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CUBIC SPHERICAL NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract: This study introduces cubic spherical neutrosophic sets as a novel approach to represent uncertainty within mathematical frameworks. By employing a spherical representation, these sets offer a comprehensive depiction of varying degrees of truth, indeterminacy and falsity associated with elements. The concept of cubic spherical neutrosophic topological space is introduced as a generalization of neutrosophic topology. Using illustrative examples, we explore fundamental theorems and characteristics of these spaces. Cubic spherical neutrosophic sets provide a flexible framework for integrating multiple perspectives and sources of uncertainty, making them suitable for modeling real-world phenomena.

Keywords and Phrases: Neutrosophic set, cubic spherical neutrosophic set, cubic spherical neutrosophic topological spaces.

2020 Mathematics Subject Classification: 57N25, 54B05.

1. Introduction and Preliminaries

In 1998, the concept of neutrosophic sets (NSs) was introduced and studied by F. Smarandache [16, 17] as a generalization of Atanassov's [1] theory of Intuitionistic fuzzy set. The concept of fuzzy topological spaces was introduced in 1968 by Chang C. L. [2]. The notion of neutrosophic topology was introduced and studied by many authors [4-6, 10, 11, 14, 15, 18, 19]. Many generalizations of neutrosophic sets were introduced and studied, which includes interval neutrosophic topological spaces [12], neutrosophic bitopological spaces [20], neutrosophic tri-topological space [4], pentapartitioned neutrosophic topological space [3, 7] and linguistic neutrosophic topology [8].

The concept of Cubic Spherical Neutrosophic Set (CSNSs) was introduced and studied by Gomathi et. al [9] as a geometric representation of collection of neutrosophic sets. Cubic spherical neutrosophic sets offer a novel approach to representing uncertainty and ambiguity within mathematical frameworks. By utilizing a spherical representation, these sets allow for a comprehensive depiction of varying degrees of truth, indeterminacy and falsity associated with elements. This geometric interpretation facilitates intuitive visualization and analysis, enabling a deeper understanding of complex data or spaces. Moreover, CSNSs provide a flexible framework for integrating multiple perspectives and sources of uncertainty, making them suitable for modeling real-world phenomena where uncertainty is prevalent. This versatility opens up new avenues in many areas.

The following are our objective and purposes: we introduce and investigate the concept of CSNSs as a novel approach for representing uncertainty and ambiguity within mathematical frameworks. The study aims to extend the existing theory of neutrosophic sets to include a geometric representation using spheres, providing a comprehensive depiction of varying degrees of truth, indeterminacy and falsity associated with elements. Furthermore, the manuscript seeks to introduce the notion of cubic spherical neutrosophic topological space as a generalization of neutrosophic topology, exploring its fundamental theorems and characteristics.

The following outcomes are illustrated in this manuscript: To contribute the field of neutrosophic mathematics by introducing CSNSs as a versatile framework for handling uncertainty in mathematical modeling and analysis. By providing a geometric interpretation and extending topological concepts to cubic spherical neutrosophic spaces, the manuscript offers new insights into the representation and analysis of uncertain data or spaces. Through illustrative examples and investigations of fundamental theorems, the manuscript aims to demonstrate the applicability and effectiveness of cubic spherical neutrosophic sets in various real-world scenarios. Additionally, the manuscript aims to lay the groundwork for future research in this emerging area, opening up new avenues for exploration and application.

Let $\mathbb{X}$ be a fixed universe and its subset δ. The collection

$$\delta_{\mathbb{R}} = \{< x, csn\mu(x), csn\eta(x), csn\nu(x); \mathbb{R} >: x \in \mathbb{X}\}$$

where $csn\mu, csn\eta, csn\nu : X \to [0,1]$ are mappings such that $csn\mu_\delta + csn\eta_\delta + csn\nu_\delta \leq 3$ and $\mathbb{R} \in [0,1]$. The radius $\mathbb{R}$ of the sphere with center $(csn\mu(x), csn\eta(x), csn\nu(x))$ inside the cube is called cubic spherical neutrosophic set (CSNS) [9] $\delta_{\mathbb{R}}$. This sphere represents the membership degree, indeterminacy degree and non-membership degree of $x \in X$.

Let $\{< \mu_{i,1}, \eta_{i,1}, \nu_{i,1} >, < \mu_{i,2}, \eta_{i,2}, \nu_{i,2} >, ..., < \mu_{i,k_i}, \eta_{i,k_i}, \nu_{i,k_i} >\}$ be a collection of NSs assigned for any x_i in $\mathbb{X}$. We construct the center of the sphere by

$$< csn\mu(x_i), csn\eta(x_i), csn\nu(x_i) >=< \frac{\sum_{j=1}^{k_i} \mu_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} \eta_{i,j}}{k_i}, \frac{\sum_{j=1}^{k_i} \nu_{i,j}}{k_i} >$$

and the radius

$$\mathbb{R}_i = \min\left\{ \max_{1 \leq j \leq k_i} \sqrt{(csn\mu(x_i) - \mu_{i,j})^2 + (csn\eta(x_i) - \eta_{i,j})^2 + (csn\nu(x_i) - \nu_{i,j})^2}, 1 \right\}.$$

The collection of all CSNSs is denoted by $CSN(\mathbb{X})$.

Definition 1.1. [9] *Let* $\delta_{\mathbb{R}_1} = \{\langle x, csn\mu_{\delta_1}, csn\eta_{\delta_1}, csn\nu_{\delta_1}; \mathbb{R}_{\delta_1}\rangle : x \in \mathbb{X}\}$ *and* $\delta_{\mathbb{R}_2} = \{\langle x, csn\mu_{\delta_2}, csn\eta_{\delta_2}, csn\nu_{\delta_2}; \mathbb{R}_{\delta_2}\rangle : x \in \mathbb{X}\}$ *be two CSNSs over the universal set* $\mathbb{X}$. *Then the following operations are defined as follows*

1. $\delta_{\mathbb{R}_1} \cup_{\max} \delta_{\mathbb{R}_2} = \{< x, \max\{csn\mu_{\delta_1}, csn\mu_{\delta_2}\}, \min\{csn\eta_{\delta_1}, csn\eta_{\delta_2}\}, \min\{csn\nu_{\delta_1}, csn\nu_{\delta_2}\}; \max\{\mathbb{R}_{\delta_1}, \mathbb{R}_{\delta_2}\} >: x \in \mathbb{X}\}$.
2. $\delta_{\mathbb{R}_1} \cap_{\min} \delta_{\mathbb{R}_2} = \{< x, \min\{csn\mu_{\delta_1}, csn\mu_{\delta_2}\}, \max\{csn\eta_{\delta_1}, csn\eta_{\delta_2}\}, \max\{csn\nu_{\delta_1}, csn\nu_{\delta_2}\}; \min\{\mathbb{R}_{\delta_1}, \mathbb{R}_{\delta_2}\} >: x \in \mathbb{X}\}$.
3. $\delta_{\mathbb{R}_1} = \delta_{\mathbb{R}_2}$ IFF $\{< x, csn\mu_{\delta_1} = csn\mu_{\delta_2}, csn\eta_{\delta_1} = csn\eta_{\delta_2}, csn\nu_{\delta_1} = csn\nu_{\delta_2}; \mathbb{R}_{\delta_1} = \mathbb{R}_{\delta_2} >: x \in \mathbb{X}\}$.
4. $\delta_{\mathbb{R}_1} \subseteq \delta_{\mathbb{R}_2}$ IFF $\{< x, csn\mu_{\delta_1} \subseteq csn\mu_{\delta_2}, csn\eta_{\delta_1} \supseteq csn\eta_{\delta_2}, csn\nu_{\delta_1} \supseteq csn\nu_{\delta_2}; \mathbb{R}_{\delta_1} \subseteq \mathbb{R}_{\delta_2} >: x \in \mathbb{X}\}$.
5. $\delta^c_{\mathbb{R}_1} = \{< x, csn\nu_{\delta_1}, csn\eta_{\delta_1}, csn\mu_{\delta_1}; \mathbb{R}_{\delta_1} >: x \in \mathbb{X}\}$.

2. Cubic Spherical Neutrosophic Topological Spaces

In this section, we study the new notion namely cubic spherical neutrosophic topology and its characterization. The cubic spherical neutrosophic $1_\odot$ and cubic spherical neutrosophic $0_\odot$ in $\mathbb{X}$ as follows $1_\odot = \langle 1,0,0;1\rangle$, and $0_\odot =\langle 0,1,1;0\rangle$.

Proposition 2.1. *Let $\delta_{\mathbb{R}_1} =< csn\mu_{\delta_1}, csn\eta_{\delta_1}, csn\nu_{\delta_1}; \mathbb{R}_{\delta_1} >$ and $\delta_{\mathbb{R}_2} =< csn\mu_{\delta_2}, csn\eta_{\delta_2}, csn\nu_{\delta_2}; \mathbb{R}_{\delta_2} >$ be two CSNSs over the universal set $\mathbb{X}$. Then the following hold:*

1. $\delta_{\mathbb{R}_1} \cup \delta_{\mathbb{R}_1} = \delta_{\mathbb{R}_1}$ and $\delta_{\mathbb{R}_1} \cap \delta_{\mathbb{R}_1} = \delta_{\mathbb{R}_1}$.
2. $\delta_{\mathbb{R}_1} \cup 0_\odot = \delta_{\mathbb{R}_1}$ and $\delta_{\mathbb{R}_1} \cap 0_\odot = 0_\odot$.
3. $\delta_{\mathbb{R}_1} \cup 1_\odot = 1_\odot$ and $\delta_{\mathbb{R}_1} \cap 1_\odot = \delta_{\mathbb{R}_1}$.
4. $(\delta^c_{\mathbb{R}_1})^c = \delta_{\mathbb{R}_1}$.

Definition 2.2. *Let $\tau_\odot \in CSN(\mathbb{X})$, then $\tau_\odot$ is called a cubic spherical neutrosophic topology on $\mathbb{X}$, if the following hold*

1. $1_\odot, 0_\odot \in \tau_\odot$.
2. $\delta_{\mathbb{R}_1}, \delta_{\mathbb{R}_2} \in \tau_\odot \Rightarrow \delta_{\mathbb{R}_1} \cap \delta_{\mathbb{R}_2} \in \tau_\odot$.
3. $\{\delta_{\mathbb{R}_i}; i \in \triangle\} \subseteq \tau_\odot \Rightarrow \bigcup \delta_{\mathbb{R}_i} \in \tau_\odot$.

The pair $(\mathbb{X}, \tau_\odot)$ is called a Cubic Spherical Neutrosophic Topological Space (CSNTS) over $\mathbb{X}$. This generalization involves considering sphere for the membership function of truth, indeterminacy and falsehood in the context of topology. Furthermore, the members of $\tau_\odot$ are said to be CSN-open sets in $\mathbb{X}$. If $\delta^c_{\mathbb{R}_1} \in \tau_\odot$, then $\delta_{\mathbb{R}_1} \in CSN(\mathbb{X})$ is said to be CSN-closed set in $\mathbb{X}$.

The CSN-interior of set $\delta_{\mathbb{R}_1}$, denoted by $int_\odot(\delta_{\mathbb{R}_1})$ is defined as the union of all CSN-open subsets of $\delta_{\mathbb{R}_1}$. Notably, $int_\odot(\delta_{\mathbb{R}_1})$ represents the largest cubic spherical neutrosophic open set over $\mathbb{X}$ that containing $\delta_{\mathbb{R}_1}$. The CSN-closure of set $\delta_{\mathbb{R}_1}$, denoted by $cl_\odot(\delta_{\mathbb{R}_1})$ is defined as the intersection of all CSN-closed supersets of $\delta_{\mathbb{R}_1}$. Notably, $cl_\odot(\delta_{\mathbb{R}_1})$ represents the smallest cubic spherical neutrosophic closed set over $\mathbb{X}$ that contains $\delta_{\mathbb{R}_1}$.

Let $\tau_\odot = \{0_\odot, 1_\odot\}$ and $\sigma_\odot = CSN(\mathbb{X})$. Then, $(\mathbb{X}, \tau_\odot)$ and $(\mathbb{X}, \sigma_\odot)$ are two trivial CSNTS over $\mathbb{X}$. Additionally, they are referred to as CSN-discrete topological space and CSN-indiscrete topological space over $\mathbb{X}$, respectively.

Example 2.3. Let $\mathbb{X}$=$\{x, y\}$ and $\delta_1, \delta_2 \in NS(\mathbb{X})$ such that $\delta_1 = \{\langle x, 0.88, 0.33, 0.22\rangle$, $\{\langle x, 0.77, 0.44, 0.11\rangle$, $\{\langle x, 0.55, 0.44, 0.22\rangle$, $\{\langle x, 0.66, 0.55, 0.33\rangle\}$ and δ_2 $= \{\langle y, 0.66, 0.22, 0.11\rangle , \langle y, 0.88, 0.11, 0.22\rangle , \langle y, 0.88, 0.33, 0.11\rangle , \langle y, 0.99, 0.44, 0.22\rangle\}$. Then

1. The CSNSs are $\delta_{\mathbb{R}_1}$= $\{\langle x, 0.72, 0.44, 0.22; 0.20\rangle : x \in \mathbb{X}\}$ and $\delta_{\mathbb{R}_2}$= $\{\langle y, 0.85, 0.28, 0.17; 0.22\rangle : y \in \mathbb{X}\}$.

2. The union of two CSNSs $\delta_{\mathbb{R}_1}$ and $\delta_{\mathbb{R}_2}$ is $\delta_{\mathbb{R}_1} \cup_{\max} \delta_{\mathbb{R}_2} = \{\langle y, 0.85, 0.28, 0.17; 0.22\rangle : y \in \mathbb{X}\}$.

3. The intersection of two CSNSs $\delta_{\mathbb{R}_1}$ and $\delta_{\mathbb{R}_2}$ is $\delta_{\mathbb{R}_1} \cap_{\min} \delta_{\mathbb{R}_2} = \{\langle x, 0.72, 0.44, 0.22; 0.20\rangle : x \in \mathbb{X}\}$.

4. The complement of a CSNS $\delta_{\mathbb{R}_1}$ is $\delta^c_{\mathbb{R}_1} = \{\langle x, 0.22, 0.44, 0.72; 0.20\rangle : x \in \mathbb{X}\}$.

5. We have that $\delta_{\mathbb{R}_1} \subset \delta_{\mathbb{R}_2}$.

6. The family $\tau_{\odot} = \{0_{\odot}, 1_{\odot}, \delta_{\mathbb{R}_1}, \delta_{\mathbb{R}_2}\}$, of CSNSs in $\mathbb{X}$ is cubic spherical neutrosophic topology.

7. The geometric representation of δ_1, δ_2, $\delta_{\mathbb{R}_1}$, and $\delta_{\mathbb{R}_2}$ are

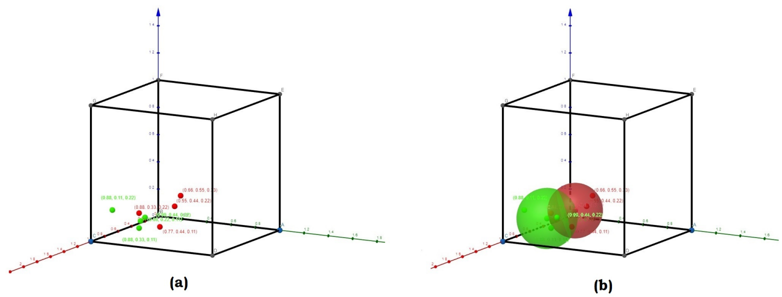

Figure 1: Geometric representation of NSs (a) and CSNSs (b)

Proposition 2.4. *Let* $(\mathbb{X}, \tau_{1\odot})$ *and* $(\mathbb{X}, \tau_{2\odot})$ *be two CSNTSs over* $\mathbb{X}$*, then* $(\mathbb{X}, \tau_{1\odot} \cap \tau_{2\odot})$ *is a CSNTS over* $\mathbb{X}$.

Proof. Let $(\mathbb{X}, \tau_{1\odot})$ and $(\mathbb{X}, \tau_{2\odot})$ be two CSNTSs over $\mathbb{X}$. It can be seen clearly that $0_{\odot}, 1_{\odot} \in \tau_{1\odot} \cap \tau_{2\odot}$. If $\delta_{\mathbb{R}_1}, \delta_{\mathbb{R}_2} \in \tau_{1\odot} \cap \tau_{2\odot}$ then, $\delta_{\mathbb{R}_1}, \delta_{\mathbb{R}_2} \in \tau_{1\odot}$ and $\delta_{\mathbb{R}_1}, \delta_{\mathbb{R}_2} \in \tau_{2\odot}$. It is given that $\delta_{\mathbb{R}_1} \cap \delta_{\mathbb{R}_2} \in \tau_{1\odot}$ and $\delta_{\mathbb{R}_1} \cap \delta_{\mathbb{R}_2} \in \tau_{2\odot}$. Thus, $\delta_{\mathbb{R}_1} \cap \delta_{\mathbb{R}_2} \in \tau_{1\odot} \cap \tau_{2\odot}$. Let $\{\delta_{\mathbb{R}_1 i} : i \in I\} \subseteq \tau_{1\odot} \cap \tau_{2\odot}$. Then, $\delta_{\mathbb{R}_1 i} \in \tau_{1\odot} \cap \tau_{2\odot}$ for all $i \in I$. Thus, $\delta_{\mathbb{R}_1 i} \in \tau_{1\odot}$ and $\delta_{\mathbb{R}_1} i \in \tau_{2\odot}$ for all $i \in I$. So, we have $\bigcap_{i \in I} \delta_{\mathbb{R}_1 i} \in \tau_{1\odot} \cap \tau_{2\odot}$.

Corollary 2.5. *Let* $\{(\mathbb{X}, \tau_{\odot i}) : i \in I\}$ *be a family of CSNTSs over* $\mathbb{X}$*. Then,* $(\mathbb{X}, \bigcap_{i \in I} \tau_{\odot i})$ *is a CSNTS over* $\mathbb{X}$.

Example 2.6. Let $\mathbb{X} = \{x, y\}$, and δ_1, δ_2, δ_3, $\delta_4 \in NS(\mathbb{X})$ such that
$\delta_1 = \{\langle x, 0.88, 0.33, 0.22\rangle, \langle x, 0.77, 0.44, 0.11\rangle, \langle x, 0.55, 0.44, 0.22\rangle, \langle x, 0.66, 0.55, 0.33\rangle\}$,
$\delta_2 = \{\langle x, 0.66, 0.22, 0.11\rangle, \langle x, 0.88, 0.11, 0.22\rangle, \langle x, 0.88, 0.33, 0.11\rangle, \langle x, 0.99, 0.44, 0.22\rangle\}$,
$\delta_3 = \{\langle y, 0.66, 0.44, 0.44\rangle, \langle y, 0.54, 0.33, 0.22\rangle, \langle y, 0.45, 0.44, 0.22\rangle, \langle y, 0.99, 0.55, 0.11\rangle\}$

and
$\delta_4, = \{\langle y, 0.66, 0.55, 0.33\rangle, \langle y, 0.54, 0.55, 0.33\rangle, \langle y, 0.45, 0.55, 0.33\rangle, \langle y, 0.88, 0.66, 0.33\rangle\}$.
Then the CSNSs are
$\delta_{\mathbb{R}_1}= \{\langle x, 0.72, 0.44, 0.22; 0.20\rangle : x \in \mathbb{X}\}$, $\delta_{\mathbb{R}_2}= \{\langle x, 0.85, 0.28, 0.17; 0.22\rangle : x \in \mathbb{X}\}$, $\delta_{\mathbb{R}_3}= \{\langle y, 0.66, 0.44, 0.25; 0.37\rangle : y \in \mathbb{X}\}$ and $\delta_{\mathbb{R}_4}= \{\langle y, 0.63, 0.58, 0.33; 0.26\rangle : y \in \mathbb{X}\}$. The family $\tau_\odot = \{0_\odot, 1_\odot, \delta_{\mathbb{R}_1}, \delta_{\mathbb{R}_2}\}$, and $\sigma_\odot = \{0_\odot, 1_\odot, \delta_{\mathbb{R}_3}, \delta_{\mathbb{R}_4}\}$, are CSNTSs but their union $\tau_\odot \cup \sigma_\odot$ is not a CSNTS, since $\delta_{\mathbb{R}_1} \cup \delta_{\mathbb{R}_3} \notin \tau_\odot \cup \sigma_\odot$.

Proposition 2.7. *Let $(\mathbb{X}, \tau_\odot)$ be a CSNTS over $\mathbb{X}$. Then*

1. *$0_\odot$ and $\mathbb{X}$ are CSN closed sets over $\mathbb{X}$.*
2. *The intersection of any number of CSN closed sets is a CSN closed set over $\mathbb{X}$.*
3. *The union of any two CSN closed sets is a CSN closed set over $\mathbb{X}$.*

Proposition 2.8. *Let $(\mathbb{X}, \tau_\odot)$ be a CSNTS over $\mathbb{X}$ and $\delta_{\mathbb{R}_1}, \delta_{\mathbb{R}_2} \in CSNS(\mathbb{X})$. Then*

1. *$int_\odot(0_\odot) = 0_\odot$ and $int_\odot(1_\odot) = 1_\odot$.*
2. *$int_\odot(\delta_{\mathbb{R}_1}) \subseteq \delta_{\mathbb{R}_1}$.*
3. *$\delta_{\mathbb{R}_1}$ is a CSN open set if and only if $\delta_{\mathbb{R}_1} = int_\odot(\delta_{\mathbb{R}_1})$.*
4. *$int_\odot(int_\odot(\delta_{\mathbb{R}_1})) = int_\odot(\delta_{\mathbb{R}_1})$.*
5. *$\delta_{\mathbb{R}_1} \subseteq \delta_{\mathbb{R}_2}$ implies $int_\odot(\delta_{\mathbb{R}_1}) \subseteq int_\odot(\delta_{\mathbb{R}_2})$.*
6. *$int_\odot(\delta_{\mathbb{R}_1}) \cup int_\odot(\delta_{\mathbb{R}_2}) \subseteq int_\odot(\delta_{\mathbb{R}_1} \cup \delta_{\mathbb{R}_2})$.*
7. *$int_\odot(\delta_{\mathbb{R}_1} \cap \delta_{\mathbb{R}_2}) = int_\odot(\delta_{\mathbb{R}_1}) \cap int_\odot(\delta_{\mathbb{R}_2})$.*

Proof. 1. and 2. are obvious.
3. If A is a cubic spherical neutrosophic open set over $\mathbb{X}$, then A is itself a cubic spherical neutrosophic open set over $\mathbb{X}$ which contains A. So, A is the largest neutrosophic open set contained in A and $int_\odot(A) = A$. Conversely, suppose that $int_\odot(A) = A$. Then $A \in \tau_\odot$.
4. Let $int_\odot(A) = B$. Then, $int_\odot(B) = B$ from 3. and then, $int_\odot(int_\odot(A)) = int_\odot(A)$.
5. Suppose that $A \subseteq B$. As $int_\odot(A) \subseteq A \subseteq B$. By definition, we have $int_\odot \subseteq int_\odot(B)$.

6. It is clear that $A \subseteq A \cup B$ and $B \subseteq A \cup B$. Thus $int_{\odot}(A) \subseteq int_{\odot}(A \cup B)$ and $int_{\odot}(B) \subseteq int_{\odot}(A \cup B)$. So we have $int_{\odot}(A) \cup int_{\odot}(B) \subseteq int_{\odot}(A \cup B)$.
7. It is known that $int_{\odot}(A \cap B) \subseteq int_{\odot}(A)$ and $int_{\odot}(A \cap B) \subseteq int_{\odot}(B)$ by 5. So that $int_{\odot}(A \cap B) \subseteq int_{\odot}(A) \cap int_{\odot}(B)$. Also, from $int_{\odot}(A) \subseteq A$ and $int_{\odot}(B) \subseteq B$, we have $int_{\odot}(A) \cap int_{\odot}(B) \subseteq A \cap B$. These imply that $int_{\odot}(A \cap B) = int_{\odot}(A) \cap int_{\odot}(B)$.

Example 2.9. Consider $(\mathbb{X}, \tau_{\odot})$ of Example 2.3. Let $A = \langle x, 0.88, 0.33, 0.23; 0.20 \rangle$, $B = \langle x, 0.77, 0.44, 0.11; 0.29 \rangle$. Then $int_{\odot}(A) = 0_{\odot}$, $int_{\odot}(B) = 0_{\odot}$, and $int_{\odot}(A \cup B) = \delta_{\mathbb{R}_1}$. Therefore, $int_{\odot}(A) \cup int_{\odot}(B) \neq int_{\odot}(A \cup B)$.

Proposition 2.10. *Let $(\mathbb{X}, \tau_{\odot})$ be a CSNTS over $\mathbb{X}$ and $\delta_{\mathbb{R}_1}, \delta_{\mathbb{R}_2} \in CSNS(\mathbb{X})$. Then*

1. $cl_{\odot}(0_{\odot}) = 0_{\odot}$ and $cl_{\odot}(1_{\odot}) = 1_{\odot}$.
2. $\delta_{\mathbb{R}_1} \subseteq cl_{\odot}(\delta_{\mathbb{R}_1})$.
3. $\delta_{\mathbb{R}_1}$ is a CSN closed set if and only if $\delta_{\mathbb{R}_1} = cl_{\odot}(\delta_{\mathbb{R}_1})$.
4. $cl_{\odot}(cl_{\odot}(\delta_{\mathbb{R}_1})) = cl_{\odot}(\delta_{\mathbb{R}_1})$.
5. $\delta_{\mathbb{R}_1} \subseteq \delta_{\mathbb{R}_2}$ implies $cl_{\odot}(\delta_{\mathbb{R}_1}) \subseteq cl_{\odot}(\delta_{\mathbb{R}_2})$.
6. $cl_{\odot}(\delta_{\mathbb{R}_1} \cup \delta_{\mathbb{R}_2}) = cl_{\odot}(\delta_{\mathbb{R}_1}) \cup cl_{\odot}(\delta_{\mathbb{R}_2})$.
7. $cl_{\odot}(\delta_{\mathbb{R}_1} \cap \delta_{\mathbb{R}_2}) \subseteq cl_{\odot}(\delta_{\mathbb{R}_1}) \cap cl_{\odot}(\delta_{\mathbb{R}_2})$.

Example 2.11. Consider $(\mathbb{X}, \sigma_{\odot})$ of Example 2.6. Let $A = \langle y, 0.25, 0.43, 0.54; 0.74 \rangle$, $B = \langle y, 0.28, 0.32, 0.64; 0.63 \rangle$. Then $cl_{\odot}(A) = 1_{\odot}$, $cl_{\odot}(B) = 1_{\odot}$, and $cl_{\odot}(A \cap B) = (\delta_{\mathbb{R}_4})^c$. Therefore, $cl_{\odot}(A) \cap cl_{\odot}(B) \neq cl_{\odot}(A \cap B)$.

Corollary 2.12. *Let $(\mathbb{X}, \tau_{\odot})$ be a CSNTS over $\mathbb{X}$ and $\delta_{\mathbb{R}_1} \in CSN(\mathbb{X})$. Then $int_{\odot}(\delta^c_{\mathbb{R}_1}) = (cl_{\odot}(\delta_{\mathbb{R}_1}))^c$. and $cl_{\odot}(\delta^c_{\mathbb{R}_1}) = (int_{\odot}(\delta_{\mathbb{R}_1}))^c$.*

Proposition 2.13. *Let $(\mathbb{X}, \tau_{\odot})$ be a CSNTS over $\mathbb{X}$, $\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$ be a CSN set on $\mathbb{X}$. Then $(\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) \cup (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})'$ is a CSN closed set.*

Proof. To prove $(\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) \cup (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})'$ is CSN closed it is enough to prove that $\left((\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) \cup (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})'\right)^{\acute{}}$ is CSN open. If $\left((\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) \cup (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})'\right)^{\acute{}} = \phi$ then it is CSN open. Let $\left((\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) \cup (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})'\right)^{\acute{}} \neq \phi$ and $x \in \left((\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) \cup (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})'\right)^{\acute{}}$ $\Rightarrow x \notin (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) \cup (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})' \Rightarrow x \notin (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})$ and $x \notin (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})' \Rightarrow$ $\exists$ an CSN open set $G * B \ni x \in (G * B)$ and $(G * B) \cap (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) = \phi \Rightarrow$

$x \in (G * B) \subseteq (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})'$
Again $x \notin (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})' \Rightarrow x \in G * B \subseteq ((\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})')'$. Therefore $x \in (G * B) \subseteq \left((\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) \cup (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})'\right)'$ and hence $\left((\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}) \cup (\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2})'\right)'$ is CSN open set.

Proposition 2.14. $(\mathbb{X}, \tau_{\odot})$ *be a CSNTS over* $\mathbb{X}$ *and* $\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$ *be an CSN set over* X *then* $\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$ *is CSN a closed iff* $\overline{\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}} = \delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$

Proof. If $\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$ is an CSN closed set then the smallest CSN closed super set of $\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$ is $\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$ itself. Therefore $\overline{\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}} = \delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$. Conversely if $\overline{\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}} = \delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$ then $\overline{\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}}$ being closed so as $\delta_{\mathbb{R}_1} * \delta_{\mathbb{R}_2}$.

3. Conclusion

In this study, we have introduced the concept of cubic spherical neutrosophic topological spaces as a generalization of neutrosophic topology. By incorporating topological characteristics into cubic spherical neutrosophic sets, we have extended the framework for dealing with uncertainty and indeterminacy in mathematical modeling. Through the investigation of fundamental theorems and characteristics of these spaces, we have demonstrated the potential for applying them in various domains. In future work, the comparative studies with existing frameworks, such as neutrosophic topology, interval-valued neutrosophic topology,could provide insights into the advantages and limitations of cubic spherical neutrosophic topological spaces and help position them within the broader context of uncertainty modeling.

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