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A NEW DISTANCE MEASURE BETWEEN FERMATEAN NEUTROSOPHIC SETS AND ITS APPLICATION IN CROP FARMING

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Abstract: The complexity of real-world scenarios often leads to uncertainty, prompting the introduction of neutrosophic theory as a tool for problem-solving. This paper aims to introduce a new distance measure on Fermatean neutrosophic sets and validate it through the axiomatic properties of distance measure. Additionally, it examines several characteristics of the distance measure and conducts a comparative analysis with existing distance measures on Fermatean neutrosophic sets. Finally, to demonstrate its practical relevance, we apply the proposed distance measure to address the problems associated with decision-making in crop farming within a Fermatean neutrosophic framework.

Keywords and Phrases: Fermatean fuzzy set, Neutrosophic set, Fermatean neutrosophic set.

2020 Mathematics Subject Classification: 03E72, 03E75, 26E50.

1. Introduction

In 1965, Zadeh [16] introduced the concept of the fuzzy set to address ambiguity and unpredictability in real-life circumstances. In 1986, Atanassov [2] developed the idea of intuitionistic fuzzy set theory as an extension of fuzzy set theory since Zadeh's fuzzy set only addressed the membership function. This helped to investigate the non-membership of the attribute. In numerous real-life scenarios, it is possible for the total of the membership degree (MD) and non-membership degree (NMD) to extend 1. In an attempt to get around these challenges, Yager [15] expanded the intuitionistic fuzzy set theory to create the Pythagorean fuzzy set (PyFS). PyFS is defined by the criterion that the sum of the squares of MD and NMD is less than or equal to 1. Later, Fermatean fuzzy set(FFS) was proposed by Senapati and Yager [10] as a method for more readily managing uncertain information. When handling uncertain information, Feramtean fuzzy set performs better than intuitionistic fuzzy sets and Pythagorean fuzzy sets in terms of flexibility and efficiency since Fermatean fuzzy set is defined by the criterion that the sum of the cubes of MD and NMD is less than or equal to 1. Smarandache [11] introduced the concept of Neutrosophic set (NS) where each element has the membership degree, non-membership degree and hesitation degree in the non-standard unit interval and no further restriction on the degree of Neutrosophic sets. C. Antony Crispin Sweety [12] proposed the concept of Fermatean neutrosophic sets and its algebraic properties in 2021.

Distance measures serve as essential tools across various domains, including decision-making, optimization, image processing, and pattern recognition, providing a quantitative way to assess similarity or dissimilarity between items. Different distance measures for the Fermatean fuzzy set are proposed. Yager and Senapati [10] introduced the Euclidean distance measure for FFSs. Afterward, the Hellinger distance and a distance measure based on triangular divergence were proposed by Zhan Deng and Jianyu Wang [4]. After that, Abdul Haseeb Ganie et al. [5] proposed a completely novel Fermatean fuzzy distance. The Tanimoto distance metric for FFSs was extended by Hongpeng Wang et al. [14]. Since the distance measure for neutroscopic sets is vital in many fields of study, numerous researchers are investigating it. Normalized hamming and Euclidean distance for NS were developed by P. Majumdar and S. K. Samanta [6]. The Hausdroff distance for NS was proposed by A. Awang et al. [3] and used to investigate coastal erosion. V. Vakkas et al. [13] created a new hybrid distance for refined neutrosophic sets. V. Antonysamy [1] suggested the Hausdorff minimum distance for the neutrosophic set, while N. Mustapha et al. [7] employed a novel distance metric in cardiovascular disease risk analysis. Afterward, N. Mustapha [8] created a generalized distance metric, which can be used to analyze coronavirus diseases. A distance measure for Fermatean neutrosophic sets was recently established by Muhammad Saeed et al. [9], and a decision-making model is constructed.

The paper is organized as follows: In Section 2, we recall the essential knowledge of intuitionistic fuzzy sets, Fermatean fuzzy sets, and Fermatean neutrosophic sets. In Section 3, we propose a novel distance measure on the Fermatean neutrosophic set and prove some properties. In order to demonstrate the efficacy and consistency of the suggested distance measurement, a comparative analysis with existing distance measures is carried out in Section 4. We demonstrate in Section 5 how the newly established distance metric is used in crop cultivation. Finally, we make a conclusion in Section 6.

2. Preliminaries

This section goes over a few key ideas about intuitionistic fuzzy set, Pythagorean fuzzy set, Fermatean fuzzy set, Neutrosophic set and Fermatean neutrosophic sets.

Definition 2.1. Intuitionistic fuzzy set [2] *Suppose that U is a non-empty set. The intuitionistic fuzzy set E in U is defined as*

$$E = \{(u, \mu_E(u), \nu_E(u)) : u \in U\}$$

where $\mu_E : U \to [0,1]$ denotes the membership degree of u in E and $\nu_E : U \to [0,1]$ denotes the non-membership degree of u in E such that $0 \leq \mu_E(u) + \nu_E(u) \leq 1$ for any u in U. Also $\pi_A(u) = 1 - \mu_E(u) - \nu_E(u)$ is called indeterminacy degree of each $u \in U$.

Definition 2.2. Fermatean fuzzy set [10] *Let U be a non-empty set. A Fermatean fuzzy set F is defined as*

$$F = \{(u, \mu_F(u), \nu_F(u)) : u \in U\}$$

where $\mu_F : U \to [0,1]$ is the membership degree and $\nu_F : U \to [0,1]$ is the non-membership degree such that $(\mu_F(u))^3 + (\nu_F(u))^3 \leq 1$ for each $u \in U$. Here the degree of indeterminacy of each $u \in U$ is $\pi_F(u) = (1 - (\mu_F(u))^3 - (\nu_F(u))^3)^{\frac{1}{3}}$.

Definition 2.3. Neutrosophic set [11] *Let U be a non-empty set. A neutrosophic set N is defined as*

$$N = \{(u, \mu_N(u), \gamma_N(u), \sigma_N(u)) : u \in U\}$$

where $\mu_N(u) : U \to [0,1]$ represents the degree of membership, $\sigma_N(u) : U \to [0,1]$ represents the degree of non-membership, $\gamma_N(u) : U \to [0,1]$ represents the degree of indeterminacy and $\mu_N(u) + \sigma_N(u) + \gamma_N(u)$ lies in $[0,3]$ for all $u \in U$.

Definition 2.4. Fermatean Neutrosophic set [12] *Let U be a non-empty set. A Feramatean neutrosophic set L on U is defined as*

$$L = \{(u, \mu_L(u), \gamma_L(u), \sigma_L(u)) : u \in U\}$$

where $\mu_L(u) : U \to [0,1]$ represents the degree of membership, $\sigma_L(u) : U \to [0,1]$ represents the degree of non-membership, $\gamma_L(u) : U \to [0,1]$ represents the degree of indeterminacy such that $0 \leq (\mu_L(u))^3 + (\sigma_L(u))^3 \leq 1$ and $0 \leq (\gamma_L(u))^3 \leq 1$. Then $0 \leq (\mu_L(u))^3 + (\gamma_L(u))^3 + (\sigma_L(u))^3 \leq 2$ for all $u \in U$. Here $\mu_L(u)$ and $\sigma_L(u)$ are dependent components and $\gamma_L(u)$ is an independent component.

Definition 2.5. [12] *Let U be a non-empty set with $H = \{(u, X_{H(u)}, \Psi_{H(u)}, \Omega_{H(u)}) : u \in U\}$ and $K = \{(u, X_{K(u)}, \Psi_{K(u)}, \Omega_{K(u)}) : u \in U\}$ are two Fermatean neutrosophic sets, then*
$H^c = \{(u, \Omega_{H(u)}, 1 - \Psi_{H(u)}, X_{H(u)}) : u \in U\}$
$H \cup K = \{(u, max(X_{H(u)}, X_{K(u)}), min(\Psi_{H(u)}, \Psi_{K(u)}), min(\Omega_{H(u)}, \Omega_{K(u)})) : u \in U\}$
$H \cap K = \{(u, min(X_{H(u)}, X_{K(u)}), max(\Psi_{H(u)}, \Psi_{K(u)}), max(\Omega_{H(u)}, \Omega_{K(u)})) : u \in U\}$.

Definition 2.6. [9] *Let $H = \{(u_i, X_{H(u_i)}, \Psi_{H(u_i)}, \Omega_{H(u_i)}) : u_i \in U\}$ and $K = \{(u_i, X_{K(u_i)}, \Psi_{K(u_i)}, \Omega_{K(u_i)}) : u_i \in U\}$ be Fermatean neutrosophic sets, then Hamming distance between H and K is defined as*
$d_{\mathbb{HM}}(H, K) = \frac{1}{3}\sum_{i=1}^{n} \omega_i(|X^3_{H(u_i)} - X^3_{K(u_i)}| + |\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)}| + |\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)}|)$
where ω_i represents the weight of each element with ω_i is non-negative for all $i = 1, 2, ..., n$ and $\sum_{i=1}^{n} \omega_i = 1$.

Definition 2.7. [9] *Let $H = \{(u_i, X_{H(u_i)}, \Psi_{H(u_i)}, \Omega_{H(u_i)}) : u_i \in U\}$ and $K = \{(u_i, X_{K(u_i)}, \Psi_{K(u_i)}, \Omega_{K(u_i)}) : u_i \in U\}$ are Fermatean neutrosophic sets, then Euclidean distance between H and K is defined as*
$d_{\mathbb{EU}}(H, K) = [\frac{1}{3}\sum_{i=1}^{n} \omega_i(|X^3_{H(u_i)} - X^3_{K(u_i)}|^2 + |\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)}|^2 + |\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)}|^2)]^{\frac{1}{2}}$
where ω_i represents the weight of each element with ω_i is non-negative for all $i = 1, 2, ..., n$ and $\sum_{i=1}^{n} \omega_i = 1$.

3. New Distance Measure on Fermatean Neutrosophic sets

We propose a novel distance measure for the Fermatean neutrosophic sets in this section and show that it satisfies the fundamental requirements of a distance metric. Additionally, we talk about a few algebraic properties of this distance measure.

Definition 3.1. *Let $U = \{u_1, u_2, ..., u_n\}$ be a non-empty set. Let H and K be two fermatean neutrosophic sets represented by $H = \{(u_i, X_{H(u_i)}, \Psi_{H(u_i)}, \Omega_{H(u_i)}) : u_i \in U\}$ and $K = \{(u_i, X_{K(u_i)}, \Psi_{K(u_i)}, \Omega_{K(u_i)}) : u_i \in U\}$, where $X_{H(u_i)}$ is the membership degree of u_i in H, $\Psi_{H(u_i)}$ is the indeterminacy degree of u_i in H, $\Omega_{H(u_i)}$ is the non-membership degree of u_i in H, $X_{K(u_i)}$ is the membership degree of u_i in K, $\Psi_{K(u_i)}$ is the indeterminacy degree of u_i in K, and $\Omega_{K(u_i)}$ is the non-membership degree of u_i in K. Then the distance measure($\ddot{d}$) between H and K is denoted by*

$$\ddot{d}(H,K) = [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2 + max\{(X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}}$$

Theorem 3.1. *Let H and K be Fermatean neutrosophic sets in U and $\ddot{d}(H,K)$ be the distance measure between H and K, then the following properties hold:*
(i) $\ddot{d}(H,K) = 0$ if and only if $H = K$
(ii) $\ddot{d}(H,K) = \ddot{d}(K,H)$
(iii)$0 \le \ddot{d}(H,K) \le 1$
(iv) If $H \subseteq K \subseteq L$ then $\ddot{d}(H,K) \le \ddot{d}(H,L)$ and $\ddot{d}(K,L) \le \ddot{d}(H,L)$

Proof. (i) $\ddot{d}(H,K) = 0$ if and only if $H = K$
Necessarily if $\ddot{d}(H,K) = 0$ then
$X^3_{H(u_i)} = X^3_{K(u_i)}, \Psi^3_{H(u_i)} = \Psi^3_{K(u_i)}, \Omega^3_{H(u_i)} = \Omega^3_{K(u_i)} \forall u_i \in U$
i.e., $X_{H(u_i)} = X_{K(u_i)}, \Psi_{H(u_i)} = \Psi_{K(u_i)}, \Omega_{H(u_i)} = \Omega_{K(u_i)} \forall u_i \in U$
i.e., $H = K$, Therefore $\ddot{d}(H,K) = 0$ then $H = K$
Conversely if $H = K$ then
$X^3_{H(u_i)} = X^3_{K(u_i)}, \Psi^3_{H(u_i)} = \Psi^3_{K(u_i)}, \Omega^3_{H(u_i)} = \Omega^3_{K(u_i)} \forall u_i \in U$

$$\ddot{d}(H,K) = [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2 + max\{(X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}} = 0$$

So we can conclude that $\ddot{d}(H,K) = 0$ if and only if $H = K$
(ii) $\ddot{d}(H,K) = \ddot{d}(K,H)$

$$\begin{aligned}\ddot{d}(H,K) &= [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2 \\ &\quad + max\{(X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}} \\ &= [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{K(u_i)} - X^3_{H(u_i)})^2 + (\Omega^3_{K(u_i)} - \Omega^3_{H(u_i)})^2 + (\Psi^3_{K(u_i)} - \Psi^3_{H(u_i)})^2 \\ &\quad + max\{(X^3_{K(u_i)} - X^3_{H(u_i)})^2, (\Omega^3_{K(u_i)} - \Omega^3_{H(u_i)})^2, (\Psi^3_{K(u_i)} - \Psi^3_{H(u_i)})^2\})]^{\frac{1}{2}} \\ &= \ddot{d}(K,H)\end{aligned}$$

(iii) $0 \le \ddot{d}(H,K) \le 1$
Obviously $\ddot{d}(H,K) \ge 0$

Since H and K are Fermatean neutrosophic set, for any $u_i \in U$,
$0 \leq X^3_{H(u_i)} + \Omega^3_{H(u_i)} \leq 1$, $0 \leq \Psi^3_{H(u_i)} \leq 1$ and $0 \leq X^3_{K(u_i)} + \Omega^3_{K(u_i)} \leq 1$, $0 \leq \Psi^3_{K(u_i)} \leq 1$
Since $0 \leq X_{H(u_i)} \leq 1$, $0 \leq X^3_{H(u_i)} \leq 1$ and $0 \leq X^3_{K(u_i)} \leq 1$
Thus $0 \leq |X^3_{H(u_i)} - X^3_{K(u_i)}| \leq 1$
Similarly, we get $0 \leq |\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)}| \leq 1$ and $0 \leq |\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)}| \leq 1$

$$
\begin{aligned}
&(X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2 \\
&\quad + max\{(X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\} \\
&\leq X^3_{H(u_i)} + \Omega^3_{H(u_i)} + 1 + 1 \\
&\leq 3 \\
&\frac{1}{3n}\sum_{i=1}^{n}((X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2 \\
&+ max\{(X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\}) \leq 1
\end{aligned}
$$

So, $\ddot{d}(H, K) \leq 1$
Therefore, $0 \leq \ddot{d}(H, K) \leq 1$
(iv) If $H \subseteq K \subseteq L$ then $\ddot{d}(H, K) \leq \ddot{d}(H, L)$ and $\ddot{d}(K, L) \leq \ddot{d}(H, L)$
Since $H \subseteq K \subseteq L$, for any $u_i \in U$,

$$
\begin{gathered}
X^3_{H(u_i)} \leq X^3_{K(u_i)} \leq X^3_{L(u_i)} \\
\Omega^3_{H(u_i)} \geq \Omega^3_{K(u_i} \geq \Omega^3_{L(u_i)} \\
\Psi^3_{H(u_i)} \leq \Psi^3_{K(u_i)} \leq \Psi^3_{L(u_i)} \\
(X^3_{H(u_i)} - X^3_{K(u_i)})^2 \leq (X^3_{H(u_i)} - X^3_{L(u_i)})^2 \\
(\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 \leq (\Omega^3_{H(u_i)} - \Omega^3_{L(u_i)})^2 \\
(\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2 \leq (\Psi^3_{H(u_i)} - \Psi^3_{L(u_i)})^2
\end{gathered}
$$

Therefore,

$$
\begin{aligned}
&(X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2 \\
&\qquad + max\{(X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\} \\
&\leq (X^3_{H(u_i)} - X^3_{L(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{L(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{L(u_i)})^2 \\
&\qquad + max\{(X^3_{H(u_i)} - X^3_{L(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{L(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{L(u_i)})^2\}
\end{aligned}
$$

Thus $\ddot{d}(H, K) \leq \ddot{d}(H, L)$
Similarly, we can prove $\ddot{d}(K, L) \leq \ddot{d}(H, L)$

Definition 3.2. *Let U be a non-empty set and H be a Fermatean neutrosophic set defined as $H = \{(u, X_{H(u)}, \Psi_{H(u)}, \Omega_{H(u)}) : u \in U\}$ then H' is a Fermatean neutrosophic set defined as*
$H' = \{(u, \Omega_{H(u)}, \Psi_{H(u)}, X_{H(u)}) : u \in U\}$

Theorem 3.2. *Let H and K are two Fermatean neutrosophic sets, then the distance measure $\ddot{d}$ has the following properties:*
(i) $\ddot{d}(H \cap K, K) \leq \ddot{d}(H, K)$
(ii) $\ddot{d}(H \cup K, K) \leq \ddot{d}(H, K)$
(iii) $\ddot{d}(H', K') = \ddot{d}(H, K)$
(iv) $\ddot{d}(H, K') = \ddot{d}(H', K)$
(v) $\ddot{d}(H, H') = 0$ *if and only if* $X_{H(u_i)} = \Omega_{H(u_i)}, \forall u_i \in U$

Proof. Let $H = \{(u_i, X_{H(u_i)}, \Psi_{H(u_i)}, \Omega_{H(u_i)}) : u_i \in U\}$ and
$K = \{(u_i, X_{K(u_i)}, \Psi_{K(u_i)}, \Omega_{K(u_i)}) : u_i \in U\}$ then,
$H' = \{(u_i, \Omega_{H(u_i)}, \Psi_{H(u_i)}, X_{H(u_i)}) : u_i \in U\}$
$K' = \{(u_i, \Omega_{K(u_i)}, \Psi_{K(u_i)}, X_{K(u_i)}) : u_i \in U\}$
$H \cap K = \{(u_i, min(X_{H(u_i)}, X_{K(u_i)}), max(\Psi_{H(u_i)}, \Psi_{K(u_i)}), max(\Omega_{H(u_i)}, \Omega_{K(u_i)})) : u_i \in U\}$
$H \cup K = \{(u_i, max(X_{H(u_i)}, X_{K(u_i)}), min(\Psi_{H(u_i)}, \Psi_{K(u_i)}), min(\Omega_{H(u_i)}, \Omega_{K(u_i)})) : u_i \in U\}$
(i) $\ddot{d}(H \cap K, K) \leq \ddot{d}(H, K)$

$$\begin{aligned}
&\ddot{d}(H \cap K, K) \\
&= [\frac{1}{3n}\sum_{i=1}^{n}(((min(X_{H(u_i)}, X_{K(u_i)}))^3 - X^3_{K(u_i)})^2 \\
&\quad + ((max(\Omega_{H(u_i)}, \Omega_{K(u_i)}))^3 - \Omega^3_{K(u_i)})^2 + ((max(\Psi_{H(u_i)}, \Psi_{K(u_i)}))^3 - \Psi^3_{K(u_i)})^2 \\
&\quad + max\{((min(X_{H(u_i)}, X_{K(u_i)}))^3 - X^3_{K(u_i)})^2, ((max(\Omega_{H(u_i)}, \Omega_{K(u_i)}))^3 - \Omega^3_{K(u_i)})^2, \\
&\quad ((max(\Psi_{H(u_i)}, \Psi_{K(u_i)}))^3 - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}} \\
&\leq [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2 \\
&\quad + max\{(X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}} \\
&\leq \ddot{d}(H, K)
\end{aligned}$$

(ii) $\ddot{d}(H \cup K, K) \leq \ddot{d}(H, K)$

$$
\begin{aligned}
&\ddot{d}(H \cup K, K)\\
&= [\frac{1}{3n}\sum_{i=1}^{n}(((max(X_{H(u_i)}, X_{K(u_i)}))^3 - X^3_{K(u_i)})^2\\
&\quad + ((min(\Omega_{H(u_i)}, \Omega_{K(u_i)}))^3 - \Omega^3_{K(u_i)})^2 + ((min(\Psi_{H(u_i)}, \Psi_{K(u_i)}))^3 - \Psi^3_{K(u_i)})^2\\
&\quad + max\{((max(X_{H(u_i)}, X_{K(u_i)}))^3 - X^3_{K(u_i)})^2, ((min(\Omega_{H(u_i)}, \Omega_{K(u_i)}))^3 - \Omega^3_{K(u_i)})^2,\\
&\quad ((min(\Psi_{H(u_i)}, \Psi_{K(u_i)}))^3 - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}}\\
&\le [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\\
&\quad + max\{(X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}}\\
&\le \ddot{d}(H, K)
\end{aligned}
$$

(iii) $\ddot{d}(H', K') = \ddot{d}(H, K)$

$$
\begin{aligned}
\ddot{d}(H', K') &= [\frac{1}{3n}\sum_{i=1}^{n}((\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\\
&\quad + max\{(\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}}\\
&= [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\\
&\quad + max\{(X^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}}\\
&= \ddot{d}(H, K)
\end{aligned}
$$

(iv) $\ddot{d}(H, K') = \ddot{d}(H', K)$

$$
\begin{aligned}
\ddot{d}(H, K') &= [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Omega^3_{H(u_i)} - X^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\\
&\quad + max\{(X^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Omega^3_{H(u_i)} - X^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}}\\
&= [\frac{1}{3n}\sum_{i=1}^{n}((\Omega^3_{H(u_i)} - X^3_{K(u_i)})^2 + (X^3_{H(u_i)} - \Omega^3_{K(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\\
&\quad + max\{(\Omega^3_{H(u_i)} - X^3_{K(u_i)})^2, (X^3_{H(u_i)} - \Omega^3_{K(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{K(u_i)})^2\})]^{\frac{1}{2}}\\
&= \ddot{d}(H', K)
\end{aligned}
$$

(v) $\ddot{d}(H, H') = 0$ if and only if $X_{H(u_i)} = \Omega_{H(u_i)}, \forall u_i \in U$

$$\begin{aligned}
&\ddot{d}(H, H') = 0 \\
&\Longleftrightarrow [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{H(u_i)} - \Omega^3_{H(u_i)})^2 + (\Omega^3_{H(u_i)} - X^3_{H(u_i)})^2 + (\Psi^3_{H(u_i)} - \Psi^3_{H(u_i)})^2 \\
&\qquad + max\{(X^3_{H(u_i)} - \Omega^3_{H(u_i)})^2, (\Omega^3_{H(u_i)} - X^3_{H(u_i)})^2, (\Psi^3_{H(u_i)} - \Psi^3_{H(u_i)})^2\})]^{\frac{1}{2}} = 0 \\
&\Longleftrightarrow X^3_{H(u_i)} = \Omega^3_{H(u_i)}, \forall u_i \in U \\
&\Longleftrightarrow X_{H(u_i)} = \Omega_{H(u_i)}, \forall u_i \in U
\end{aligned}$$

Remark. *The theorem 3.2 will not hold if we replace H' with H^c. For example, consider $H = \{(a, 0.2, 0.3, 0.5)\}$ and $K = \{(a, 0.6, 0.4, 0.1)\}$ then $H^c = \{(a, 0.5, 0.7, 0.2)\}$ and $K^c = \{(a, 0.1, 0.6, 0.6)\}$. Using definition 3.1, we get $\ddot{d}(H, K) = 0.1855$, $\ddot{d}(H^c, K^c) = 0.1984$, $\ddot{d}(H, K^c) = 0.1631$ and $\ddot{d}(H^c, K) = 0.2338$. In the same way, if $H = \{(a, 0.5, 0.2, 0.5)\}$ then $\ddot{d}(H, H^c) \neq 0$, even though $X_{H(a)} = \Omega_{H(a)} = 0.5$.*

4. Comparative Study

In this section, we demonstrate the superiority of the suggested method by comparing it with the existing distance measures on Fermatean neutrosophic sets [9]. Let H_i and K_i are two Fermatean neutrosophic sets in the universe of discourse $U = \{u_1, u_2\}$ and are listed in Table 1. Further, Table 2 displays the distances produced by the various techniques on the above Fermatean neutrosophic sets (take weightage $\omega_i = 0.5$ for $i = 1, 2$). Here $H_1 = H_2$, $K_1 \neq K_2, H_3 = H_4$ and $K_3 \neq K_4$ in Table 1. While examining the Table 2, the results produced by Hamming distance($d_{\mathbb{HM}}$) for case 1 and case 2 are same, which is contradictory and illogical. Besides analyzing the results of Euclidean distance($d_{\mathbb{EU}}$) in case 3 and case 4 are irrational due to the same values. Therefore, it is clear that the proposed method is more responsive to variation in Fermatean neutrosophic sets and strongly reflects the degree of discrimination, which is far more effective than other methods.

Table 1: Fermatean neutrosophic sets H_i and K_i under various cases

	Case 1	Case 2
H_i	$\{(u_1, 0.6, 0.5, 0.4), (u_2, 0.4, 0.3, 0.6)\}$	$\{(u_1, 0.6, 0.5, 0.4), (u_2, 0.4, 0.3, 0.6)\}$
K_i	$\{(u_1, 0.7, 0.6, 0.8), (u_2, 0.36, 0.2, 0.8)\}$	$\{(u_1, 0.4, 0.35, 0.6), (u_2, 0.4, 0.7, 0.8)\}$
	Case 3	Case 4
H_i	$\{(u_1, 0.5, 0.8, 0.6), (u_2, 0.7, 0.8, 0.2)\}$	$\{(u_1, 0.5, 0.8, 0.6), (u_2, 0.7, 0.8, 0.2)\}$
K_i	$\{(u_1, 0.4, 0.3, 0.3), (u_2, 0.7, 0.3, 0.2)\}$	$\{(u_1, 0.3, 0.4, 0.85), (u_2, 0.2, 0.7, 0.3)\}$

Table 2: Distance measures using various methods

	Case 1	Case 2	Case 3	Case 4
Hamming distance($d_{\mathbb{HM}}$) [9]	**0.16664**	**0.16664**	0.2033	0.2445
Euclidean distance($d_{\mathbb{EU}}$) [9]	0.2285	0.2002	**0.2915**	**0.2915**
Proposed distance measure($\ddot{d}$)	0.3167	0.2461	0.4042	0.3703

A Case Study

Selecting the best institution is an important choice that can significantly affect a student's life and mold their future in terms of their academic, professional and personal goals. Let us examine a group of four institutions $\mathfrak{C}_1$, $\mathfrak{C}_2$, $\mathfrak{C}_3$ and $\mathfrak{C}_4$ in a locality. Faculty expertise($\mathfrak{A}_1$), facilities and resources($\mathfrak{A}_2$), internship and career opportunities($\mathfrak{A}_3$), financial aid and affordability($\mathfrak{A}_4$) and Reputation($\mathfrak{A}_5$) are the factors taken into consideration for assessing the institution and give equal weightage for each factor. Let $\mathfrak{I}$ represents the ideal institution. Table 3 provides the Fermatean neutrosophic representation of each institution against each factor. The results of the different methods used to determine which institution is best are shown in Table 4. In Table 4, the lowest distance between them is indicated by bold digits. Thus for both Hamming and Euclidean distance, we discovered that the distance between $\mathfrak{C}_1$ and $\mathfrak{I}$ is the smallest. The outcome is also consistent with our proposed metric. Thus we say that out of all the institutions $\mathfrak{C}_1$ is the best.

Table 3: Fermatean neutrosophic representation of institutions

	$\mathfrak{A}_1$	$\mathfrak{A}_2$	$\mathfrak{A}_3$	$\mathfrak{A}_4$	$\mathfrak{A}_5$
$\mathfrak{C}_1$	$(0.9, 0.3, 0.3)$	$(0.7, 0.2, 0.2)$	$(0.8, 0.3, 0.4)$	$(0.6, 0.2, 0.1)$	$(0.8, 0.2, 0.2)$
$\mathfrak{C}_2$	$(0.7, 0.4, 0.5)$	$(0.8, 0.4, 0.3)$	$(0.6, 0.4, 0.6)$	$(0.7, 0.3, 0.4)$	$(0.7, 0.3, 0.5)$
$\mathfrak{C}_3$	$(0.7, 0.6, 0.4)$	$(0.4, 0.3, 0.5)$	$(0.7, 0.5, 0.4)$	$(0.7, 0.5, 0.2)$	$(0.6, 0.5, 0.4)$
$\mathfrak{C}_4$	$(0.5, 0.2, 0.2)$	$(0.5, 0.2, 0.4)$	$(0.9, 0.2, 0.3)$	$(0.6, 0.4, 0.4)$	$(0.7, 0.3, 0.5)$
$\mathfrak{I}$	$(1, 0, 0)$	$(1, 0, 0)$	$(1, 0, 0)$	$(1, 0, 0)$	$(1, 0, 0)$

Table 4: Distance measure obtained by different methods

	$\delta(\mathfrak{C}_1, \mathfrak{I})$	$\delta(\mathfrak{C}_2, \mathfrak{I})$	$\delta(\mathfrak{C}_3, \mathfrak{I})$	$\delta(\mathfrak{C}_4, \mathfrak{I})$
Hamming distance($d_{\mathbb{HM}}$) [9]	**0.1916**	0.2697	0.3089	0.2577
Euclidean distance($d_{\mathbb{EU}}$) [9]	**0.3269**	0.3868	0.4403	0.4227
Proposed distance measure($\ddot{d}$)	**0.4618**	0.5411	0.6161	0.5962

5. Application of New Distance Measure for Fermatean Neutrosophic sets in Crop Cultivation

The relevance of the proposed distance measure in crop farming is suggested in this section. Here, we would determine which crop would produce a high yield in each place by using the distance measure. We take into account four places and five crops such as rice, tapioca, banana, apple and cabbage.

Let us consider the crops such as Cabbage C_1, Rice C_2, Banana C_3, Tapioca C_4, Apple C_5 and four different areas A_1, A_2, A_3 and A_4. The factors that affect cultivation in each area are temperature, humidity, soil pH, soil fertility and the availability of water. In Table 5, each factor is described by its membership degree X, non-membership degree Ω and degree of indeterminacy Ψ. Table 5 defines the factors affecting the area where each crop is grown.

	Water availability	Temperature	Soil fertility	Humidity	Soil pH
Cabbage(C_1)	(0.5,0.7,0.7)	(0.2,0.35,0.55)	(0.85,0.1,0.1)	(0.5,0.2,0.3)	(0.55,0.5,0.3)
Rice(C_2)	(0.8,0.1,0.2)	(0.7,0.4,0.6)	(0.9,0.4,0.15)	(0.6,0.7,0.6)	(0.45,0.3,0.4)
Banana(C_3)	(0.5,0.7,0.5)	(0.6,0.55,0.65)	(0.8,0.6,0.4)	(0.7,0.4,0.4)	(0.45,0.4,0.45)
Tapioca(C_4)	(0.4,0.6,0.4)	(0.55,0.6,0.7)	(0.75,0.2,0.5)	(0.65,0.3,0.35)	(0.45,0.35,0.5)
Apple(C_5)	(0.6,0.7,0.8)	(0.3,0.3,0.4)	(0.8,0.2,0.1)	(0.5,0.4,0.45)	(0.5,0.45,0.35)

Table 5

Table 6 describe A_1, A_2, A_3 and A_4 with the factors affecting cultivation.

	Water availability	Temperature	Soil fertility	Humidity	Soil pH
A_1	(0.6,0.4,0.3)	(0.5,0.2,0.4)	(0.8,0.1,0.4)	(0.6,0.3,0.1)	(0.5,0.2,0.6)
A_2	(0.8,0.2,0.4)	(0.7,0.5,0.6)	(0.9,0.3,0.5)	(0.65,0.2,0.5)	(0.4,0.3,0.4)
A_3	(0.5,0.75,0.5)	(0.6,0.5,0.6)	(0.7,0.4,0.2)	(0.8,0.4,0.4)	(0.55,0.4,0.5)
A_4	(0.7,0.5,0.8)	(0.4,0.2,0.4)	(0.65,0.5,0.1)	(0.5,0.2,0.3)	(0.65,0.3,0.4)

Table 6

Using the distance measure $\ddot{d}$ between areas and crops we get,

$$\ddot{d}(A_l, C_j) = [\frac{1}{3n}\sum_{i=1}^{n}((X^3_{A_l(u_i)} - X^3_{C_j(u_i)})^2 + (\Omega^3_{A_l(u_i)} - \Omega^3_{C_j(u_i)})^2 + (\Psi^3_{A_l(u_i)} - \Psi^3_{C_j(u_i)})^2$$
$$+ max\{(X^3_{A_l(u_i)} - X^3_{C_j(u_i)})^2, (\Omega^3_{A_l(u_i)} - \Omega^3_{C_j(u_i)})^2, (\Psi^3_{A_l(u_i)} - \Psi^3_{C_j(u_i)})^2\})]^{\frac{1}{2}}$$

where $l = 1, 2, 3, 4$ and $j = 1, 2, 3, 4, 5$

Here n=5, therefore equation becomes

$$\ddot{d}(A_l, C_j) = [\frac{1}{15}\sum_{i=1}^{5}((X^3_{A_l(u_i)} - X^3_{C_j(u_i)})^2 + (\Omega^3_{A_l(u_i)} - \Omega^3_{C_j(u_i)})^2 + (\Psi^3_{A_l(u_i)} - \Psi^3_{C_j(u_i)})^2$$
$$+ max\{(X^3_{A_l(u_i)} - X^3_{C_j(u_i)})^2, (\Omega^3_{A_l(u_i)} - \Omega^3_{C_j(u_i)})^2, (\Psi^3_{A_l(u_i)} - \Psi^3_{C_j(u_i)})^2\})]^{\frac{1}{2}}$$

where $l = 1, 2, 3, 4$ and $j = 1, 2, 3, 4, 5$

	Cabbage (C_1)	Rice (C_2)	Banana (C_3)	Tapioca (C_4)	Apple (C_5)
A_1	0.17401	0.21539	0.2013	0.14586	0.21148
A_2	0.26122	0.19127	0.20123	0.22524	0.25885
A_3	0.21291	0.27496	0.1077	0.13952	0.18112
A_4	0.298	0.31477	0.23459	0.2426	0.14174

Table 7

The results are displayed in Table 7. Each cultivation area is better for the crop with the lowest distance. Thus, we get areas A_1, A_2, A_3 and A_4 are suitable for cultivating Tapioca, Rice, Banana and Apple respectively.

6. Conclusion

In this article, we proposed a novel distance measure to handle the decision-making challenges in the Fermatean neutrosophic environment. Also, some properties of the distance measure were examined. A comparative analysis is conducted to demonstrate the effectiveness and superiority of the suggested distance metric. Then a cultivation based numerical example is given to assist farmers in selecting the best crop for a given location. In this case, we make decision based on calculating the distances between a specific location and all crops, taking into consideration of all variables that may have an impact on the crop's cultivation.

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