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SUPRA R-OPEN SOFT SETS AND SUPRA R-CONTINUOUS (R^*-CONTINUOUS) SOFT MAP

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Abstract: In the paper a new class of generalized supra open soft sets called supra R-open soft sets are introduced and investigated the properties of supra R-open (R-closed) soft sets and supra soft R-interior (closure). The relationships between some generalized supra open soft sets and this class are investigated and illustrated with examples. Also, new types of supra continuous soft maps called supra R-continuous (R^*-continuous) soft maps are studied depending on the concept of supra R-open soft sets.

Keywords and Phrases: Soft topological space, supra R-open soft set, supra soft R-interior, supra soft R-closure, supra R-continuous soft map, supra R^*-continuous soft map.

2020 Mathematics Subject Classification: 54A99.

1. Introduction

The classical mathematical approaches are often insufficient for modelling problems with uncertain data. There are several theories such as fuzzy sets, theory of intuitionistic fuzzy sets, theory of vague sets and theory of rough sets which can be considered as tools for dealing with uncertainties. But these theories have their own difficulties due to the inadequacy of the parametrization tool of the theories as pointed out by Molodstov. In 1999, Molodtsov [10] introduced soft set theory as a mathematical tool for solving complex problems dealing with uncertainties.

Recently research works on soft set theory and its applications in various fields are advancing rapidly.

In the year 2011, Shabir and Naz [11] introduced soft topological spaces which are defined over an initial universal set with a fixed set of parameters. Sabir Huzzain and Bashir Ahmad [5] discussed properties of soft interior, soft closure, soft exterior and soft boundary in 2011. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [11]. They also introduced new different types of subsets of supra soft topological spaces and study the relations between them in detail.

In this work, we first present the notion of supra R-open soft sets as a new class of generalized supra open sets. Then we introduce the concept of supra R-continuous and supra R* continuous maps. We begin with some basic definitions and results which are essential for our study.

2. Preliminaries

Definition 2.1. [11] *Let τ be collection of soft sets (F, A) over X. Then τ is said to be a soft topology on X if*

1 ϕ_A, $\tilde{X}_A \in \tau$

2 Soft union of any number of soft sets in τ belongs to τ

3 Soft intersection of two soft sets in τ belongs to τ

Then (X, τ, A) is called soft a topological space over X. The members of τ are called soft open sets.

Definition 2.2. [11] *A soft set (F, A) over X is said to be a soft closed set if its relative complement $(F, A)' \in \tau$.*

Proposition 2.1. [11] *Let (X, τ, A) be a soft topological space over X. Then*

1. *ϕ_A, $\tilde{X}_A$ are soft closed sets over X*
2. *The soft intersection of any number of soft closed sets is a soft closed set*
3. *The soft union of two soft closed set is a soft closed set.*

Proposition 2.2. [11] *Let (X, τ, A) be soft a topological space over X. Then $\tau_\alpha = \{F(\alpha) / (F, A) \in \tau\}, \forall \alpha \in A$ is a topology on X.*

Definition 2.3. [11] *Let (X, τ, A) be a soft topological space over X and (F, A) be a soft set over X. Then soft closure of (F, A), denoted by $\overline{(F, A)}$, is the intersection* *of all soft closed sets containing* (F, A).

Theorem 2.4. [11] *Let* (X, τ, A) *be a soft topological space over* X *and let* (F, A) *and* (G, A) *are soft sets over* X

1 $\overline{\phi_A} = \phi_A$ *and* $\overline{\tilde{X_A}} = \tilde{X_A}$

2 $(F, A) \subseteq \overline{(F, A)}$

3 (F, A) *is a soft closed set if and only if* $\overline{(F, A)} = (F, A)$

4 $\overline{\overline{(F, A)}} = \overline{(F, A)}$

5 $(F, A) \subseteq (G, A) \Rightarrow \overline{(F, A)} \subseteq \overline{(G, A)}$

6 $\overline{(F, A) \cup (G, A)} = \overline{(F, A)} \cup \overline{(G, A)}$

7 $\overline{(F, A) \cap (G, A)} \subseteq \overline{(F, A)} \cap \overline{(G, A)}$.

Definition 2.5. [11] *Let* (X, τ, A) *be a soft topological space over* X *and let* (F, A) *be a soft set over* X. *Then* $\left(\overline{F}, A\right)$ *is a soft set over* X *defined by* $\overline{F}(\alpha) = \overline{F(\alpha)}, \forall\, \alpha \in A$, *where* $\overline{F(\alpha)}$ *is the closure of* $F(\alpha)$ *in* τ_α.

Proposition 2.3. [11] *Let* (X, τ, A) *be a soft topological space over* X *and let* (F, A) *be a soft set over*X. *Then* $\left(\overline{F}, A\right) \subseteq \overline{(F, A)}$.

Corollary 2.6. [11] $\left(\overline{F}, A\right) = \overline{(F, A)}$ *if and only if* $\left(\overline{F}, A\right)' \in \tau$.

Definition 2.7. [11] *Let* (X, τ, A) *be a soft topological space over* X *and let* (F, A) *be a soft set over* X *and* $x \in X$.*Then* x *is said to be a soft interior point of* (F, A) *if there exists a soft open set* (G, A) *such that* $x \in (G, A) \subseteq (F, A)$.

Definition 2.8. [11] *Let* (X, τ, A) *be a soft topological space over* $X, (F, A)$ *be a soft set over* X *and* $x \in X$. *Then* (F, A) *is said to be soft neighbourhood of* x *if there exists a soft open set* (G, A) *such that* $x \in (G, A) \subseteq (F, A)$.

Proposition 2.4. [11] *Let* (X, τ, A) *be a soft topological space over* X, *let* (F, A) *be a soft set over* X *and* $x \in X$. *Then if* x *is a soft interior point of* (F, A) *then* x *is an interior point of* $F(\alpha)$ *in* $(X, \tau_\alpha), \forall\, \alpha \in A$.

Definition 2.9. [5] *Let* (X, τ, A) *be a soft topological space over* X *and let* (F, A) *be a soft set over* X. *Then soft interior of* (F, A) *is the soft union of all soft open set contained in* (F, A). *Soft interior of* (F, A) *is denoted by* $(F, A)^o$.
Clearly, $(F, A)^o$ *is the largest soft open set contained in* (F, A).

Theorem 2.10. [5] *Let (X, τ, A) be a soft topological space over X and let (F, A) and (G, A) be soft sets over X. Then*

1 $\phi_A^o = \phi_A, \tilde{X_A}^o = \tilde{X_A}$

2 $(F, A)^o \subseteq (F, A)$

3 $[(F, A)^o]^o = (F, A)^o$

4 (F, A) *is soft open set if and only if* $(F, A)^o = (F, A)$.

5 $(F, A)^o \cap (G, A)^o = [(F, A) \cap (G, A)]^o$

6 $(F, A)^o \cup (G, A)^o \subseteq [(F, A) \cup (G, A)]^0$

7 $(F, A) \subseteq (G, A) \Rightarrow (F, A)^o \subseteq (G, A)^o$.

Definition 2.11. [5] *Let (X, τ, A) be a soft topological space over X and let (F, A) be a soft set over X. Then (F^o, A) is a soft set defined by $F^o(\alpha) = (F(\alpha))^o$, where $(F(\alpha))^o$ is the interior of $F(\alpha)$ in $\tau_\alpha, \forall\, \alpha \in A$.*

Proposition 2.5. *Let (X, τ, A) be a soft topological space over X and let (F, A) be a soft set over X. Then $(F, A)^o \subseteq (F^o, A)$.*

Corollary 2.12. $(F, A)^o = (F^o, A)$ *if and only if* $(F^o, A) \in \tau$.

Definition 2.13. [2] *Let $S_E(X)$ and $S_E(Y)$ be families of soft sets over X and Y respectively. Let $u : X \mapsto Y$ and $p : E \mapsto K$ be mappings. We define a soft mapping $f_{pu} : S_E(X) \mapsto S_K(Y)$ as follows:*

- *If $(F, E) \in S_E(X)$, then the image of (F, E) under f_{pu}, written as $f_{pu}(F, E)$ is a soft set in $S_K(Y)$ such that:*

$$f_{pu}(F, E)(k) = \begin{cases} \cup\{u[F(e] : e \in p^-1(k)\} & if \quad p^-1(k) \neq \phi \\ \phi & if \quad p^-1(k) = \phi \end{cases}$$

- *If $(H, K) \in S_K(Y)$, then the inverse of (H, K) under f_{pu}, written as $f_{pu}^{-1}(H, K)$ is a soft set in $S_E(X)$ such that*

$$f_{pu}^{-1}(H, K)(e) = u^{-1}[H(p(e)] \text{ for all } e \in E$$

Definition 2.14. [13] *Let* (X, τ_1, E) *and* (Y, τ_2, K) *be soft topological spaces and let* $f_{pu} : S_E(X) \mapsto S_K(Y)$ *be a soft map. Then* f_{pu} *is called continuous soft map if* $f_{pu}^{-1}(G, K) \in \tau_1,$ *for all* $(G, K) \in \tau_2$.

Definition 2.15. [8] *Let* (X, τ, E) *be soft topological space over* X *and let* $(F, E) \in S_E(X)$. *Then,*

1. *Pre open soft set if* $(F, E) \subseteq Int(Cl(F, E))$.
2. *Semi open soft set if* $(F, E) \subseteq Cl(Int(F, E))$.
3. α*-open soft set if* $(F, E) \subseteq Int(Cl(Int(F, E)))$.
4. β*-open soft set if* $(F, E) \subseteq Cl(Int(Cl(F, E)))$.
5. *b-open soft set if* $(F, E) \subseteq Cl(Int(F, E)) \cup Int(Cl(F, E))$.

Definition 2.16. [4] *Let* $S_E(X)$ *be collection of all soft sets over a universe* X *with a fixed set of parameters* E. *Then* $\mu \subseteq S_E(X)$ *is called supra soft topology on* X *with fixed set of parameters* E *if,*

1. $\tilde{\phi},\ \tilde{X} \in \mu$.
2. *The soft union of any number of members in* μ *belongs to* μ.

Then the triplet (X, μ, E) *is called supra soft topological space and the members of* μ *are called the supra open soft sets over* X.

Definition 2.17. [4] *Let* (X, μ, E) *be supra soft topological space over* X *and let* $(F, E) \in S_E(X)$. *Then the supra soft interior of* (F, E), *denoted by* $Int^*(F, E)$, *is the soft union of all supra open soft subsets of* (F, E). *Clearly,* $Int^*(F, E)$ *is the largest supra soft open set over* X *contained in* (F, E).

Definition 2.18. [4] *Let* (X, μ, E) *be supra soft topological space over* X *and let* $(F, E) \in S_E(X)$. *Then the supra soft closure of* (F, E), *denoted by* $Cl^*(F, E)$, *is the soft intersection of all supra closed soft super sets of* (F, E). *Clearly,* $Cl^*(F, E)$ *is the smallest supra soft closed set over* X *containing* (F, E).

3. R-open soft set

In this section we introduce the notion of supra R-open soft set and introduce its relationships with other generalized supra open sets. Also we explicit some of its basic properties.

Definition 3.1. *Let* $(X, A, \tau^*,)$ *be supra soft topological space. A non empty soft*

set (F, A) over X is called supra R-open soft set if there exist a non empty supra open soft set (G,A) such that $(G, A) \subseteq Cl^*(F, A)$. Empty soft set is supra R-open soft set.
The complement of supra R-open soft set (G, A) is called supra R-closed soft set.

Theorem 3.2. *Let (X, A, τ^*) be a supra soft topological space. A soft subset (F, A) over X is supra R-closed soft set if and only if there exist a supra closed soft set (G, A) such that $Int^*(F, A) \subseteq (G, A)$.*
Proof. Suppose (F, A) over X is a supra R-closed soft set. Then by definition there exists a supra open soft set (G, A) such that

$$\begin{aligned} &(G, A) \subseteq Cl^*(F, A)^{'}. \\ \Longrightarrow\ &(G, A)^{'} \supseteq (Cl^*(F, A)^{'})^{'}. \\ \Longrightarrow\ &Int^*(F, A) \subseteq (G, A)^{'}. \end{aligned}$$

Conversely, assume there exists a supra closed soft set (G, A) such that $Int^*(F, A) \subseteq (G, A)$. Then $(G, A)^{'} \subseteq (Int^*(F, A))^{'} = Cl^*(F, A)^{'}$ so that (F, A) is a supra R-closed soft set.

Theorem 3.3. *Every supra open soft sets are supra R-open soft sets.*
Proof. The proof is trivial from the definition.

Remark 3.1. *Every supra R-open set need not be supra open soft set.*

Example 1. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and let $\tau^* = \{\tilde{\phi_E}, \tilde{X}_E, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$, where

$$\begin{aligned} (F_1, E) &= \{(e_1, \{x_1\}), (e_2, \{x_2\}\} \\ (F_2, E) &= \{(e_1, \{x_1, x_2\}), (e_2, \{x_3\})\}. \\ (F_3, E) &= \{(e_1, \{x_2\}), (e_2, \{x_1, x_3\})\}. \\ (F_4, E) &= \{(e_1, \{x_1, x_2\}), (e_2, X)\} \\ (F_5, E) &= \{(e_1, \{x_1, x_2\}), (e_2\{x_2, x_3\})\} \\ (F_6, E) &= \{(e_1, \{x_1, x_2\}), (e_2\{x_1, x_3\})\} \end{aligned}$$

Then (X, τ^*, E) is supra soft topological space. We can see that $(F, E) = \{(e_1, \{x_2, x_3\}), (e_2, \{x_1, x_3\})\}$ is a supra R-open soft set, but not a supra open soft set.

Theorem 3.4. *Let (X, μ, E) be supra soft topological space over X. Then*

1. *Every supra b-open soft set is supra R-open soft set.*

2 *Every supra β-open soft sets are supra R-open*

Proof. Let (F,A) be soft set over X.

1. Let (F,A) be supra b-open soft set. Then

$$Int^*(Cl^*(F,A)) \subseteq Int^*(Cl^*(F,A)) \cup Cl^*(Int^*(F,A)) \subseteq Cl^*(F,A).$$

Hence (F,A) is supra R-open soft set.

2. Let (F,A) be supra β-open soft set.Then

$$\begin{aligned} &(F,A) \subseteq Cl^*\big(Int^*(Cl^*(F,A))\big) \\ \Longrightarrow\ &Int^*(Cl^*(F,A)) \subseteq Cl^*(F,A) \subseteq Cl^*(Int^*(Cl^*(F,A))). \end{aligned}$$

Hence (F,A) is supra R-open soft set.

Remark 3.2. *In* [1] *the authors proved that every supra pre-open, supra semi open, and supra α-open soft sets are supra b-open soft set. Hence we can deduce that they are supra R-open soft set.*

Remark 3.3. *But every supra R-open soft set need not be a supra b-open soft set.*

Example 2. In the Example 1, the soft set $(F,E) = \{(e_1,\{x_2,x_3\}),(e_2,\{x_1\})\}$ is supra R-open soft set, but not a supra b-open soft set.

Remark 3.4. *Every supra R open sets are need not be β-open soft set.*

Example 3. Let $X = \{x,y,z\}$, $E = \{e_1,e_2\}$ and let $\tau^* = \{\tilde{\phi_E}, \tilde{X_E}, (F_1,E)$, $(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E)\}$, where

$$\begin{aligned} (F_1,E) &= \{(e_1,\{x\}),(e_2\{x\})\} \\ (F_2,E) &= \{(e_1,\{y\}),(e_2,\{y\})\} \\ (F_3,E) &= \{(e_1,\{x,y\}),(e_2,\{x,y\})\} \\ (F_4,E) &= \{(e_1,\{y,z\}),(e_2,\{y,z\})\} \\ (F_5,E) &= \{(e_1,\{z\}),(e_2,X)\} \\ (F_6,E) &= \{(e_1,\{x,z\}),(e_2,X)\} \\ (F_7,E) &= \{(e_1,\{y,z\}),(e_2,X)\} \end{aligned}$$

Then τ^* is a supra soft topology on X. We can see that $(F,E) = \{(e_1,\{x\}),(e_2,\{z\})\}$ is a supra R-open soft set, but not a supra β-open soft set.

Definition 3.5. *Let (X,τ^*,A) be a supra soft topological space and let $x \in X$. Then (F,A) is a supra soft neighbourhood of (x_e,A), if there exits a supra open* *soft set (G, A) such that $(x_e, A) \in (G, A) \subseteq (F, A)$.*

Theorem 3.6. *Every supra neighbourhood of a soft point is supra soft R-open.*
Proof. Let (F, A) be a soft neighbourhood of the soft point (x_e, A). Then there exists a supra open set (G, A) such that

$$\begin{aligned} & (x_e, A) \in (G, A) \subseteq (F, A)\,. \\ \implies & (G, A) \subseteq Cl^*\,(G, A) \subseteq Cl^*\,(F, A)\,. \\ \implies & (F, A) \quad \text{is a supra R-open soft set.} \end{aligned}$$

Remark 3.5. *The converse of the above theorem need not be true.*

Example 4. In Example 1, $(F, E) = \{(e_1, \{x_2\}), (e_2, \{x_3\})\}$ is a supra R-open soft set, but not a supra neighbourhood of any soft point.

Theorem 3.7. *Let (F, A) be a supra R-open set on a supra soft topological space (X, τ^*, A). Then every proper soft super set of (F, A) is a supra R-open soft set.*
Proof. The proof is trivial from the definition.

Theorem 3.8. *Let (X, τ^*, A) be a supra soft topological space. Then arbitrary union of supra R-open sets is a supra R-open soft set.*
Proof. Let $\{(F_i, A) : i \in I\}$ be a family of supra R-open soft sets in X. Then there exists a supra open set (G, A) such that

$$(G, A) \subseteq Cl^*\,(F_i, A) \subseteq Cl^* \cup (F_i, A)\,.$$

Hence $\cup\,(F_i, A)$ is a supra R-open soft set.

Remark 3.6. *Intersection of finite supra R-open soft sets need not be supra R-open.*

Example 5. In Example 1, $(G_1, E) = \{(e_1, \{x_2, x_3\}), (e_2, \{x_3\})\}$ and $(G_2, E) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2\})\}$ are supra R-open soft sets. But $(G_1, E) \cap (G_2, E) = \{(e_1, \{x_3\}), (e_2, \phi)\}$ is not a supra R-open soft set.

Theorem 3.9. *Let (X, τ^*, A) be a supra soft topological space over X. Then the intersection of arbitrary supra R-closed soft sets is a supra R-closed soft set.*
Proof. Let $\{(F_i, A)\}$ be a family of supra R-closed soft sets. Then,

$$[\cap\,(F_i, A)]' = \cup\,(F_i, A)'$$

is a supra R-closed soft set. Hence $\cap\,(F_i, A)$ is a supra R-closed soft set.

Definition 3.10. *Let (X, τ^*, A) be a supra soft topological space over X and let (F, A) be a soft set over X. Then,*

(1) *The supra soft R-interior of (F, A) is the union of all supra R-open soft sets contained in (F, A) and is denoted by $int^s_R(F, A)$.*

(2) *The supra soft R-closure of (F, A) is the intersection of all supra R-closed soft sets containing (F, A) and is denoted by $Cl^s_R(F, A)$.*

Theorem 3.11. *Let (X, τ^*, A) be a supra soft topological space over X. Then*

(1) *$Int^s_R(F, A) \subseteq (F, A)$ and $(F, A) = Int^s_R(F, A)$ if and only if (F, A) is a supra R-open soft set.*

(2) *$Cl^s_R(F, A) \supseteq (F, A)$ and $(F, A) = Cl^s_R(F, A)$ if and only if (F, A) is a supra soft R-closed set.*

(3) *$X - Int^s_R(F, A) = Cl^s_R(F, A)^{'}$.*

Proof. Proof of [(1)] and [(2)] are trivial from the definition.

(3)

$$\begin{aligned}
& Int^s_R(F, A) \subseteq (F, A) \\
& \Longrightarrow [Int^s_R(F, A)]^{'} \supseteq (F, A)^{'} \\
& \Longrightarrow [Int^s_R(F, A)]^{'} \supseteq Cl^s_R(F, A)^{'} \\
\text{Also we have, } & (F, A)^{'} \subseteq Cl^s_R(F, A)^{'} \\
& \Longrightarrow (F, A) \supseteq \left[Cl^s_R(F, A)^{'}\right]^{'} \\
& \Longrightarrow \left[Cl^s_R(F, A)^{'}\right]^{'} \subseteq Int^s_R(F, A) \\
& \Longrightarrow Cl^s_R(F, A)^{'} \supseteq [Int^s_R(F, A)]_{'}.
\end{aligned}$$

Hence $[Int^s_R(F, A)]^{'} = Cl^s_R(F, A)^{'}$.

Definition 3.12. *Let (X, τ^*, A) be a supra soft topological space. Let (F, A) be a soft set and e_H be a soft point. Then*

(1) *e_H is called a supra R interior soft point of (F, A) if there exists a supra R-open soft set (G, A) such that $e_H \in (G, A) \subseteq (F, A)$.*

(2) *e_H is called a supra R-closure soft point of (F, A) if $(G, A) \cap (F, A) \neq \phi_A$, for every supra R-open soft set (G, A) containing e_H.*

Theorem 3.13. *Let (X, τ^*, A) be a supra soft topological space over X. Then*

(1) The set of all supra R-interior soft points of (F, A) is the supra soft R-interior of (F, A).

(2) The set of all supra R-closure soft points of (F, A) is the supra soft R-closure of (F, A).

Proof.

(1) Let e_H be a supra R-interior soft point of (F, A). Then there exists a supra R-open soft set (G, A) such that $e_H \in (G, A) \subseteq (F, A)$. Clearly $e_H \in Int^s_R(F, A)$.
Conversely, assume e_H be a soft point in Int^s_R. Then Int^s_R is a supra R-open soft set such that $e_H \in Int^s_R \subseteq (F, A)$, so that e_H is supra soft R-interior point of (F, A).

(2) Let e_H be a supra R-closure soft point of (F, A).Then for any supra R-open soft set (G, A) containing e_H, $(G, A) \cap (F, A) \neq \phi_A$. Let (M, A) be a supra R-closed soft set such that $(F, A) \subseteq (M, A)$. Then $(M, A)^{'}$ is a supra R-open soft set such that $(M, A)^{'} \cap (F, A) = \phi$. Therefore, $e_H \in (M, A)$, so that $e_H \in Cl^s_R(F, A)$.
Conversely, assume $e_H \in Cl^s_R(F, A)$. Let (G, A) be any supra R-open soft set containing e_H. Then $(F, A) \nsubseteq (G, A)^{'}$, so that $(F, A) \cap (G, A) \neq \phi_A$.

From the definition of supra soft R-interior and supra soft R-closure, we can prove the following theorems trivially.

Theorem 3.14. *Let (X, τ^*, A) be a supra soft topological space and $(F, A), (G, A)$ be two soft sets over X. Then*

(1) $Int^s_R\left(\tilde{X}_A\right) = \tilde{X}_A$ and $Int^s_R(\phi_A) = \phi_A$.

(2) If $(F, A) \subseteq (G, A)$, then $Int^s_R(F, A) \subseteq Int^s_R(G, A)$.

(3) $Int^s_R\big(Int^s_R(F, A)\big) = Int^s_R(F, A)$.

(4) $Int^s_R(F, A) \cup Int^s_R(G, A) \subseteq Int^s_R\big((F, A) \cup (G, A)\big)$.

(5) $Int^s_R\big((F, A) \cap (G, A)\big) \subseteq Int^s_R(F, A) \cap Int^s_R(G, A)$.

Theorem 3.15. *Let (X, τ^*, A) be a supra soft topological space and $(F, A), (G, A)$ be two soft sets over X. Then*

(1) $Cl_R^s\left(\tilde{X}_A\right) = \tilde{X}_A$ *and* $Cl_R^s(\phi_A) = \phi_A$.

(2) *If* $(F,A) \subseteq (G,A)$, *then* $Cl_R^s(F,A) \subseteq Cl_R^s(G,A)$.

(3) $Cl_R^s\big(Cl_R^s(F,A)\big) = Cl_R^s(F,A)$.

(4) $Cl_R^s(F,A) \cup Cl_R^s(G,A) \subseteq Cl_R^s\big((F,A) \cup (G,A)\big)$.

(5) $Cl_R^s\big((F,A) \cap (G,A)\big) \subseteq Cl_R^s(F,A) \cap Cl_R^s(G,A)$.

Remark 3.7. *In theorem 3.14, inclusions in* (4) *and* (5) *cannot be replaced by equality.*

Example 6. In Example 3, let $(F,E) = \{(e_1,\{z\}),(e_2,\{z\})\}$ and $(G,E) = \{(e_1,\{x\}),(e_2,\{x\})\}$. Then

$$Int_R^s(F,E) \cup Int_R^s(G,E) = \{(e_1,\{x\}),(e_2,\{x\})\}$$

$$Inr_R^s\big((F,E) \cup (G,E)\big) = \{(e_1,\{x,z\}),(e_2,\{x,z\})\}.$$

Example 7. In Example 1, let $(G_1,E) = \{(e_1,\{x_2,x_3\}),(e_2,\{x_3\})\}$ and $(G_2,E) = \{(e_1,\{x_1,x_3\}),(e_2,\{x_2\})\}$. Since they are supra R-open soft sets,

$$Int_R^s(G_1,E) \cap Int_R^s(G_2,E) = \{(e_1,\{x_3\}),(e_2,\phi)\}$$

. But, $Int_R^s\big((G_1,E) \cap (G_2,E)\big) = Int_R^s\{(e_1,\{x_3\},(e_2,\phi)\} = \tilde{\phi}_E$.

4. Supra R-continuous soft function

The aim of this section is to introduce supra R-continuous(supra R*-continuous) soft map and some characterization of these concepts are investigated.

Definition 4.1. *Let* $(X,\tau_1,E),(Y,\tau_2,K)$ *be two soft topological spaces,* μ_1,μ_2 *be supra soft topological spaces associated with* τ_1,τ_2 *respectively and let* $f_{pu}: S_E(X) \mapsto S_K(Y)$ *be a soft function. Then,*

(1) *the soft function* f_{pu} *is called R-continuous soft function if for every* $f_{pu}^{-1}(G,K)$ *is supra R-open soft set in* X*, for every* $(G,K) \in \tau_2$.

(2) *the soft function* f_{pu} *is called* R^**-continuous soft function if* $f_{pu}^{-1}(G,K)$ *is supra R-open soft set in* X*, for every supra R-open soft set* (G,K) *in* Y.

Theorem 4.2. *Every supra continuous soft map is always supra R-continuous.*
Proof. Suppose $f_{pu}: S_E(X) \mapsto S_K(Y)$ is supra continuous soft map. Then

$f_{pu}^{-1}(G, K)$ is supra soft open set for any $(G, K) \in \tau_2$. Since every supra open soft sets are supra R-open soft set, it follows that f_{pu} is a supra R-continuous soft map.

But the converse of the above need not be true.

Example 8. Let $X = \{a, b, c\}, Y = \{x, y, z\}, E = \{e_1, e_2\}$ & $K = \{k_1, k_2\}$. Define $u : X \mapsto Y$ and $p : E \mapsto K$ as follows:

$$u(a) = x, u(b) = z, u(c) = y$$
$$p(e_1) = k_2, p(e_2) = k_1$$

Let (X, τ_1, E) be soft topological space over X, where $\tau_1 = \{\tilde{X}, \tilde{\phi}, (F_1, E)\}$, where $(F_1, E) = \{(e_1, \{a, b\}\}, (e_2, \{a, b\}\}$. Let $\mu 1 = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ be the supra soft topology associated with τ_1, where

$$(F_1, E) = \{(e_1, \{a, b\}, (e_2, \{a, b\})\}$$
$$(F_2, E) = \{(e_1, \{a\}, (e_2, \{a\})\}$$
$$(F_3, E) = \{(e_1, \{b, c\}), (e_2, \{b, c\})\}$$

Let (Y, τ_2, K) be soft topological space over Y, where $\tau_2 = \{\tilde{X}, \tilde{\phi}, (G_1, K)\}$, where $(G_1, K) = \{(k_1, \{x, y\}), (e_2, \{x, y\})\}$. Let $f_{pu} : S_E(X) \mapsto S_K(Y)$ be a soft map. Then $f_{pu}^{-1}(G_1, K) = \{(e_1, \{a, c\}), (e_2, \{a, c\})\}$ is supra R-open set, but not supra open soft set. Hence f_{pu} is supra R-continuous soft map, but not supra continuous soft map.

Theorem 4.3. *Every supra R*-continuous soft map is supra R-continuous soft map.*

Proof. Straight forward from the definition.

But the converse need not be true.

Example 9. In Example 8, let $\mu_2 = \{\tilde{X}, \tilde{\phi}, (G_1, K), (G_2, K), (G_3, K)\}$ be supra soft topology associated with τ_2, where

$$(G_1, K) = \{(k_1, \{x, y\}), (k_2, \{x, y\})\}$$
$$(G_2, K) = \{(k_1, \{z\}), (k_2, \{z\})\}$$
$$(G_3 K) = \{k_1 \{y, z\}), (k_2, \{y, z\})\}$$

Then $(G, K) = \{(k_1, \{y\}), (k_2, \{y\})\}$ is supra R-open soft set in Y, but $f_{pu}^{-1}(G, K) = (F_1, E) = \{(e_1, \{c\}), (e_2, \{c\})\}$ is not supra R-open soft set in X. Hence f_{pu} is supra R-open soft map, but it is not supra R*-open soft map.

Theorem 4.4. *Every supra b-continuous soft map is supra R-continuous.*

Proof. Proof is immediate from Theorem 3.4.

But the converse need not be true.

Theorem 4.5. *Let $(X, \tau_1, E), (Y, \tau_2, K)$ be two soft topological spaces, μ_1 be associated supra soft topology with τ_1 and $f_{pu} : S_E(X) \mapsto S_K(Y)$ be a soft function. Then the following are equivalent.*

(1) f_{pu} supra R-continuous soft function.

(2) For any supra closed soft (H, K) in Y, $f_{pu}^{-1}(H, K)$ is a supra R-closed soft set.

(3) For any soft set (F, E) over X, $f_{pu}\big(Cl^s_R(F, E)\big) \subseteq Cl_{\tau_2}\big(f_{pu}(F, E)\big)$.

(4) For any soft set (H, K) over Y, $Cl^s_R\big(f_{pu}^{-1}(H, K)\big) \subseteq f_{pu}^{-1}\big(Cl_{\tau_2}(H, K)\big)$.

(5) For any soft set (H, K) over Y, $f_{pu}^{-1}\big(Int_{\tau_2}(H, K)\big) \subseteq Int^s_R\big(f_{pu}^{-1}(H, K)\big)$.

Proof. Since complement of supra R-open soft set is supra R-closed soft set and vice versa and since $f_{pu}^{-1}(F, E)' = \big(f_{pu}^{-1}(F, E)\big)'$, it follows that (1) and (2) are equivalent.

(2) $\Longrightarrow$ (3)

Let (G, E) be a soft set over X. We have

$$(G, E) \subseteq f_{pu}^{-1}\big(f_{pu}(G, E)\big) \subseteq f_{pu}^{-1}\big[Cl_{\tau_2}\big(f_{pu}(G, E)\big)\big]$$

Since $f_{pu}^{-1}\big[Cl_{\tau_2}\big(f_{pu}(G, E)\big)\big]$ is a supra R-closed soft set in X, we have $Cl^s_R(G, E) \subseteq f_{pu}^{-1}\big[Cl_{\tau_2}\big(f_{pu}(G, E)\big)\big]$. So $f_{pu}\big(Cl^s_R(G, E)\big) \subseteq Cl_{\tau_2}\big(f_{pu}(G, E)\big)$.

(3) $\Longrightarrow$ (4)

Let (H, E) be a soft set over Y. We have

$$f_{pu}\big[Cl^s_R\big(f_{pu}^{-1}(H, E)\big)\big] \subseteq Cl_{\tau_2}\big[f_{pu}\big(f_{pu}^{-1}(H, E)\big)\big] \subseteq Cl_{\tau_2}(H, E).$$

So $Cl^s_R\big(f_{pu}^{-1}(H, E)\big) \subseteq f_{pu}^{-1}\big(Cl_{\tau_2}(H, E)\big)$.

(4) $\Longrightarrow$ (2)

Let (H, E) be a closed soft set over Y. Then,

$$Cl^s_R\big(f_{pu}^{-1}(H, E)\big) \subseteq f_{pu}^{-1}\big(Cl_{\tau_2}(H, E)\big) = f_{pu}^{-1}(H, E).$$

Hence it follows that $f_{pu}^{-1}(H, E) = Cl^s_R\big(f_{pu}^{-1}(H, E)\big)$. So $f_{pu}^{-1}(H, E)$ is a supra R-closed soft set over X.

(1) $\Longrightarrow$ (5)

Since $f_{pu}^{-1}\big(Int_{\tau_2}(H,K)\big) \subseteq f_{pu}^{-1}(H,K)$ and since f_{pu} is a supra R-continuous soft function we have,

$$f_{pu}^{-1}\big(Int_{\tau_2}(H,K)\big) \subseteq Int_R^s\big(f_{pu}^{-1}(H,K)\big) \subseteq f_{pu}^{-1}(H,K)$$

.

(5) $\implies$ (1)

Let (H,K) be an open soft set in Y. We have

$$f_{pu}^{-1}(H,K) = f_{pu}^{-1}\big(Int_{\tau_2}(H,K)\big) \subseteq Int_R^s\big(f_{pu}^{-1}(H,K)\big)$$

. Hence it follows that $Int_R^s\big(f_{pu}^{-1}(H,K)\big) = f_{pu}^{-1}(H,k)$. So f_{pu} is a supra R-continuous soft function.

Theorem 4.6. *Let $(X,\tau_1,E),(Y,\tau_2,K)$ be two soft topological spaces, μ_1 and μ_2 be associated supra soft topology with τ_1 and τ_2 respectively and let $f_{pu}: S_E(X) \mapsto S_K(Y)$ be a soft function. Then the following are equivalent.*

(1) f_{pu} supra R^*-continuous soft function.

(2) For any supra R-closed soft set (H,K) in Y, $f_{pu}^{-1}(H,K)$ is a supra R-closed soft set.

(3) For any soft set (F,E) over X, $f_{pu}\big(Cl_R^s(F,E)\big) \subseteq Cl_R^s\big(f_{pu}(F,E)\big)$.

(4) For any soft set (H,K) over Y, $Cl_R^s\big(f_{pu}^{-1}(H,K)\big) \subseteq f_{pu}^{-1}\big(Cl_R^s(H,K)\big)$.

(5) For any soft set (H,K) over Y, $f_{pu}^{-1}\big(Int_R^s(H,K)\big) \subseteq Int_R^s\big(f_{pu}^{-1}(H,K)\big)$.

Proof. The proof is same as above.

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