{tr}}} \left(f_{N_{\text{tr}}}(K)\right)\right) = f_{N_{\text{tr}}}(K)$. Since $f_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-open, $f_{N_{\text{tr}}}(K)$ is $N_{\text{tr}}\Lambda_P$-open in V and hence $g_{N_{\text{tr}}}$ is $N_{\text{tr}}\Lambda_P$-continuous.

4. Neutrosophic$\Lambda_{\mathbf{P}}$-homeomorphism

Definition 4.1. *A bijective map* $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ *is said to be a* ***neutrosophic*$\Lambda_{\mathbf{P}}$*-homeomorphism*** *if* $f_{N_{tr}}$ *and* $f^{-1}_{N_{tr}}$ *are* $N_{tr}\Lambda_P$*-continuous.*

Example 4.2. Let $U = \{a, b\}\,,\ V = \{x, y\}\,, \tau_{N_{\text{tr}}} = \{0_{N_{\text{tr}}}, 1_{N_{\text{tr}}}, K_1, K_2\}\,,$
$\rho_{N_{\text{tr}}} = \{0_{N_{\text{tr}}}, 1_{N_{\text{tr}}}, M\}$ where $K_1 = \{< a, 0.3, 0.4, 0.5 >< b, 0.6, 0.5, 0.6 >\}\,,$
$K_2 = \{< a, 0.3, 0.3, 0.8 >< b, 0.5, 0.2, 0.6 >\}$ and
$M = \{< x, 0.2, 0.3, 0.8 >< y, 0.3, 0.2, 0.9 >\}\,.$

Consider the collections $\mathcal{A}=\{A\ :0_{N_{\rm tr}}\subset A\subset K\},\mathcal{B}=\{B\ :K\subset B\subset K^c\}$ and $\mathcal{C}=\{C:C\subsetneq K,K\subsetneq C,\ C\subset K^c\}$ of neutrosophic sets in U and the collections $\mathcal{P}=\{P\ :0_{N_{\rm tr}}\subset P\subset K_2\},\mathcal{Q}=\{Q\ :K_1\subset Q\subset K_1^C\}$ and $\mathcal{R}=\{R\ :R\subsetneq K_1;K_1\subsetneq R;\ R\subset K_1^C\}$ of neutrosophic sets in V.
Then $N_{\rm tr}\Lambda_P O(U,\tau_{N_{\rm tr}})=\{0_{N_{\rm tr}},K_1,K_2,K_1^C,\mathcal{A},\mathcal{B},\mathcal{C},1_{N_{\rm tr}}\}$ and $N_{\rm tr}\Lambda_P O\,(V,\ \rho_{N_{\rm tr}})=\{0_{N_{\rm tr}},M,M^c,\mathcal{P},\mathcal{Q},\mathcal{R},1_{N_{\rm tr}}\}$. Define $f_{N_{\rm tr}}:(U,\tau_{N_{\rm tr}})\longrightarrow(V,\rho_{N_{\rm tr}})$ as $f_{N_{\rm tr}}(a)=x$ and $f_{N_{\rm tr}}(b)=y$. Then $f_{N_{\rm tr}}^{-1}:(V,\rho_{N_{\rm tr}})\longrightarrow(U,\tau_{N_{\rm tr}})$ is defined as $f_{N_{\rm tr}}^{-1}(x)=a$ and $f_{N_{\rm tr}}^{-1}(y)=b$. Now, $f_{N_{\rm tr}}^{-1}(M)=\{<a,0.2,\ 0.3,\ 0.8><b,0.3,0.2,0.9>\}\in\mathcal{A}$,$(f_{N_{\rm tr}}^{-1})^{-1}(K_1)=\{<x,0.3,\ 0.4,\ 0.5><y,0.6,0.5,0.6>\}\in\mathcal{Q}$ and $(f_{N_{\rm tr}}^{-1})^{-1}(K_2)=\{<x,0.3,\ 0.3,\ 0.8><y,0.5,0.2,0.6>\}\in\mathcal{R}$. Clearly, $f_{N_{\rm tr}}$ is a bijection and both $f_{N_{\rm tr}}$ and $f_{N_{\rm tr}}^{-1}$ are $N_{\rm tr}\Lambda_P$-continuous. Hence $f_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$-homeomorphism.

Theorem 4.3. *Every N_{tr}-homeomorphism is a $N_{tr}\Lambda_P$-homeomorphism.*
Proof. Let a bijective map $f_{N_{\rm tr}}:(U,\tau_{N_{\rm tr}})\longrightarrow(V,\rho_{N_{\rm tr}})$ be a $N_{\rm tr}$-homeomorphism. Then both $f_{N_{\rm tr}}$ and $f_{N_{\rm tr}}^{-1}$ are $N_{\rm tr}$-continuous. By theorem 2.13, both $f_{N_{\rm tr}}$ and $f_{N_{\rm tr}}^{-1}$ are $N_{\rm tr}\Lambda_P$-continuous. Hence $f_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$-homeomorphism.

Remark 4.4. *The converse of theorem 4.3 does not hold in general.*

Example 4.5. Let $U=\{a,b\},\ V=\{x,y\},\tau_{N_{\rm tr}}=\{0_{N_{\rm tr}},1_{N_{\rm tr}},K\}$ and $\rho_{N_{\rm tr}}=\{0_{N_{\rm tr}},1_{N_{\rm tr}},M\}$ Where$K=\{<a,0.3,0.4,0.9><b,0.2,0.3,0.8>\}$ and
$M=\{<a,0.4,0.5,0.8><b,0.3,0.5,\ 0.4>\}$.

Consider the collections $\mathcal{A}=\{A\ :0_{N_{\rm tr}}\subset A\subset K\},\mathcal{B}=\{B\ :K\subset B\subset K^C\}$,$\mathcal{C}=\{C\ :C\subsetneq K;K\subsetneq C;\ C\subset K^C\}$of neutrosophic sets in U and the collections $\mathcal{P}=\{P\ :0_{N_{\rm tr}}\subset P\subset M\},\mathcal{Q}=\{Q\ :M\subset Q\subset M^C\},\mathcal{R}=\{R\ :R\subsetneq M;M\subsetneq R;\ R\subset M^C\}$ of neutrosophic sets in V.

Then, $N_{\rm tr}\Lambda_P O(U,\tau_{N_{\rm tr}})=\{0_{N_{\rm tr}},K,K^c,\mathcal{A},\mathcal{B},\mathcal{C},1_{N_{\rm tr}}\}$,
$N_{\rm tr}\Lambda_P O\,(V,\ \rho_{N_{\rm tr}})=\{0_{N_{\rm tr}},M,M^c,\mathcal{P},\mathcal{Q},\mathcal{R},1_{N_{\rm tr}}\}$. Define $f_{N_{\rm tr}}:(U,\tau_{N_{\rm tr}})\longrightarrow(V,\rho_{N_{\rm tr}})$ as $f_{N_{\rm tr}}(a)=y$ and $f_{N_{\rm tr}}(b)=x$. Then $f_{N_{\rm tr}}^{-1}:(V,\rho_{N_{\rm tr}})\longrightarrow(U,\tau_{N_{\rm tr}})$ is defined as $f_{N_{\rm tr}}^{-1}(x)=b$ and $f_{N_{\rm tr}}^{-1}(y)=a$.
Now, $f_{N_{\rm tr}}^{-1}(M)=\{<a,0.3,\ 0.5,\ 0.4><b,0.4,0.5,0.8>\}\in\mathcal{B}$ and $(f_{N_{\rm tr}}^{-1})^{-1}(K)=\{<x,0.2,\ 0.3,\ 0.8><y,0.3,0.4,0.9>\}\in\mathcal{P}$.Clearly, $f_{N_{\rm tr}}$is a bijection and both $f_{N_{\rm tr}}$ and $f_{N_{\rm tr}}^{-1}$ are $N_{\rm tr}\Lambda_P$-continuous but not $N_{\rm tr}$-continuous. Hence $f_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$-homeomorphism but not $N_{\rm tr}$-homeomorphism.

Theorem 4.6. *Let $f_{N_{tr}}:(U,\tau_{N_{tr}})\longrightarrow(V,\rho_{N_{tr}})$ be a bijective $N_{tr}\Lambda_P$-continuous function between two neutrosophic topological spaces. Then, the following are equivalent* *i.* $f_{N_{tr}}$ *is a* $N_{tr}\Lambda_P$*-open map.*

ii. $f_{N_{tr}}$ *is a* $N_{tr}\Lambda_P$*-closed map.*

iii. $f_{N_{tr}}$ *is a* $N_{tr}\Lambda_P$*-homeomorphism.*

Proof. Proof follows from theorem 3.21.

Remark 4.7. *The composition of two* $N_{tr}\Lambda_P$*-homeomorphisms need not be* $N_{tr}\Lambda_P$*-homeomorphism.*

Example 4.8. Let $U = \{a, b\}$, $V = \{x, y\}$and $W = \{p, q\}$. Consider the neutrosophic topologies $\tau_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, K_1, K_2\}$, $\rho_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, M\}$ and $\omega_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, N\}$ where $K_1 = \{< a, 0.4, 0.5, 0.1 >< b, 0.6, 0.7, 0.2 >\}$, $K_2 = \{< a, 0.1, 0.5, 0.4 >< b, 0.2, 0.3, 0.6 >\}$, $M = \{< x, 0.2, 0.4, 0.7 >< y, 0.2, 0.3, 0.8 >\}$ and $N = \{< p, 0.1, 0.4, 0.7 >< q, 0.3, 0.7, 0.8 >\}$.
Consider $\mathcal{A} = \{A : 0_{N_{\rm tr}} \subset A \subset K_2\}$, the collection of neutrosophic sets in U, $\mathcal{X} = \{X : 0_{N_{\rm tr}} \subset X \subset M\}$, $\mathcal{Y} = \{Y : M \subset Y \subset M^C\}$,
$\mathcal{Z} = \{Z : Z \subsetneq M; M \subsetneq Z; Z \subset M^C\}$ the collection of neutrosophic sets in V and $\mathcal{P} = \{P : 0_{N_{\rm tr}} \subset P \subset N\}$, $\mathcal{Q} = \{Q : N \subset Q \subset N^C\}$,
$\mathcal{R} = \{R : R \subsetneq N; N \subsetneq R; R \subset N^C\}$, the collection of neutrosophic sets in W. Then, $N_{\rm tr}\Lambda_P O(U, \tau_{N_{\rm tr}}) = \{0_{N_{\rm tr}}, K_1, K_2, \mathcal{A}, 1_{N_{\rm tr}}\}$,
$N_{\rm tr}\Lambda_P O(V, \rho_{N_{\rm tr}}) = \{0_{N_{\rm tr}}, M, M^c, \mathcal{X}, \mathcal{Y}, \mathcal{Z}, 1_{N_{\rm tr}}\}$ and
$N_{\rm tr}\Lambda_P O(W, \omega_{N_{\rm tr}}) = \{0_{N_{\rm tr}}, N, N^c, \mathcal{P}, \mathcal{Q}, \mathcal{R}, 1_{N_{\rm tr}}\}$. Define $f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (V, \rho_{N_{\rm tr}})$ as $f_{N_{\rm tr}}(a) = x$ and $f_{N_{\rm tr}}(b) = y$. Then
$f_{N_{\rm tr}}(K_1) = \{< x, 0.4, 0.5, 0.1 >< y, 0.6, 0.7, 0.2 >\}$
and $f_{N_{\rm tr}}(K_2) = \{< x, 0.1, 0.5, 0.4 >< y, 0.2, 0.3, 0.6 >\} \in \mathcal{Y}$.

Also, $f^{-1}_{N_{\rm tr}}(M) = \{< a, 0.2, 0.4, 0.7 >< b, 0.2, 0.3, 0.8 >\} \in \mathcal{A}$. Now, define $g_{N_{\rm tr}} : (V, \rho_{N_{\rm tr}}) \longrightarrow (W, \omega_{N_{\rm tr}})$ as $g_{N_{\rm tr}}(x) = p$ and $g_{N_{\rm tr}}(y) = q$. Then $g_{N_{\rm tr}}(M) = \{< p, 0.2, 0.4, 0.7 >< q, 0.2, 0.3, 0.8 >\} \in \mathcal{P}$ and
$g^{-1}_{N_{\rm tr}}(N) = \{< x, 0.1, 0.4, 0.7 >< y, 0.3, 0.7, 0.8 >\} \in \mathcal{Y}$. This implies that both $f_{N_{\rm tr}}$ and $g_{N_{\rm tr}}$ are $N_{\rm tr}\Lambda_P$-homeomorphism. Now, let $g_{N_{\rm tr}} \circ f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (W, \omega_{N_{\rm tr}})$ be the composition of two $N_{\rm tr}\Lambda_P$-homeomorphisms. Then, $g_{N_{\rm tr}} \circ f_{N_{\rm tr}}$ is not a $N_{\rm tr}\Lambda_P$-homeomorphism since $(g_{N_{\rm tr}} \circ f_{N_{\rm tr}})^{-1}(N) = f^{-1}_{N_{\rm tr}}(g^{-1}_{N_{\rm tr}}(N)) = \{< a, 0.1, 0.4, 0.7 >< b, 0.3, 0.7, 0.8 >\}$is not $N_{\rm tr}\Lambda_P$-open in $(U, \tau_{N_{\rm tr}})$.

Theorem 4.9. *Let*$f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ *and* $g_{N_{tr}} : (V, \rho_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ *be* $N_{tr}\Lambda_P$*-homeomorphisms where* $(V, \rho_{N_{tr}})$ *is a* $N_{tr}T_{\Lambda_P}$*-space. Then, their composition* $g_{N_{tr}} \circ f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ *is also a* $N_{tr}\Lambda_P$*-homeomorphism.*
Proof. Let K be a neutrosophic set in W. Since $g_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$-homeomorphism, $g_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-continuous. This implies $g^{-1}_{N_{\rm tr}}(K)$ is $N_{\rm tr}\Lambda_P$-open in V. By hypothesis,

$g_{N_{\rm tr}}^{-1}(K)$ is $N_{\rm tr}$-open in V. Also, since $f_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$-homeomorphism, $f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-continuous. Hence $(g_{N_{\rm tr}} \circ f_{N_{\rm tr}})^{-1} = f_{N_{\rm tr}}^{-1}(g_{N_{\rm tr}}^{-1}(K))$ is $N_{\rm tr}\Lambda_P$-open in U. Therefore $g_{N_{\rm tr}} \circ f_{N_{\rm tr}}$ is $N_{\rm tr}\Lambda_P$-continuous. Similarly, $(g_{N_{\rm tr}} \circ f_{N_{\rm tr}})^{-1}$ is $N_{\rm tr}\Lambda_P$-continuous. Hence $g_{N_{\rm tr}} \circ f_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$-homeomorphism.

5. Neutrosophic $\Lambda_{\mathbf{P}}$-i homeomorphism

Definition 5.1. *A bijective map $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ is said to be a* ***neutrosophic $\Lambda_{\mathbf{P}}$-i homeomorphism*** *if $f_{N_{tr}}$ and $f_{N_{tr}}^{-1}$ are $N_{tr}\Lambda_P$-irresolute.*

Example 5.2. Let $U = \{a, b\}$, $V = \{x, y\}$, $\tau_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, K\}$ and $\rho_{N_{\rm tr}} = \{0_{N_{\rm tr}}, 1_{N_{\rm tr}}, M\}$ Where $K = \{< a,\ 0.7, 0.8, 0.2 >< b, 0.9, 0.6, 0.4 >\}$ and $M = \{< x,\ 0.9, 0.6, 0.4 >< y, 0.7, 0.8, 0.2 >\}$. Consider the collection
$\mathcal{A} = \{A\ :\ K^C \subset A \subset K\}$ and $\mathcal{B} = \{B\ :\ K \subset B \subset 1_{N_{\rm tr}}\}$ of neutrosophic sets in U and V respectively. Here, $N_{\rm tr}\Lambda_P O(U, \tau_{N_{\rm tr}}) = \{0_{N_{\rm tr}}, K, \mathcal{A}, 1_{N_{\rm tr}}\}$ and $N_{\rm tr}\Lambda_P O\,(V,\ \rho_{N_{\rm tr}}) = \{0_{N_{\rm tr}}, M, \mathcal{B}, 1_{N_{\rm tr}}\}$. Define $f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (V, \rho_{N_{\rm tr}})$ as $f_{N_{\rm tr}}(a) = y$ and $f_{N_{\rm tr}}(b) = x$. Then $f_{N_{\rm tr}}^{-1} : (V, \rho_{N_{\rm tr}}) \longrightarrow (U, \tau_{N_{\rm tr}})$ is defined as $f_{N_{\rm tr}}^{-1}(x) = b$ and $f_{N_{\rm tr}}^{-1}(y) = a$. Now, $f_{N_{\rm tr}}^{-1}(M) = \{< a, 0.7,\ 0.8,\ 0.2 >< b, 0.9, 0.6, 0.4 >\} = K$ and for every neutrosophicset $B \in \mathcal{B}$, there exists some$A \in \mathcal{A}$ such that $f_{N_{\rm tr}}^{-1}(B) = A$. Also, $(f_{N_{\rm tr}}^{-1})^{-1}(K) = \{< x, 0.9,\ 0.6,\ 0.4 >< y, 0.7, 0.8, 0.2 >\} = M$ and for every neutrosophic set $A \in \mathcal{A}$, there exists some $B \in \mathcal{B}$ such that $(f_{N_{\rm tr}}^{-1})^{-1}(A) = B$. Clearly, $f_{N_{\rm tr}}$is a bijection and both $f_{N_{\rm tr}}$ and $f_{N_{\rm tr}}^{-1}$ are $N_{\rm tr}\Lambda_P$-irresolute. Hence $f_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$-i homeomorphism.

Theorem 5.3. *Every $N_{tr}\Lambda_P$-i homeomorphism is a $N_{tr}\Lambda_P$-homeomorphism.*
Proof. Let $f_{N_{\rm tr}} : (U, \tau_{N_{\rm tr}}) \longrightarrow (V, \rho_{N_{\rm tr}})$ be a $N_{\rm tr}\Lambda_P$-i homeomorphism. Then, both $f_{N_{\rm tr}}$ and $f_{N_{\rm tr}}^{-1}$ are $N_{\rm tr}\Lambda_P$-irresolute. Now, by theorem 2.15, both $f_{N_{\rm tr}}$ and $f_{N_{\rm tr}}^{-1}$ are $N_{\rm tr}\Lambda_P$-continuous. Hence $f_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$-homeomorphism.

Remark 5.4. *The converse of theorem 5.3 does not hold in general.*

Example 5.5. Consider the topological spaces and bijection $f_{N_{\rm tr}}$ defined in example 4.2. Here both $f_{N_{\rm tr}}$ and $f_{N_{\rm tr}}^{-1}$ are $N_{\rm tr}\Lambda_P$-continuous. However $f_{N_{\rm tr}}$ is not $N_{\rm tr}\Lambda_P$-irresolute since $f_{N_{\rm tr}}^{-1}(M^c) = \{< a, 0.8, 0.7, 0.2 >< b, 0.9, 0.8, 0.3 >\} \notin N_{\rm tr}\Lambda_P O\,(U, \tau_{N_{\rm tr}})$. Hence $f_{N_{\rm tr}}$ is a $N_{\rm tr}\Lambda_P$- homeomorphism but not a $N_{\rm tr}\Lambda_P$-i homeomorphism. We denote the family of all $N_{\rm tr}$-homeomorphisms (resp. $N_{\rm tr}\Lambda_P$-homeomorphism, $N_{\rm tr}\Lambda_P$-i homeomorphism) from the topological space $(U, \tau_{N_{\rm tr}})$ into $(U, \tau_{N_{\rm tr}})$ by $N_{\rm tr}h\,(U, \tau_{N_{\rm tr}})$ (resp.$N_{\rm tr}\Lambda_P h\,(U, \tau_{N_{\rm tr}})$, $N_{\rm tr}\Lambda_P{\rm ih}\,(U, \tau_{N_{\rm tr}})$).

Theorem 5.6. *Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ be a bijection between two neutrosophic topological spaces. Then, the following are equivalent:*

i. $f_{N_{tr}}$ *is a* $N_{tr}\Lambda_P$*-ihomeomorphism.*

ii. $f^{-1}_{N_{tr}}\left(N_{tr}\Lambda_P int(K)\right) = N_{tr}\Lambda_P int\left(f^{-1}_{N_{tr}}(K)\right)$ *for every neutrosophic set* K *in* V.

iii. $f_{N_{tr}}\left(N_{tr}\Lambda_P int(K)\right) = N_{tr}\Lambda_P int\left(f_{N_{tr}}(K)\right)$ *for every neutrosophic set* K *in* U.

iv. $N_{tr}\Lambda_P cl\left(f_{N_{tr}}(K)\right) = f_{N_{tr}}\left(N_{tr}\Lambda_P cl(K)\right)$ *for every neutrosophic set* K *in* U.

v. $N_{tr}\Lambda_P cl\left(f^{-1}_{N_{tr}}(K)\right) = f^{-1}_{N_{tr}}\left(N_{tr}\Lambda_P cl(K)\right)$ *for every neutrosophic set* K *in* V.

Proof. **(i)**$\Longrightarrow$**(ii)** Since$f_{N_{\mathrm{tr}}}$ is a $N_{\mathrm{tr}}\Lambda_P$-$i$homeomorphism, $f_{N_{\mathrm{tr}}}$ is $N_{\mathrm{tr}}\Lambda_P$-irresolute. Now, let K be a neutrosophic set in V. Then $N_{\mathrm{tr}}\Lambda_P int(K)$ is $N_{\mathrm{tr}}\Lambda_P$-open set in V. Since $f_{N_{\mathrm{tr}}}$ is $N_{\mathrm{tr}}\Lambda_P$- irresolute, $f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P int(K)\right)$ is $N_{\mathrm{tr}}\Lambda_P$-open in U. Also, $f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P int(K)\right) \subseteq f^{-1}_{N_{\mathrm{tr}}}(K)$. Thus, $f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}(K)\right) \subseteq N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)$ for every neutrosophic set K in V. Again, since $f_{N_{\mathrm{tr}}}$ is a $N_{\mathrm{tr}}\Lambda_P$-$i$homeomorphism, $f^{-1}_{N_{\mathrm{tr}}} : (V, \rho_{N_{\mathrm{tr}}}) \longrightarrow (U, \tau_{N_{\mathrm{tr}}})$ is $N_{\mathrm{tr}}\Lambda_P$-irresolute. Now, for any neutrosophic set K in V, $N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)$is $N_{\mathrm{tr}}\Lambda_P$-open in U.Then, $\left(f^{-1}_{N_{\mathrm{tr}}}\right)^{-1}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)\right) = f_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)\right)$ is $N_{\mathrm{tr}}\Lambda_P$-open in V. Also, $f_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)\right) \subseteq f_{N_{\mathrm{tr}}}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right) \subseteq K$. Therefore, $f_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)\right) \subseteq N_{\mathrm{tr}}\Lambda_P int(K) \Longrightarrow N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right) \subseteq f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}(K)\right)$ for every neutrosophic set K in V. Hence $f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}(K)\right) = N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)$ for every neutrosophic set K in V.

(ii)$\Longrightarrow$**(iii)** Let $M = f_{N_{\mathrm{tr}}}(K)$ be a neutrosophic set in V.

By (ii), $f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f_{N_{\mathrm{tr}}}(K)\right)\right) = f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}(M)\right) = N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}(M)\right) = N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f^{-1}_{N_{\mathrm{tr}}}\left(f_{N_{\mathrm{tr}}}(K)\right)\right) = N_{\mathrm{tr}}\Lambda_P \mathrm{int}(K)$.
This implies $N_{\mathrm{tr}}\Lambda_P \mathrm{int}\left(f_{N_{\mathrm{tr}}}(K)\right) = f_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{int}(K)\right)$ for every neutrosophic set K in U.

(iii)$\Longrightarrow$**(iv)** For any neutrosophic set K in U,$N_{\mathrm{tr}}\Lambda_P \mathrm{cl}(K) = (N_{\mathrm{tr}}\Lambda_P int(K^C))^C$. Then, $f_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{cl}(K)\right) = f_{N_{\mathrm{tr}}}(N_{\mathrm{tr}}\Lambda_P int(K^C))^C = (f_{N_{\mathrm{tr}}}(N_{\mathrm{tr}}\Lambda_P int(K^C)))^C$
$=(N_{\mathrm{tr}}\Lambda_P int(f_{N_{\mathrm{tr}}}\left(K^C\right))^C = N_{\mathrm{tr}}\Lambda_P \mathrm{cl}\left(f_{N_{\mathrm{tr}}}\left(K^C\right)^C\right) = N_{\mathrm{tr}}\Lambda_P \mathrm{cl}\left(f_{N_{\mathrm{tr}}}(K)\right)$.
Hence $f_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{cl}(K)\right) = N_{\mathrm{tr}}\Lambda_P \mathrm{cl}\left(f_{N_{\mathrm{tr}}}(K)\right)$ for every neutrosophic set K in U.

(iv)$\Longrightarrow$**(v)** Let $K = f_{N_{\mathrm{tr}}}(M)$ be a neutrosophic set in V. By (iv),
$f_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{cl}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)\right) = N_{\mathrm{tr}}\Lambda_P \mathrm{cl}\left(f_{N_{\mathrm{tr}}}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)\right) = N_{\mathrm{tr}}\Lambda_P cl(K)$ implies
$N_{\mathrm{tr}}\Lambda_P \mathrm{cl}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right) = f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{cl}(K)\right)$ for every neutrosophic set K in V.

(v)$\Longrightarrow$**(i)** By theorem 2.16,$f_{N_{\mathrm{tr}}}$ is $N_{\mathrm{tr}}\Lambda_P$-irresolute if
$N_{\mathrm{tr}}\Lambda_P \mathrm{cl}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right) \subseteq f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{cl}(K)\right)$ for every neutrosophic set K in V. Similarly, $f^{-1}_{N_{\mathrm{tr}}}$ is $N_{\mathrm{tr}}\Lambda_P$-irresolute if $f^{-1}_{N_{\mathrm{tr}}}\left(N_{\mathrm{tr}}\Lambda_P \mathrm{cl}(K)\right) \subseteq N_{\mathrm{tr}}\Lambda_P \mathrm{cl}\left(f^{-1}_{N_{\mathrm{tr}}}(K)\right)$ for every

neutrosophic set K in V. Now, by assumption, both $f_{N_{\text{tr}}}$ and $f^{-1}_{N_{\text{tr}}}$ are $N_{\text{tr}}\Lambda_P$-irresolute. Hence $f_{N_{\text{tr}}}$ is a $N_{\text{tr}}\Lambda_P$-i homeomorphism.

Theorem 5.7. *Let $f_{N_{tr}} : (U, \tau_{N_{tr}}) \longrightarrow (V, \rho_{N_{tr}})$ and $g_{N_{tr}} : (V, \rho_{N_{tr}}) \longrightarrow (W, \omega_{N_{tr}})$ be two $N_{tr}\Lambda_P$-i homeomorphisms. Then, their composition $g_{N_{tr}} \circ f_{N_{tr}}$ is also a $N_{tr}\Lambda_P$-i homeomorphism.*

Proof. By hypothesis, the functions $f_{N_{\text{tr}}}$, $g_{N_{\text{tr}}}$ and $f^{-1}_{N_{\text{tr}}}$, $g^{-1}_{N_{\text{tr}}}$ are all $N_{\text{tr}}\Lambda_P$-irresolute. Then, by theorem 2.17, both $g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}$ and $(g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}})^{-1} = f^{-1}_{N_{\text{tr}}} \circ g^{-1}_{N_{\text{tr}}}$ are $N_{\text{tr}}\Lambda_P$-irresolute. Hence $g_{N_{\text{tr}}} \circ f_{N_{\text{tr}}}$ is a $N_{\text{tr}}\Lambda_P$-i homeomorphism.

6. Conclusion

This article defined and examined some of the properties of $N_{\text{tr}}\Lambda_P$- homeomorphism and $N_{\text{tr}}\Lambda_P$-i homeomorphism in neutrosophic topological spaces. Additionally, the study was expanded to include discussion of $N_{\text{tr}}\Lambda_P$-open and $N_{\text{tr}}\Lambda_P$-closed mappings. Numerous instances are provided to support the findings. This concept can be used to drive few more new results of $N_{\text{tr}}\Lambda_P$-connectedness and compactness in neutrosophic topological spaces. Also, this study will be extended to separation axioms, normal and regular spaces using $N_{\text{tr}}\Lambda_P$-open sets in neutrosophic topological spaces

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