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ON NANO SOFT $^{s}(\mathscr{I})\beta\alpha$- REGULAR SPACES AND NORMAL SPACES

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Abstract: In this paper, we introduce the idea of Nano Soft $^{s}(\mathscr{I})\beta\alpha$- Regular Spaces (RS) and Normal Spaces (NS). Further we define Nano Soft $^{s}(\mathscr{I})\alpha$- Regular and Normal Spaces, Nano Soft $^{s}(\mathscr{I})\beta$- Regular and Normal Spaces, Nano Soft $^{s}(\mathscr{I})$ Semi- Regular and Normal Spaces, Nano Soft $^{s}(\mathscr{I})$ Pre- Regular and Normal Spaces. Also their features and characterization are explored with an example.

Keywords and Phrases: Nano Soft $^{s}(\mathscr{I})\alpha$- Regular Spaces and Normal Spaces, Nano Soft $^{s}(\mathscr{I})\beta$- Regular Spaces and Normal Spaces, Nano Soft $^{s}(\mathscr{I})\beta\alpha$- Regular Spaces and Normal Spaces, Nano Soft $^{s}(\mathscr{I})\alpha\beta$- Regular Spaces and Normal Spaces.

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1. Introduction

The soft set theory was developed by Molodstov [18] in 1999 to solve the problem in a mathematical model to the uncertainty. M. Shabir and M. Naz [21] introduced the soft topological spaces (TS). The nano topology was produced by Lellis Thivagar [11] in 2013. Jankovic and Hamlett [9] was developed the ideal topological space in 1990. The concept of α- open sets are presented by O. Njasted [19] in 1965. The ideas of α- closed sets in TS were first presented in 1983 by A. M. Mashhour et al. [13]. The method of β- open sets and β- continuity in topology was produced by M. E. Abd ElMonsef et al. [1] in 1983. The notion of α- normal spaces was presented by Benchalli et al. [4] and β- normal spaces was produced by Mahmoud et al. [14]. The concept of $\beta\alpha$- RS and $\beta\alpha$- NS in topology was introduced by Govindappa Navalagi [6]. The nano soft ideal topology was introduced by S. P. R. Priyalatha et al. [20]. In this paper we present the method of Nano soft ${}^{s}(\mathscr{I})\beta\alpha$- Regular spaces and Normal spaces. Also we introduced Nano Soft ${}^{s}(\mathscr{I})\alpha$- Regular and Normal Spaces, Nano Soft ${}^{s}(\mathscr{I})\beta$- Regular and Normal Spaces, Nano Soft ${}^{s}(\mathscr{I})\alpha\beta$- Regular and Normal Spaces, Nano Soft ${}^{s}(\mathscr{I})$ Semi-Regular and Normal Spaces, Nano Soft ${}^{s}(\mathscr{I})$ Pre- Regular and Normal Spaces and their characteristics are investigated.

2. Preliminaries

Definition 2.1. [11] *Let $\tilde{U}$ be a non empty finite set of objects called the universe, R be an equivalence relation on $\tilde{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\tilde{U}, R)$ is said to be approximation space. Let $X \subseteq \tilde{U}$.*

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \left\{ \bigcup_{x\in\tilde{\mathcal{U}}} \{R(x) : R(x) \subseteq X\} \right\}$, where $R(x)$ denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be for possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \left\{ \bigcup_{x\in\tilde{\mathcal{U}}} \{R(x) : R(x) \cap X \neq \emptyset\} \right\}$.

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [11, 12] *Let $\tilde{U}$ be the universe, R be an equivalence relation on $\tilde{U}$ and $\tau_R(x) = \left\{\tilde{U}, \tilde{\emptyset}, L_R(X), U_R(X), B_R(X)\right\}$ where $X \subseteq \tilde{U}$ and $\tau_R(X)$ satisfies the following axioms.*

(i) $\tilde{U}$ and $\tilde{\emptyset} \in \tau_R(X)$.

(ii) The union of the elements of any subcollection $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on $\tilde{U}$ and it is called as the nano topology on $\tilde{U}$ with respect to X. The elements of $\tau_R(X)$ are called as nano open sets.

Definition 2.3. [11] *If $(\tilde{U}, \tilde{\tau}_R(X))$ is a nano topological space and $A \subseteq \tilde{U}$. Then A is said to be*

(i) Nano semi- open if $A \subseteq NCl(NInt(A))$

(ii) Nano Pre- open (briefly nano p-open) if $A \subseteq NInt(NCl(A))$

(iii) Nano α- open if $A \subseteq NInt(NInt(NCl(A)))$.

Definition 2.4. [15, 18] *A soft set $\mathscr{F}_{\mathscr{A}}$ on the universe $\tilde{U}$ is defined by the set of ordered pairs $\mathscr{F}_{\mathscr{A}} = \left\{(e, F(e)) : e \in E, F(e) \in P(\tilde{U})\right\}$, where $F : E \to P(\tilde{U})$ such that $F(e) = \emptyset$, if $e \notin \mathscr{A}$ and $\mathscr{A} \subseteq E$.*

Definition 2.5. [21] *Let $\tilde{\tau}$ be the collection of soft sets over $\tilde{U}$, then $\tilde{\tau}$ is said to be soft topology on $\tilde{U}$ if*

(i) $\tilde{U}, \tilde{\emptyset} \in \tilde{\tau}$

(ii) Union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

(iii) Intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet $(\tilde{U}, \tilde{\tau}, E)$ is called soft topological space over $\tilde{U}$. The members in $\tilde{\tau}$ are said to be soft open sets in $\tilde{U}$.

Definition 2.6. [21] *A soft subset $\mathscr{F}_{\mathscr{A}}$ of a soft topological space $\tilde{U}$ is said to be soft closed if $\tilde{U} - \mathscr{F}_{\mathscr{A}}$ is soft open.*

Definition 2.7. [21] *Let $(\tilde{U}, \tilde{\tau}, E)$ be a soft topological space over $\tilde{U}$ and $\mathscr{F}_{\mathscr{A}}$ be a soft set over $\tilde{U}$. Then the soft closure of $\mathscr{F}_{\mathscr{A}}$ is the intersection of all soft closed supersets of $\mathscr{F}_{\mathscr{A}}$.*

Definition 2.8. [21] *Let $(\tilde{U}, \tilde{\tau}, E)$ be a soft topological space over $\tilde{U}$, $\mathscr{F}_{\mathscr{A}}$ be a soft set over $\tilde{U}$ and $u \in \tilde{U}$. Then u is said to be soft interior of $\mathscr{F}_{\mathscr{A}}$ if there exists a soft open set $\mathscr{G}_{\mathscr{A}}$ such that $u \in \mathscr{G}_{\mathscr{A}} \subset \mathscr{F}_{\mathscr{A}}$.*

Definition 2.9. [16] *A soft set $\mathscr{F}_{\mathscr{A}}$ of a soft topological space $(\tilde{U}, \tilde{\tau}, E)$ is called soft α- open set if $\mathscr{F}_{\mathscr{A}} \subset Int(ClInt\mathscr{F}_{\mathscr{A}})$. The complement of soft α- open set is* *called soft α- closed set.*

Definition 2.10. [2] *A soft set $\mathscr{F}_{\mathscr{A}}$ in a soft topological space $(\tilde{U}, \tilde{\tau}, E)$ is said to be soft β- open if $\mathscr{F}_{\mathscr{A}} \subseteq Cl(Int(Cl(\mathscr{F}_{\mathscr{A}})))$ and soft β- closed if $Int(Cl(Int(\mathscr{F}_{\mathscr{A}}))) \subseteq \mathscr{F}_{\mathscr{A}}$.*

Definition 2.11. [5] *A soft set $\mathscr{F}_{\mathscr{A}}$ in a soft topological space $(\tilde{U}, \tilde{\tau}, E)$ is said to be soft semi- open if there exists a soft open set $\mathscr{G}_{\mathscr{A}}$ such that $\mathscr{G}_{\mathscr{A}} \subset \mathscr{F}_{\mathscr{A}} \subset Cl(\mathscr{G}_{\mathscr{A}})$.*

Definition 2.12. [17] *Let $\mathscr{F}_{\mathscr{A}}$ be any soft set of a soft topological space $(\tilde{U}, \tilde{\tau}, E)$. $\mathscr{F}_{\mathscr{A}}$ is called*

(i) soft pre- open set of $\tilde{U}$ if $\mathscr{F}_{\mathscr{A}} \subset IntCl(\mathscr{F}_{\mathscr{A}})$.

(ii) soft pre- closed set of $\tilde{U}$ if $IntCl(\mathscr{F}_{\mathscr{A}}) \subset \mathscr{F}_{\mathscr{A}}$.

Definition 2.13. [21] *Let $(\tilde{U}, \tilde{\tau}, E)$ be a soft topological space over $\tilde{U}$, $\mathscr{F}_{\mathscr{A}}$ be a soft closed set in $\tilde{U}$ and $u \in \tilde{U}$ such that $u \notin \mathscr{F}_{\mathscr{A}}$. If there exists soft open sets $\mathscr{M}_{\mathscr{A}}$ and $\mathscr{N}_{\mathscr{A}}$ such that $u \in \mathscr{M}_{\mathscr{A}}$, $\mathscr{F}_{\mathscr{A}} \subset \mathscr{N}_{\mathscr{A}}$ and $(\mathscr{M}_{\mathscr{A}}) \cap (\mathscr{N}_{\mathscr{A}}) = \emptyset$ then $(\tilde{U}, \tilde{\tau}, E)$ is called a soft regular space.*

Definition 2.14. [21] *Let $(\tilde{U}, \tilde{\tau}, E)$ be a soft topological space over $\tilde{U}$, $\mathscr{F}_{\mathscr{A}}$ and $\mathscr{G}_{\mathscr{A}}$ are soft closed sets over $\tilde{U}$ such that $\mathscr{F}_{\mathscr{A}} \cap \mathscr{G}_{\mathscr{A}} = \emptyset$. If there exists soft open sets $\mathscr{M}_{\mathscr{A}}$ and $\mathscr{N}_{\mathscr{A}}$ such that $\mathscr{F}_{\mathscr{A}} \subset \mathscr{M}_{\mathscr{A}}$, $\mathscr{G}_{\mathscr{A}} \subset \mathscr{N}_{\mathscr{A}}$ and $(\mathscr{M}_{\mathscr{A}}) \cap (\mathscr{N}_{\mathscr{A}}) = \emptyset$ then $(\tilde{U}, \tilde{\tau}, E)$ is called a soft normal space.*

Definition 2.15. [10] *An ideal $\mathscr{I}$ on a topological space $(\tilde{U}, \tilde{\tau})$ is a non empty collection of subsets of $\tilde{U}$ which satisfies*

(i) $A \in \mathscr{I}$ and $B \subseteq A$ imply $B \in \mathscr{I}$ and

(ii) $A \in \mathscr{I}$ and $B \in \mathscr{I}$ imply $A \cup B \in \mathscr{I}$.

Definition 2.16. [3] *Let $\mathscr{I}$ be the non empty collection of soft sets over $\tilde{U}$, with the same set of parameters E. Then $\mathscr{I} \subseteq SS(\tilde{U})_E$ is called a soft ideal on $\tilde{U}$ with the same set E if,*

(i) $\mathscr{F}_E \in \mathscr{I}$ and $\mathscr{G}_E \in \mathscr{I}$ implies $\mathscr{F}_E \cup \mathscr{G}_E \in \mathscr{I}$.

(ii) $\mathscr{F}_E \in \mathscr{I}$ and $\mathscr{G}_E \subseteq \mathscr{F}_E$ implies $\mathscr{G}_E \in \mathscr{I}$.

Definition 2.17. [8] *An ideal topological space $(\tilde{U}, \tilde{\tau}, \mathscr{I})$ is said to be $\mathscr{I}$- normal if for each pair of disjoint $\mathscr{I}$- closed sets A and B, there exist disjoint open sets V and W in $\tilde{U}$ such that $A \subseteq V$ and $B \subseteq W$.*

Definition 2.18. [8] *An ideal topological space $(\tilde{U}, \tilde{\tau}, \mathscr{I})$ is said to be $\mathscr{I}$- regular if for each pair consisting of a point u and a closed set B not containing u, there exists disjoint $\mathscr{I}$- open sets V and W such that $u \in V$ and $B \subseteq W$.*

Definition 2.19. [20] *Let $\tilde{U}$ be a non empty finite set of objects called the universe, $\mathscr{F}_{\mathscr{A}} \subseteq \mathscr{G}_{\mathscr{A}}$ is an soft set over $\tilde{U}$ and $\mathscr{I}$ is an ideal on $\mathscr{G}_{\mathscr{A}}$. Then $(\tilde{U}, \mathscr{F}_{\mathscr{A}}, \mathscr{I})$ is an triplet ordered pair of soft ideal approximation space and $\tilde{\tau}_R(\mathscr{I}) = \{\tilde{U}, \emptyset, L_R(\mathscr{I}), U_R(\mathscr{I}), B_R(\mathscr{I})\}$ where $\mathscr{I} \subseteq \mathscr{G}_{\mathscr{A}}$ and $\tilde{\tau}_R(\mathscr{I})$ satisfies the following axiom*

(i) $\tilde{U}, \emptyset \in \tilde{\tau}_R(\mathscr{I})$

(ii) *The union of the elements of any subcollection of soft ideal $\tilde{\tau}_R(\mathscr{I})$is in $\tilde{\tau}_R(\mathscr{I})$.*

(iii) *The intersection of the elements of any finite subcollection of soft ideal $\tilde{\tau}_R(\mathscr{I})$is in $\tilde{\tau}_R(\mathscr{I})$.*

That is, $\tilde{\tau}_R(\mathscr{I})$ forms a soft ideal topology on $\tilde{U}$ having atmost five elements of soft ideal and four ordered pair $(\tilde{U}, \tilde{\tau}_R, E, \mathscr{I})$ is called a nano soft ideal topological space over $\tilde{U}$ with respect to $\mathscr{I}$, then the members of $\tilde{\tau}_R$ are said to be nano soft ideal open sets in $\tilde{U}$.

Definition 2.20. [20] *Let $(\tilde{U}, \tilde{\tau}_R, E, \mathscr{I})$ be a nano soft ideal topological space over $\tilde{U}$ and soft set $\mathscr{F}_{\mathscr{A}} \in (\tilde{U}, E)$ is said to be*

(i) *nano soft ideal α- open set if $\mathscr{F}_{\mathscr{A}} \subseteq \mathscr{N}_{SI}Int(\mathscr{N}_{SI}Cl(\mathscr{N}_{SI}Int(\mathscr{F}_{\mathscr{A}})).$*

(ii) *nano soft ideal pre- open set if $\mathscr{F}_{\mathscr{A}} \subseteq \mathscr{N}_{SI}Int(\mathscr{N}_{SI}Cl(\mathscr{F}_{\mathscr{A}}).$*

(iii) *nano soft ideal semi- open set if $\mathscr{F}_{\mathscr{A}} \subseteq \mathscr{N}_{SI}Cl(\mathscr{N}_{SI}Int(\mathscr{F}_{\mathscr{A}}).$*

(iv) *nano soft ideal β- open set if $\mathscr{F}_{\mathscr{A}} \subseteq \mathscr{N}_{SI}Cl(\mathscr{N}_{SI}Int(\mathscr{N}_{SI}Cl(\mathscr{F}_{\mathscr{A}})).$*

Definition 2.21. [20] *Let $(\tilde{U}, \tilde{\tau}, E, \mathscr{I})$ be a nano soft ideal topological space over $\tilde{U}$. Then nano soft ideal closure of soft set $\mathscr{F}_E$ over $\tilde{U}$ is denoted by $\mathscr{N}_{SI}Cl(\mathscr{F}_E)$. Thus $\mathscr{N}_{SI}Cl(\mathscr{F}_E)$ is the smallest nano soft ideal closed set which containing $\mathscr{F}_E$ and is defined as the intersection of all nano soft ideal closed supersets of $\mathscr{F}_E$.*

Definition 2.22. [7] *A topological space $\tilde{U}$ is said to be β^*- regular if for each β- closed set $\mathscr{F}$ and for each $u \in \tilde{U} - \mathscr{F}$, there exists disjoint β- open sets V and W such that $u \in V$ and $\mathscr{F} \subset W$.*

Definition 2.23. [7] *A topological space $\tilde{U}$ is said to be strongly β^*- regular if for each β- closed set $\mathscr{F}$ and each $u \notin \mathscr{F}$, there exists disjoint open sets V and W such that $\mathscr{F} \subset V$ and $u \in W$.*

Definition 2.24. [6] *A topological space $\tilde{U}$ is said to be $\beta\alpha$- regular if for each β-closed set $\mathscr{F}$ of $\tilde{U}$ and each point u in $\tilde{U} - \mathscr{F}$, there exist disjoint α- open sets V and W such that $u \in V$ and $\mathscr{F} \subset W$.*

Definition 2.25. [6] *A topological space $\tilde{U}$ is said to be $\alpha\beta$- regular if for each α-closed set $\mathscr{F}$ of $\tilde{U}$ and each point $u \in \tilde{U} - \mathscr{F}$, there exist disjoint β- open sets V and W such that $u \in V$ and $\mathscr{F} \subset W$.*

Definition 2.26. [6] *A topological space $\tilde{U}$ is said to be $\beta\alpha$- normal if for any pair of disjoint β- closed set $\mathscr{F}$ and $\mathscr{G}$ of $\tilde{U}$, there exist disjoint α- open sets V and W such that $\mathscr{F} \in V$ and $\mathscr{G} \subset W$.*

Definition 2.27. [6] *A topological space $\tilde{U}$ is said to be $\alpha\beta$- normal if for any pair of disjoint α- closed set $\mathscr{F}$ and $\mathscr{G}$ of $\tilde{U}$, there exist disjoint β- open sets V and W such that $\mathscr{F} \in V$ and $\mathscr{G} \subset W$.*

3. Nano Soft ${}^{s}(\mathscr{I})\beta\alpha$- Regular Spaces and Normal Spaces

In this section we define Nano Soft ${}^{s}(\mathscr{I})\beta\alpha$- Regular Spaces and Normal Spaces and go through their characteristics. Throughout this paper we represent Regular Spaces by RS, Normal Spaces by NS and Topological Space by TS.

Definition 3.1. *Let $(\tilde{Y}, \tilde{\iota}_R({}^{s}(\mathscr{I})))$ be a nano soft ${}^{s}(\mathscr{I})$ topological space and $\mathscr{F}_A$ be a soft closed set over $\tilde{Y} \ni \mathscr{E} \notin \mathscr{F}_A$ for $y \in \tilde{Y}$. If $\exists$ disjoint soft open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{F}_{\mathscr{A}} \subset \mathscr{V}_{\mathscr{A}}$, then $(\tilde{Y}, \tilde{\iota}_R({}^{s}(\mathscr{I})))$ is called a nano soft ${}^{s}(\mathscr{I})$-regular space.*

Example 3.2. (i) Let $(\tilde{Y}, \tilde{\iota}_R({}^{s}(\mathscr{I})))$ be a nano soft ${}^{s}(\mathscr{I})$ TS where $\tilde{Y} = \{a, b\}, \mathscr{A} = \{p_1, p_2\}, \mathscr{F}_{\mathscr{A}} = \{(p_1, \{a\}), (p_2, \{b\}), (p_1, \{a, b\}), (p_2, \{a, b\})\}$ where ${}^{s}(\mathscr{I}) = \{\emptyset, (p_2, \{b\}), (p_2, \{a, b\})\}$ such that $\mathscr{F}(p_1) = \{a\}, \mathscr{F}(p_2) = \{b\}, \mathscr{F}(p_1) = \{a, b\}, \mathscr{F}(p_2) = \{a, b\}$ and $\mathscr{R} = \{\mathscr{F}(p_1) \times \mathscr{F}(p_1), \mathscr{F}(p_2) \times \mathscr{F}(p_2), \mathscr{F}(p_1) \times \mathscr{F}(p_2), \mathscr{F}(p_2) \times \mathscr{F}(p_1)\}, \tilde{\tau}_R{}^{s}(\mathscr{I}) = \{\tilde{Y}, \emptyset, (p_1, \{a\}), (p_2, \{b\}), \{(p_1, \{a\}), (p_2, \{b\})\}$ where $\mathscr{L}_{\mathscr{R}}({}^{s}(\mathscr{I})) = \{p_2, \{b\}\}, \mathscr{U}_{\mathscr{R}}({}^{s}(\mathscr{I})) = \{p_1, \{a\}, p_2, \{b\}\}, \mathscr{B}_{\mathscr{R}}({}^{s}(\mathscr{I})) = \{p_1, \{a\}\}$. Then $(\tilde{Y}, \tilde{\iota}_R({}^{s}(\mathscr{I})))$ is a nano soft ${}^{s}(\mathscr{I})$ - regular space.
(ii) Let $(\tilde{Y}, \tilde{\iota}_R({}^{s}(\mathscr{I})))$ be a nano soft ${}^{s}(\mathscr{I})$ topological space where $\tilde{Y} = \{a, b, c\}, \mathscr{A} = \{p, q\}, \tilde{\iota}_R({}^{s}(\mathscr{I})) = \{\tilde{Y}, \emptyset, (p, \{b\}), (q, \{a\})\}$. Then $(\tilde{Y}, \tilde{\iota}_R({}^{s}(\mathscr{I})))$ is not a nano soft ${}^{s}(\mathscr{I})$- regular space.

Definition 3.3. *A nano soft ${}^{s}(\mathscr{I})$ TS $(\tilde{Y}, \tilde{\iota}_R({}^{s}(\mathscr{I})))$ is called a nano soft ${}^{s}(\mathscr{I})\alpha$-regular if $\forall$ soft closed set $\mathscr{F}_A$ and $\forall$ $\mathscr{E} \in \tilde{Y} - \mathscr{F}_A$, $\exists$ two disjoint soft α- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{F}_A \subset \mathscr{V}_A$.*

Example 3.4. From the example 3.2, we take two disjoint soft α- open sets $\mathscr{U}_A = (p_1, \{a\})$ and $\mathscr{V}_A = (p_2, \{a, b\})$, then $(\tilde{Y}, \tilde{\iota}_R({}^{s}(\mathscr{I})))$ is a nano soft ${}^{s}(\mathscr{I})\alpha$ -

regular space.

Definition 3.5. *A nano soft $^s(\mathscr{I})$ topological space $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is called a nano soft $^s(\mathscr{I})\beta$- regular if $\forall$ soft closed set $\mathscr{F}_A$ and $\forall$ $\mathscr{E} \in \tilde{Y} - \mathscr{F}_A$, $\exists$ two disjoint soft β- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{F}_A \subset \mathscr{V}_A$.*

Definition 3.6. *A nano soft $^s(\mathscr{I})$ topological space $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is called a nano soft $^s(\mathscr{I})\beta\alpha$- regular if $\forall$ soft β- closed set $\mathscr{F}_A$ of $\tilde{Y}$ and $\forall$ point $\mathscr{E} \in \tilde{Y} - \mathscr{F}_A$, $\exists$ two disjoint soft α- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{F}_A \subset \mathscr{V}_A$.*

Definition 3.7. *A nano soft $^s(\mathscr{I})$ TS $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is called a nano soft $^s(\mathscr{I})$ semi- regular if $\forall$ soft closed set $\mathscr{F}_A$ and $\forall$ $\mathscr{E} \in \tilde{Y} - \mathscr{F}_A$ $\exists$ two disjoint soft semi- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{F}_A \subset \mathscr{V}_A$.*

Definition 3.8. *A nano soft $^s(\mathscr{I})$ TS $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is referred to as nano soft $^s(\mathscr{I})\alpha$ semi - regular if $\forall$ soft α- closed set $\mathscr{F}_A$ of $\tilde{Y}$ and $\forall$ point $\mathscr{E} \in \tilde{Y} - \mathscr{F}_A$, $\exists$ two disjoint soft semi- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{F}_A \subset \mathscr{V}_A$.*

Definition 3.9. *A nano soft $^s(\mathscr{I})$ TS $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is referred to as nano soft $^s(\mathscr{I})\beta$ semi - regular if $\forall$ soft β- closed set $\mathscr{F}_A$ of $\tilde{Y}$ and $\forall$ point $\mathscr{E} \in \tilde{Y} - \mathscr{F}_A$, $\exists$ two disjoint soft semi- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{F}_A \subset \mathscr{V}_A$.*

Definition 3.10. *A nano soft $^s(\mathscr{I})$ TS $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is referred to as nano soft $^s(\mathscr{I})$ pre - regular if $\forall$ soft closed set $\mathscr{F}_A$ of $\tilde{Y}$ and $\forall$ point $\mathscr{E} \in \tilde{Y} - \mathscr{F}_A$, $\exists$ two disjoint soft pre- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{F}_A \subset \mathscr{V}_A$.*

Theorem 3.11. *Let $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a nano soft $^s(\mathscr{I})$ TS, then the preceding statements are equivalent.*

(a) $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is nano soft $^s(\mathscr{I})\beta\alpha$- regular.

(b) $\forall$ $\mathscr{E} \in \tilde{Y}$ and $\forall$ soft β- open set $\mathscr{U}_A$ containing $\mathscr{E}$ $\exists$ a soft α- open set $\mathscr{V}_A$ containing $\mathscr{E} \ni \mathscr{E} \in \mathscr{V}_A \subset \alpha Cl(\mathscr{V}_A) \subset \mathscr{U}_A$.

(c) $\forall$ soft β- closed set $\mathscr{F}_A$ of $\tilde{Y}$, $\cap \left\{\alpha Cl(\mathscr{V}_A)/\mathscr{F}_A \subset \mathscr{V}_A and \mathscr{V}_A \in \alpha O(\tilde{Y})\right\} = \mathscr{F}_A$.

(d) $\forall$ nonempty soft subset $\mathscr{G}_A$ of $\tilde{Y}$ and $\forall$ soft set $\mathscr{U}_A \in \beta O(\tilde{Y})$ if $\mathscr{G}_A \cap \mathscr{U}_A \neq \emptyset$, then $\exists$ soft set $\mathscr{V}_A \in \alpha O(\tilde{Y}) \ni \mathscr{G}_A \cap \mathscr{V}_A \neq \emptyset$ and $\alpha Cl(\mathscr{V}_A) \subset \mathscr{U}_A$.

(e) $\forall$ nonempty soft subset $\mathscr{G}_A$ of $\tilde{Y}$ and $\forall$ soft set $\mathscr{F}_A \in \beta\mathscr{F}_A(\tilde{Y})$ if $\mathscr{G}_A \cap \mathscr{F}_A = \emptyset$, then there exists soft set $\mathscr{V}_A, \mathscr{W}_A \in \alpha O(\tilde{Y})$ such that $\mathscr{G}_A \cap \mathscr{V}_A \neq \emptyset$, $\mathscr{F}_A \subset \mathscr{W}_A$ and $\mathscr{V}_A \cap \mathscr{W}_A = \emptyset$.

Proof. (a)$\Rightarrow$(b) Let $(\tilde{Y}, \tilde{\tau}_R(^s(\mathscr{I})))$ be a NSI $\beta\alpha$- RS. Let $\mathscr{E} \in \tilde{Y}$ and $\mathscr{U}_A$ be soft β- open set containing $\mathscr{E}$ implies $\tilde{Y} - \mathscr{U}_A$ is soft β- closed set $\ni \mathscr{E} \notin \tilde{Y} - \mathscr{U}_A$.

Therefore by given condition $\exists$ two soft α- open sets $\mathscr{V}_A$ and $\mathscr{W}_A \ni \mathscr{E} \in \mathscr{V}_A$ and $(\tilde{Y}-\mathscr{U}_A) \subset \mathscr{W}_A$ implies $(\tilde{Y}-\mathscr{W}_A) \subset \mathscr{U}_A$. Since $\mathscr{V}_A \cap \mathscr{W}_A = \emptyset$ implies $\alpha Cl(\mathscr{V}_A) \cap \mathscr{W}_A = \emptyset \Rightarrow \alpha Cl(\mathscr{V}_A) \subset (\tilde{Y}-\mathscr{W}_A) \subset \mathscr{U}_A$. Therefore $\mathscr{E} \in \mathscr{V}_A \subset \alpha Cl(\mathscr{V}_A) \subset \mathscr{U}_A$.

(b)$\Rightarrow$(c) Consider $\mathscr{F}_A$ be a soft β- closed subset of $\tilde{Y}$ and $\mathscr{E} \notin \mathscr{F}_A$ then $(\tilde{Y}-\mathscr{F}_A)$ is soft β- open set containing $\mathscr{E}$. By the given condition (b) $\exists$ soft α- open set $\mathscr{U}_A \ni \mathscr{E} \in \mathscr{U}_A \subset \alpha cl(\mathscr{U}_A) \subset (\tilde{Y}-\mathscr{F}_A)$ implies $\mathscr{F}_A \subset \tilde{Y} - \alpha cl(\mathscr{U}_A) \subset \tilde{Y} - \mathscr{U}_A$. That is $\mathscr{F}_A \subset \mathscr{V}_A \subset \tilde{Y} - \mathscr{U}_A$ where $\mathscr{V}_A = \tilde{Y} - \alpha cl(\mathscr{U}_A) \in \alpha O(\tilde{Y})$ and $\mathscr{E} \notin \mathscr{V}_A$ implies that $\mathscr{E} \notin \alpha Cl(\mathscr{V}_A)$ implies $\mathscr{E} \notin \cap \left\{\alpha Cl(\mathscr{V}_A)/\mathscr{F}_A \subset \mathscr{V}_A \in \alpha O(\tilde{Y})\right\}$. Hence $\left\{\alpha Cl(\mathscr{V}_A)/\mathscr{F}_A \subset \mathscr{V}_A \in \alpha O(\tilde{Y})\right\} = \mathscr{F}_A$.

(c)$\Rightarrow$(d) Consider $\mathscr{G}_A$ be a soft subset of $\tilde{Y}$ and $\mathscr{U}_A \in \beta O\tilde{Y} \ni \mathscr{G}_A \cap \mathscr{U}_A \neq \emptyset$ then $\exists\ \mathscr{E}_0 \in \tilde{Y}$ such that $\mathscr{E}_0 \in \mathscr{G}_A \cap \mathscr{U}_A$. Therefore $(\tilde{Y}-\mathscr{U}_A)$ is soft β- closed set not containing $\mathscr{E}_0$ implies $\mathscr{E}_0 \notin \beta Cl(\tilde{Y}-\mathscr{U}_A)$. By the given condition (c) there exists $\mathscr{W}_A \in \alpha O(\tilde{Y})$ such that $(\tilde{Y}-\mathscr{U}_A) \subset \mathscr{W}_A$ implies $\mathscr{E}_0 \notin \alpha Cl(\mathscr{W}_A)$. Put $\mathscr{V}_A = (\tilde{Y} - \alpha Cl(\mathscr{W}_A))$, then $\mathscr{V}_A$ is soft α-open set containing $\mathscr{E}_0 \Rightarrow \mathscr{G}_A \cap \mathscr{V}_A \neq \emptyset$ and $\alpha Cl(\mathscr{V}_A) \subset \alpha Cl(\tilde{Y} - Cl(\mathscr{W}_A)) \subset \alpha Cl(\tilde{Y}-\mathscr{W}_A)$. Therefore $\alpha Cl(\mathscr{V}_A) \subset \alpha Cl(\tilde{Y} - \mathscr{W}_A) \subset \mathscr{U}_A$. Then $\alpha Cl(\mathscr{V}_A) \subset \mathscr{U}_A$.

(d)$\Rightarrow$(e) Consider $\mathscr{G}_A$ be a non empty soft subset of $\tilde{Y}$ and $\mathscr{F}_A$ be soft β- closed set $\ni \mathscr{G}_A \cap \mathscr{F}_A = \emptyset$. Then $(\tilde{Y}-\mathscr{F}_A)$ is soft β- open in $\tilde{Y}$ and $\mathscr{G}_A \cap (\tilde{Y}-\mathscr{F}_A) \neq \emptyset$. Then by the given condition (d), there exist $\mathscr{V}_A \in \alpha O(\tilde{Y})$ such that $\mathscr{G}_A \cap \mathscr{V}_A \neq \emptyset$ and $\alpha Cl(\mathscr{V}_A) \subset (\tilde{Y} - \mathscr{F}_{\mathscr{A}})$. Put $\mathscr{W}_A = (\tilde{Y} - \alpha Cl(\mathscr{V}_A))$ then $\mathscr{W}_A \in \alpha O(\tilde{Y})$ such that $\mathscr{F}_A \subset \mathscr{W}_A$ and $\mathscr{W}_A \cap \mathscr{V}_A = \emptyset$.

(e)$\Rightarrow$(a) Let $\mathscr{E} \in \tilde{Y}$ be initial point and $\mathscr{F}_A$ be soft β- closed set not containing $\mathscr{E}$. Let $\mathscr{G}_A = (\tilde{Y} - \mathscr{F}_A)$ be a non empty soft β- open set containing $\mathscr{E}$ then by the given condition (e), $\exists$ disjoint soft α- open sets $\mathscr{V}_A$ and $\mathscr{W}_A \ni \mathscr{F}_A \subset \mathscr{W}_A$ and $\mathscr{G}_A \cap \mathscr{V}_A \neq \emptyset$ implies $\mathscr{E} \in \mathscr{V}_A$. Hence, $(\tilde{Y}, \tilde{\tau}_R({}^s(\mathscr{I})))$ is a nano soft ${}^s(\mathscr{I})\beta\alpha$- regular.

Definition 3.12. *A nano soft* ${}^s(\mathscr{I})$ *TS* $(\tilde{Y}, \tilde{\iota}_R({}^s(\mathscr{I})))$ *is called a nano soft* ${}^s(\mathscr{I})\beta^*$*-regular if* $\forall$ *soft* β*- closed set* $\mathscr{F}_A$ *and* $\forall\ \mathscr{E} \in \tilde{Y} - \mathscr{F}_A$*,* $\exists$ *two disjoint soft* β*- open sets* $\mathscr{U}_A$ *and* $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ *and* $\mathscr{F}_A \subset \mathscr{V}_A$*.*

Definition 3.13. *A nano soft* ${}^s(\mathscr{I})$ *TS* $(\tilde{Y}, \tilde{\iota}_R({}^s(\mathscr{I})))$ *is called a nano soft* ${}^s(\mathscr{I})$ *strongly* β^**- regular if* $\forall$ *soft* β*- closed set* $\mathscr{F}_A$ *and* $\forall\ \mathscr{E} \notin \mathscr{F}_A$*,* $\exists$ *two disjoint soft open sets* $\mathscr{U}_A$ *and* $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ *and* $\mathscr{F}_A \subset \mathscr{V}_A$*.*

Remark 3.14.

(i) Every nano soft ${}^s(\mathscr{I})\beta\alpha$*- regular is nano soft* ${}^s(\mathscr{I})\alpha$*- regular.*

(ii) *Every nano soft $^{s}(\mathscr{I})\beta\alpha$- regular is nano soft $^{s}(\mathscr{I})\beta^{*}$- regular.*

(iii) *Every nano soft $^{s}(\mathscr{I})$ strongly β^{*}- regular is nano soft $^{s}(\mathscr{I})\beta\alpha$- regular.*

Lemma 3.15. *Let $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ be a nano soft $^{s}(\mathscr{I})$ TS. Then the succeeding assertions are true.*

(i) *Suppose $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ is nano soft $^{s}(\mathscr{I})\beta^{*}$- regular space then it is nano soft $^{s}(\mathscr{I})\beta$- regular space.*

(ii) *If $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ is nano soft $^{s}(\mathscr{I})$ strongly β^{*}- regular space then it is nano soft $^{s}(\mathscr{I})\beta^{*}$- RS.*

Lemma 3.16. *Let $\mathscr{F}_A$ and $\mathscr{G}_A$ be soft subsets of $\tilde{Y}$. If $\mathscr{F}_A \in PO(\tilde{Y})$ and $\mathscr{G}_A \in \alpha O(\tilde{Y})$ then $\mathscr{F}_A \cap \mathscr{G}_A$ is soft α- open set in the subspace $\mathscr{F}_A$.*

Theorem 3.17. *Let $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ be a nano soft $^{s}(\mathscr{I})\beta\alpha$- regular space and $\mathscr{G}_{\mathscr{A}} \in PO(\tilde{Y})$ then $\mathscr{G}_A$ is nano soft $^{s}(\mathscr{I})\beta\alpha$- regular as subspace.*

Proof. Given $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ be a nano soft$^{s}(\mathscr{I})\beta\alpha$- regular space. Let $\mathscr{F}_A$ be a soft β- closed set of $\mathscr{G}_A$ and $\mathscr{E} \in \mathscr{G}_A - \mathscr{F}_A$ then $\exists$ a soft β- closed set $\mathscr{H}_A$ of $\tilde{Y} \ni \mathscr{F}_A = \mathscr{G}_A \cap \mathscr{H}_A$ and $\mathscr{E} \notin \mathscr{H}_A$. Since $\tilde{Y}$ is nano soft $^{s}(\mathscr{I})\beta\alpha$- regular, therefore $\forall$ soft β- closed set $\mathscr{H}_A$ of $\tilde{Y}$ and $\mathscr{E} \notin \mathscr{H}_A$, $\exists$ soft α- open sets $\mathscr{U}_A$ and $\mathscr{V}_A$ of $\tilde{Y} \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{H}_A \subset \mathscr{V}_A$ with $\mathscr{U}_A \cap \mathscr{V}_A = \emptyset$. Now put $\mathscr{M}_A = \mathscr{U}_A \cap \mathscr{G}_A$ and $\mathscr{N}_A = \mathscr{V}_A \cap \mathscr{G}_A$ then $\mathscr{M}_A$ and $\mathscr{N}_A$ are soft α- open subsets of $\mathscr{G}_A$(by the lemma 3.15) $\ni \mathscr{E} \in \mathscr{M}_A$ and $\mathscr{F}_A \subset \mathscr{N}_A$ with $\mathscr{M}_A \cap \mathscr{N}_A = \emptyset$. This shows that $\mathscr{G}_A$ is nano soft $^{s}(\mathscr{I})\beta\alpha$- regular space.

Definition 3.18. *A nano soft $^{s}(\mathscr{I})$ TS $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I}))$ is referred to as nano soft $^{s}(\mathscr{I})$- normal if every set of disjoint soft closed sets $\mathscr{F}_A$ and $\mathscr{G}_A$ of $\tilde{Y}$, $\exists$ disjoint soft open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{F}_A \subset \mathscr{U}_A$ and $\mathscr{G}_A \subset \mathscr{V}_A$.*

Example 3.19. Let $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ be a nano soft $^{s}(\mathscr{I})$ TS where $\tilde{Y} = \{l, m\}, \mathscr{A} = \{x_1, x_2\}, \mathscr{F}_{\mathscr{A}} = \{(x_1, \{l\}), (x_2, \{m\}), (x_1, \{l, m\}), (x_2, \{l, m\})\}$ where $^{s}(\mathscr{I}) = \{\emptyset, (x_2, \{m\}), (x_2, \{l, m\})\}$ such that $\mathscr{F}(x_1) = \{l\}, \mathscr{F}(x_2) = \{m\}, \mathscr{F}(x_1) = \{l, m\}, \mathscr{F}(x_2) = \{l, m\}$ and $\mathscr{R} = \{\mathscr{F}(x_1) \times \mathscr{F}(x_1), \mathscr{F}(x_2) \times \mathscr{F}(x_2), \mathscr{F}(x_1) \times \mathscr{F}(x_2), \mathscr{F}(x_2) \times \mathscr{F}(x_1)\}$, $\tilde{\iota}_R{}^{s}(\mathscr{I}) = \{\tilde{Y}, \emptyset, (x_1, \{l\}), (x_2, \{m\}), \{(x_1, \{l\}), (x_2, \{m\})\}$ where $\mathscr{L}_{\mathscr{R}}(^{s}(\mathscr{I})) = \{x_2, \{m\}\}, \mathscr{U}_{\mathscr{R}}(^{s}(\mathscr{I})) = \{x_1, \{l\}, x_2, \{m\}\}, \mathscr{B}_{\mathscr{R}}(^{s}(\mathscr{I})) = \{x_1, \{l\}\}$. Then $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ is a nano soft $^{s}(\mathscr{I})$ - normal space.

Definition 3.20. *A nano soft $^{s}(\mathscr{I})$ topological space $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ is called a nano soft $^{s}(\mathscr{I})\alpha$- normal if every set of disjoint soft closed sets $\mathscr{F}_A$ and $\mathscr{G}_A$ of $\tilde{Y}$, $\exists$ disjoint soft α- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{F}_A \subset \mathscr{U}_A$ and $\mathscr{G}_A \subset \mathscr{V}_A$.*

Definition 3.21. *A nano soft $^s(\mathscr{I})$ topological space $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is called a nano soft $^s(\mathscr{I})\beta$- normal if every set of disjoint soft closed sets $\mathscr{F}_A$ and $\mathscr{G}_A$ of $\tilde{Y}$, $\exists$ disjoint soft β- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{F}_A \subset \mathscr{U}_A$ and $\mathscr{G}_A \subset \mathscr{V}_A$.*

Definition 3.22. *A nano soft $^s(\mathscr{I})$ TS $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is called a nano soft $^s(\mathscr{I})$ semi-normal if every set of disjoint soft closed sets $\mathscr{F}_A$ and $\mathscr{G}_A$ of $\tilde{Y}$, $\exists$ disjoint soft semi- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{F}_A \subset \mathscr{U}_A$ and $\mathscr{G}_A \subset \mathscr{V}_A$.*

Definition 3.23. *A nano soft $^s(\mathscr{I})$ topological space $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is referred to as nano soft $^s(\mathscr{I})\beta\alpha$- normal if every set of disjoint soft β- closed sets $\mathscr{F}_A$ and $\mathscr{G}_A$ of $\tilde{Y}$, $\exists$ disjoint soft α- open sets $\mathscr{U}_A$ and $\mathscr{V}_A$ such that $\mathscr{F}_A \subset \mathscr{U}_A$ and $\mathscr{G}_A \subset \mathscr{V}_A$.*

Example 3.24. By the example 3.19, we take disjoint soft α- open sets $\mathscr{U}_A = (x_2, \{m\})$ and $\mathscr{V}_A = (x_1, \{l\})$ such that $\mathscr{F}_A \subset \mathscr{U}_A$ and $\mathscr{G}_A \subset \mathscr{V}_A$, then $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is nano soft $^s(\mathscr{I})\beta\alpha$- normal.

Remark 3.25. *Every nano soft $^s(\mathscr{I})\beta\alpha$- normal is nano soft $^s(\mathscr{I})\alpha$- normal.*

Theorem 3.26. *Let $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a nano soft $^s(\mathscr{I})\beta\alpha$- NS and $\mathscr{F}_A \in P(O(\tilde{Y})$, then $\mathscr{F}_A$ is nano soft $^s(\mathscr{I})\beta\alpha$- NS.*

Proof. Let $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a nano soft $^s(\mathscr{I})\beta\alpha$- NS and $\mathscr{F}_A$ be an soft pre-open set of $\tilde{Y}$. Consider $\mathscr{U}_A$ and $\mathscr{V}_A$ be disjoint soft β- closed subsets of $\mathscr{F}_A$. Then $\mathscr{U}_A = \mathscr{F}_A \cap \mathscr{M}_A$ and $\mathscr{F}_A \cap \mathscr{N}_A$ where $\mathscr{M}_A$ and $\mathscr{N}_A$ are disjoint soft β- closed sets of $\tilde{Y}$. As a result, $(\tilde{Y}, \tilde{\tau}_R(^s(\mathscr{I})))$ is nano soft $^s(\mathscr{I})\beta\alpha$- normal, $\exists$ disjoint soft α- open sets $\mathscr{G}_A$ and $\mathscr{H}_A$ of $\tilde{Y} \ni \mathscr{U}_A \subset \mathscr{G}_A$ and $\mathscr{V}_A \subset \mathscr{H}_A$ implies that $(\mathscr{F}_A \cap \mathscr{M}_A) \subset (\mathscr{F}_A \cap \mathscr{G}_A)$, $(\mathscr{F}_A \cap \mathscr{N}_A) \subset (\mathscr{F}_A \cap \mathscr{H}_A)$ where $(\mathscr{F}_A \cap \mathscr{G}_A)$ and $(\mathscr{F}_A \cap \mathscr{H}_A)$ are soft α- open subsets of $\mathscr{F}_A$ with $(\mathscr{F}_A \cap \mathscr{G}_A) \cap (\mathscr{F}_A \cap \mathscr{H}_A) = \emptyset$. Then $\mathscr{F}_A$ is nano soft $^s(\mathscr{I})$ $\beta\alpha$- normal space.

4. Nano Soft $^s(\mathscr{I})\alpha\beta$- Regular Spaces and Normal Spaces

In this section we define $\alpha\beta$- regular and normal spaces in nano soft $^s(\mathscr{I})$ topological space and their properties are discussed.

Definition 4.1. *A nano soft $^s(\mathscr{I})$ topological space $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is called a nano soft $^s(\mathscr{I})\alpha\beta$- regular if $\forall$ soft α- closed set $\mathscr{F}_A$ of $\tilde{Y}$ and $\forall$ point $\mathscr{E} \in \tilde{Y} - \mathscr{F}_A$, $\exists$ two disjoint soft β- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{E} \in \mathscr{U}_A$ and $\mathscr{F}_A \subset \mathscr{V}_A$.*

Example 4.2. Let $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a nano soft $^s(\mathscr{I})$ TS where $\tilde{Y} = \{x, y\}, \mathscr{A} = \{q_1, q_2\}, \mathscr{F}_{\mathscr{A}} = \{(q_1, \{x\}), (q_2, \{y\}), (q_1, \{x, y\}), (q_2, \{x, y\})\}$ where $^s(\mathscr{I}) = \{\emptyset, (q_2, \{y\}), (q_2, \{x, y\})\}$ such that $\mathscr{F}(q_1) = \{x\}, \mathscr{F}(q_2) = \{y\}, \mathscr{F}(q_1) = \{x, y\}, \mathscr{F}(q_2) = \{x, y\}$ and $\mathscr{R} = \{\mathscr{F}(q_1) \times \mathscr{F}(q_1), \mathscr{F}(q_2) \times \mathscr{F}(q_2), \mathscr{F}(q_1) \times \mathscr{F}(q_2), \mathscr{F}(q_2) \times \mathscr{F}(q_1)\}$, $\tilde{\tau}_R{}^s(\mathscr{I}) = \{\tilde{Y}, \emptyset, (q_1, \{x\}), (q_2, \{y\}), \{(q_1, \{x\}), (q_2, \{y\})\}$ where $\mathscr{L}_{\mathscr{R}}(^s(\mathscr{I})) = \{q_2, \{y\}\}$, $\mathscr{U}_{\mathscr{R}}(^s(\mathscr{I})) = \{q_1, \{x\}, q_2, \{y\}\}, \mathscr{B}_{\mathscr{R}}(^s(\mathscr{I})) = \{q_1, \{x\}\}$. Take $\mathscr{U}_A = \{q_1, \{x\}\}$

and $\mathscr{V}_A = \{q_2, \{y\}\}$. Then $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is a nano soft $^s(\mathscr{I})\alpha\beta$ - regular space.

Remark 4.3. *Every nano soft $^s(\mathscr{I})\beta^*$ regular is nano soft $^s(\mathscr{I})\alpha\beta$- regular.*

Definition 4.4. *A nano soft $^s(\mathscr{I})$ topological space $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is called a nano soft $^s(\mathscr{I})\alpha\beta$-normal if every set of disjoint soft α- closed sets $\mathscr{F}_A$ and $\mathscr{G}_A$ of $\tilde{Y}$, $\exists$ disjoint soft β- open sets $\mathscr{U}_A$ and $\mathscr{V}_A \ni \mathscr{F}_A \subset \mathscr{U}_A$ and $\mathscr{G}_A \subset \mathscr{V}_A$.*

Theorem 4.5. *Assume $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a nano soft $^s(\mathscr{I})$ TS, then the preceding statements are equivalent.*

(a) $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ *is nano soft $^s(\mathscr{I})\alpha\beta$- regular.*

(b) $\forall$ $\mathscr{E} \in \tilde{Y}$ *and* $\forall$ *soft α- open set $\mathscr{U}_A$ containing $\mathscr{E}$ $\exists$ a soft β- open set $\mathscr{V}_A$ containing $\mathscr{E} \ni \mathscr{E} \in \mathscr{V}_A \subset \beta Cl(\mathscr{V}_A) \subset \mathscr{U}_A$.*

(c) $\forall$ *soft α- closed set $\mathscr{F}_A$ of $\tilde{Y}$,* $\cap \left\{\beta Cl(\mathscr{V}_A)/\mathscr{F}_A \subset \mathscr{V}_A and \mathscr{V}_A \in \beta O(\tilde{Y})\right\} = \mathscr{F}_A$.

(d) $\forall$ *nonempty soft subset $\mathscr{G}_A$ of $\tilde{Y}$ and $\forall$ soft set $\mathscr{U}_A \in \alpha O(\tilde{Y})$ if $\mathscr{G}_A \cap \mathscr{U}_A \neq \emptyset$, then there exists soft set $\mathscr{V}_A \in \beta O(\tilde{Y}) \ni \mathscr{G}_A \cap \mathscr{V}_A \neq \emptyset$ and $\beta Cl(\mathscr{V}_A) \subset \mathscr{U}_A$.*

(e) $\forall$ *nonempty soft subset $\mathscr{G}_A$ of $\tilde{Y}$ and $\forall$ soft set $\mathscr{F}_A \in \alpha\mathscr{F}_A(\tilde{Y})$ if $\mathscr{G}_A \cap \mathscr{F}_A = \emptyset$, then there exists soft set $\mathscr{V}_A, \mathscr{W}_A \in \beta O(\tilde{Y})$ such that $\mathscr{G}_A \cap \mathscr{V}_A \neq \emptyset$, $\mathscr{F}_A \subset \mathscr{W}_A$ and $\mathscr{V}_A \cap \mathscr{W}_A = \emptyset$.*

Proof. This proof is similar to the theorem 3.11.

5. Conclusion

This paper presented the idea of Nano Soft $^s(\mathscr{I})\beta\alpha$- Regular Spaces and Normal Spaces via soft ideal sets. Further we define Nano Soft $^s(\mathscr{I})\alpha$- Regular and Normal Spaces, Nano Soft $^s(\mathscr{I})\beta$- Regular and Normal Spaces, Nano Soft $^s(\mathscr{I})\alpha\beta$- Regular and Normal Spaces, Nano Soft $^s(\mathscr{I})$ Semi- Regular and Normal Spaces, Nano Soft $^s(\mathscr{I})$ Pre- Regular and Normal Spaces. Likewise, their features are explored with illustration.

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