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INTUITIONISTIC FUZZY SEMI γ^* GENERALIZED IRRESOLUTE MAPPING

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Abstract: In this paper we have introduced intuitionistic fuzzy semi γ^* generalized irresolute mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy semi γ^* generalized irresolute mappings.

Keywords and Phrases: Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, Intuitionistic fuzzy semi γ^* generalized closed sets, intuitionistic fuzzy semi γ^* generalized irresolute mappings.

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1. Introduction

The concept of fuzzy set is introduced by Zadeh [7] in 1965 and later Atanassov [4] generalized this idea to intuitionistic fuzzy sets. Coker [5] has introduced intuitionistic fuzzy topological space using the notion of intuitionistic fuzzy sets. Abinaya and Jayanthi [1, 2] introduced intuitionistic fuzzy semi γ^* generalized closed sets and intuitionistic fuzzy semi γ^* generalized continuous mappings. In this paper, we have introduced intuitionistic fuzzy semi γ^* generalized irresolute mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy semi γ^* generalized irresolute mappings.

2. Preliminaries

Definition 2.1. [4] *An **intuitionistic fuzzy set (IFS)** A is of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

Where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and degree of non membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2. [4] *Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,*

(a) $A \subseteq B$ *if and only if* $\mu_A(x) \le \mu_B(x)$ *and* $\nu_A(x) \ge \nu_B(x)$ *for all* $x \in X$,

(b) $A = B$ *if and only if* $A \subseteq B$ *and* $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3. [5] *An intuitionistic fuzzy topology (IFT) on X is a family τ of IFSs in X satisfying the following axioms:*

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ *for any family* $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4. [6] *An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an*

(i) ***intuitionistic fuzzy γ closed set (IFγCS)*** *if* $cl(int(A)) \cap int(cl(A)) \subseteq A$

(ii) ***intuitionistic fuzzy γ open set (IFγOS)*** *if* $A \subseteq cl(int(A)) \cup int(cl(A))$.

Definition 2.5. [1] *An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy semi γ^* generalized closed set (IF semi γ^*GCS) if $int(cl(A)) \cap cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFSOS in (X, τ).*

Definition 2.6. [2] *A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy semi γ^* generalized (IF semi γ^*G) continuous mapping if $f^{-1}(V)$ is an IF semi γ^*GCS in (X, τ) for every IFCS V of (Y, σ).*

Definition 2.7. [1] *An IFTS (X, τ) is an intuitionistic fuzzy semi $\gamma^*_c T_{1/2}$ (IF semi $\gamma^*_c T_{1/2}$) space if every IF semi γ^*GCS is an IFCS in X.*

Definition 2.8. [1] *An IFTS (X, τ) is an intuitionistic fuzzy semi $\gamma^*_\gamma T_{1/2}$ (IF semi $\gamma^*_\gamma T_{1/2}$) space if every IF semi γ^*GCS is an IFγCS in X.*

Definition 2.9. [3] *A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy almost semi γ^* generalized (IF almost semi γ^*G) continuous mapping if $f^{-1}(V)$ is an IF semi γ^*GCS in (X, τ) for every IFRCS V of (Y, σ).*

Proposition 2.10. [1] *Every IFCS is an IF semi γ^*GCS in (X, τ).*
Proof. Let A be an IFCS in (X, τ). Let $A \subseteq U$ where U is an IFSOS in (X, τ). Since A is an IFCS, $cl(A) = A$. Now $int(cl(A)) \cap cl(int(A)) = int(A) \cap cl(int(A)) \subseteq A \cap cl(A) = A \cap A = A \subseteq U$. Hence A is an IF semi $\gamma^* GCSin(X, \tau)$.

3. Intuitionistic fuzzy semi γ^* generalized irresolute mapping

In this section we have introduced intuitionistic fuzzy semi γ^* generalized irresolute mappings and investigated some of their properties.

Definition 3.1. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy semi γ^* generalized (IF semi γ^*G) irresolute mapping if $f^{-1}(V)$ is an IF semi γ^*GCS in (X, τ) for every IF semi γ^*GCS V of (Y, σ).*

Example 3.2. Let $X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle, G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here,
$IFSO(X) = \{0_\sim, 1_\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1]/0.4 \leq \mu_a \leq 0.5, 0.4 \leq \mu_b \leq 0.6, 0.5 \leq \nu_a \leq 0.6, 0.4 \leq \nu_b \leq 0.7$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.
$IFSO(Y) = \{0_\sim, 1_\sim, \mu_u \in [0, 1], \mu_v \in [0, 1], \nu_u \in [0, 1], \nu_v \in [0, 1]/\mu_u \geq 0.5, \mu_v \geq 0.6, \nu_u \leq 0.5, \nu_v \leq 0.4$ and $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$.

Let $A = \langle y, (0.3_u, 0.6_v), (0.7_u, 0.4_v)\rangle$. Then A is an IF semi γ^*GCS in Y and $f^{-1}(A) = \langle x, (0.3_a, 0.6_b), (0.7_a, 0.4_b)\rangle$ is also an IF semi γ^*GCS in (X, τ). Therefore f is an IF semi γ^*G irresolute mapping.

Theorem 3.3. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IF semi γ^*G irresolute mapping, then f is an IF semi γ^*G continuous mapping but not conversely in general.*

Proof. Let f be an IF semi γ^*G irresolute mapping. Let V be any IFCS in Y. Then V is an IF semi γ^*GCS and by hypothesis $f^{-1}(V)$ is an IF semi γ^*GCS in X. Hence f is an IF semi γ^*G continuous mapping.

Example 3.4. Let $X = \{a, b\}, Y = \{u, v\}, G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, and $G_2 = \langle y, (0.8_u, 0.7_v), (0.2_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here,

$IFSO(X) = \{0_\sim, 1_\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1]/\mu_a \geq 0.5, \mu_b \geq 0.6, \nu_a \leq 0.5, \nu_b \leq 0.4$ and $0 \leq \mu_a + \mu_b \leq 1$ and $0 \leq \mu_a + \mu_b \leq 1\}$.

$IFSO(Y) = \{0_\sim, 1_\sim, \mu_u \in [0, 1], \mu_v \in [0, 1], \nu_u \in [0, 1], \nu_v \in [0, 1]/\mu_u \geq 0.8, \mu_v \geq 0.7, \nu_u \leq 0.2, \nu_v \leq 0.3$ and $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$.

Then f is an IF semi γ^*G continuous mapping but not an IF semi γ^*G irresolute mapping, since the IFS $A = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ is an IF semi γ^*GCS in Y but $f^{-1}(A) = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ is not an IF semi γ^*GCS in X, as $int(cl(f^{-1}(A))) \cap cl(int(f^{-1}(A))) = 1_\sim \subsetneq G_1$, but $f^{-1}(A) \subseteq G_1$.

Theorem 3.5. *A mapping $f : (X, \tau) \to (Y, \sigma)$ be an IF semi γ^*G irresolute mapping if and only if the inverse image of each IF semi γ^*GOS in Y is an IF semi γ^*GOS in X.*

Proof. Necessity: Let A be an IF semi γ^*GOS in Y. This implies A^c is an IF semi γ^*GCS in Y. Then $f^{-1}(A^c)$ is an IF semi γ^*GCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c, f^{-1}(A)$ is an IF semi γ^*GOS in X.

Sufficiency: Let A be an IF semi γ^*GCS in Y. Then A^c is an IF semi γ^*GOS in Y. By hypothesis $f^{-1}(A^c)$ is an IF semi γ^*GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c, (f^{-1}(A))^c$ is an IF semi γ^*GOS in X. Therefore $f^{-1}(A)$ is an IF semi γ^*GCS in X. Hence f is an IF semi γ^*G irresolute mapping.

Theorem 3.6. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two IF semi γ^*G irresolute mappings, then g o $f : (X, \tau) \to (Z, \delta)$ is an IF semi γ^*G irresolute mapping.*

Proof. Let V be an IF semi γ^*GCS in Z. Then $g^{-1}(V)$ is an IF semi γ^*GCS in Y, by hypothesis. Since f is an IF semi γ^*G irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IF semi γ^*GCS in X. Hence g o f is an IF semi γ^*G irresolute mapping.

Theorem 3.7. *Composition of IF semi γ^*G irresolute mapping and IF semi γ^*G continuous mapping is an IF semi γ^* continuous mapping.*
Proof. Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IF semi γ^*GCS in Y. Since f is an IF semi γ^*G irresolute, $f^{-1}(g^{-1}(V))$ is an IF semi γ^*GCS in X. Hence $g\ o\ f$ is an IF semi γ^*G continuous mapping.

Theorem 3.8. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IF semi γ^*G continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an IF semi γ^*G irresolute mapping, then $g\ o\ f : (X, \tau) \to (Z, \delta)$ is an IF semi γ^*G irresolute mapping if Y is an IF semi $\gamma_c^*T_{1/2}$ space.*
Proof. Let V be an IF semi γ^*GCS in Z. Then $g^{-1}(V)$ is an IF semi γ^*GCS in Y as g is an IF semi γ^*G irresolute mapping. Since Y is an IF semi $\gamma_c^*T_{1/2}$ space, $g^{-1}(V)$ is an IFCS in Y. Since f is an IF semi γ^*G continuous mapping, $f^{-1}(g^{-1}(V))$ is an IF semi γ^*GCS in X. Hence $g\ o\ f$ is an IF semi γ^*G irresolute mapping.

Theorem 3.9. *If (Y, σ) is an IF semi $\gamma_c^*T_{1/2}$ space and $f : (X, \tau) \to (Y, \sigma)$ be an IF semi γ^*G continuous mapping, then f is an IF semi γ^*G irresolute mapping.*
Proof. Let A be an IF semi γ^*GCS in Y. Since Y is an IF semi $\gamma_c^*T_{1/2}$ space, A is an IFCS in Y. By hypothesis $f^{-1}(A)$ is an IF semi γ^*GCS in X. Therefore f is an IF semi γ^*G irresolute mapping.

Theorem 3.10. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following conditions are equivalent if X and Y are IF semi $\gamma_\gamma^*T_{1/2}$ spaces:*

(i) f is an IF semi γ^*G irresolute mapping,

(ii) $f^{-1}(B)$ is an IF semi γ^*GOS in X for each IF semi γ^*GOS B in Y

(iii) $f^{-1}(\gamma int(B)) \subseteq \gamma int(f^{-1}(B))$ for each IFS B of Y,

(iv) $\gamma cl(f^{-1}(B)) \subseteq f^{-1}(\gamma cl(B))$ for each IFS B of Y.

Proof. (i)$\Leftrightarrow$(ii) is obvious from Theorem 3.5.
(ii)$\Rightarrow$ (iii) Let B be any IFS in Y and $\gamma int(B) \subseteq B$. Also $f^{-1}(\gamma int(B)) \subseteq f^{-1}(B)$. Since $\gamma int(B)$ is an IFγOS in Y, it is an IF semi γ^*GOS in Y. Therefore $f^{-1}(\gamma int(B))$ is an IF semi γ^*GOS in X, by hypothesis. Since X is an IF semi $\gamma_\gamma^*T_{1/2}$ spaces, $f^{-1}(\gamma int(B))$ is an IFγOS in X. Hence $f^{-1}(\gamma int(B)) = \gamma int(f^{-1}(\gamma int(B))) \subseteq \gamma int(f^{-1}(B))$.
(iii) $\Rightarrow$ (iv) is obvious by taking complement in (iii).
(iv)$\Rightarrow$ (i) Let B be an IF semi γ^*GCS in Y. Since Y is an IF semi $\gamma_\gamma^*T_{1/2}$ space, B is an IFγCS in Y and $\gamma cl(B) = B$. Hence $f^{-1}(B) = f^{-1}(\gamma cl(B)) \supseteq \gamma cl(f^{-1}(B)) \supseteq f^{-1}(B)$. Therefore $\gamma cl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFγCS and

hence it is an IF semi γ^*GCS in X. Thus f is an IF semi γ^*G irresolute mapping.

Theorem 3.11. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IF semi γ^*G irresolute mapping. Then $f^{-1}(B) \subseteq \gamma int(f^{-1}(int(cl(B)) \cup cl(int(B))))$ for every IF semi γ^*GOS B in Y, if X and Y are IF semi $\gamma^*_\gamma T_{1/2}$ spaces.*

Proof. Let B an IF semi γ^*GOS in Y. Then by hypothesis $f^{-1}(B)$ is an IF semi γ^*GOS in X. Since X is an IF semi $\gamma^*_\gamma T_{1/2}$ space, $f^{-1}(B)$ is an IFγOS in X. Therefore $\gamma int(f^{-1}(B)) = f^{-1}(B)$. Since Y is an IF semi $\gamma^*_\gamma T_{1/2}$ space, B is an IFγOS in Y, $B \subseteq int(cl(B)) \cup cl(int(B))$. Now $f^{-1}(B) = \gamma int(f^{-1}(B)) \subseteq \gamma int(f^{-1}(int(cl(B)) \cup cl(int(B))))$.

Theorem 3.12. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IF semi γ^*G irresolute mapping and $g : (Y, \sigma) \to (Z, \delta)$ be an IF almost semi γ^*G continuous mapping, then $g \ o \ f : (X, \tau) \to (Z, \delta)$ is an IF almost semi γ^*G continuous mapping.*

Proof. Let V be an IFRCS in Z. Then $g^{-1}(V)$ is an IF semi γ^*GCS in Y. Since f is an IF semi γ^*G irresolute, $f^{-1}(g^{-1}(V))$ is an IF semi γ^*GCS in X. Hence $g \ o \ f$ is an IF almost semi γ^*G continuous mapping.

Theorem 3.13. *Let $f : (X, \tau) \to (Y, \sigma)$ be an IF semi γ^*G irresolute mapping. Then $f^{-1}(B) \subseteq \gamma int(f^{-1}(int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B)))))$ for every IF semi γ^*GOS B in Y, if X and Y are IF semi $\gamma^*_\gamma T_{1/2}$ spaces.*

Proof. Let B an IF semi γ^*GOS in Y. Then by hypothesis $f^{-1}(B)$ is an IF semi γ^*GOS in X. Since X is an IF semi $\gamma^*_\gamma T_{1/2}$ space, $f^{-1}(B)$ is an IFγOS in X. Therefore $\gamma int(f^{-1}(B)) = f^{-1}(B)$ and $f^{-1}(B) \subseteq int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B)))$. Hence $f^{-1}(B) = \gamma int(f^{-1}(B)) \subseteq \gamma int(int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B))))$.

4. Conclusion

In this paper, we have introduced intuitionistic fuzzy semi γ^* generalized irresolute mappings and investigated some of their properties. In future we have analyzed completeness in intuitionistic fuzzy semi γ^* generalized continuous mapping.

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