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MAPPINGS ON FERMATEAN FUZZY SOFT CLASSES

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Abstract: This paper defined mappings on Fermatean fuzzy soft classes and established some results related to images and inverse images of Fermatean fuzzy soft sets under these mappings. Further Fermatean fuzzy soft continuous , Fermatean fuzzy soft open and Fermatean fuzzy soft closed mappings in Fermatean fuzzy soft topological spaces are created and established some theorems related their properties and characterizations.

Keywords and Phrases: Fermatean fuzzy soft sets, Fermatean fuzzy soft mappings and Fermatean fuzzy soft continuity.

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1. Introduction

Vagueness and ambiguity in information are crucial factors in decision making process. Crisp set theory is insufficient to handle the complex MADM problems involving vague and imprecise information. To handle the impreciseness and uncertainty of complex problems, Zadeh [19] in 1965, created $\mathcal{FS}$s as an extension of crisp sets. After the occurrence of Zadeh [19] paper many generalizations of $\mathcal{FS}$ such as $\mathcal{IFS}$ [1], $\mathcal{SS}$ [9], $\mathcal{PFS}$ [17] and $\mathcal{FFS}$ [13] have been appeared in the literature. Due to the parameterizations tools in $\mathcal{SS}$s, many hybridized classes with $\mathcal{SS}$s such as $\mathcal{FSS}$s [6], $\mathcal{IFSS}$s [7], $\mathcal{PFSS}$s [12] and $\mathcal{FFSS}$s [15] have been invented and studied. In 1968, Chang [2] published fuzzy topological spaces and extended many topological notions to $\mathcal{FS}$s. In the recent past topological structures over these classes of fuzzy sets have been invented and studied [3, 14, 16, 5, 10, 18, 4, 11].

The organization of this paper is as follows. Section 2, reviewed the basic operations and properties of $\mathcal{FFSS}$. Section 3, defined the mappings on $\mathcal{FFSS}$s, and their properties are presented. Section 4 is devoted to the study of Fermatean fuzzy soft continuity of mappings. Section 5 created and studied Fermatean fuzzy soft open and Fermatean fuzzy soft closed mappings in Fermatean fuzzy soft topological spaces.

2. Preliminaries

Definition 2.1. [19] *A fuzzy set ($\mathcal{FS}$) ν on a universe of discourse $\mathbb{P}$ is a structure $\nu = \{< p, m_\nu(p) >: p \in \mathbb{P}\}$, where $m_\nu : \mathbb{P} \to [0,1]$ called membership function of fuzzy set ν.*

Definition 2.2. *Let $\mathbb{P}$ be an initial universal set. A structure $\nu = \{< p, m_\nu(p), n_\nu(p) >: p \in \mathbb{P}\}$ where $m_\nu : \mathbb{P} \to [0,1]$ and $n_\nu : \mathbb{P} \to [0,1]$ denotes the degree of membership and the degree of nonmembership of each $p \in \mathbb{P}$ to ν is called :*

(a) Intuitionistic fuzzy set ($\mathcal{IFS}$) [1] *in $\mathbb{P}$ if $0 \le m_\nu(p) + n_\nu(p) \le 1,\ \forall\ p \in \mathbb{P}$.*

(b) Pythagorean fuzzy set ($\mathcal{PFS}$) [17] *in $\mathbb{P}$ if $0 \le m_\nu^2(p) + n_\nu^2(p) \le 1,\ \forall\ p \in \mathbb{P}$.*

(c) Fermatean fuzzy set ($\mathcal{FFS}$) [13] *in $\mathbb{P}$ if $0 \le m_\nu^3(p) + n_\nu^3(p) \le 1,\ \forall\ p \in \mathbb{P}$.*

The set of all $\mathcal{FS}$s (resp. $\mathcal{IFS}$s, $\mathcal{PFS}$s, $\mathcal{FFS}$s) on $\mathbb{P}$ will be denoted by $\mathcal{FS}(\mathbb{P})$ (resp. $\mathcal{IFS}(\mathbb{P})$, $\mathcal{PFS}(\mathbb{P})$, $\mathcal{FFS}(\mathbb{P})$).

Remark 2.3. [15] *In general $\mathcal{FS} \Rightarrow \mathcal{IFS} \Rightarrow \mathcal{PFS} \Rightarrow \mathcal{FFS}$. However, the reverse implications are not true.*

Definition 2.4. *Let $\mathbb{P}$ be a universe of discourse, Σ be the set of parameters and $\Upsilon \subseteq \Sigma$. A pair (ξ, Υ) is called :*

(a) a soft set ($\mathcal{SS}$) [9] *over $\mathbb{P}$, where $\xi : \Sigma \to \mathfrak{P}(\mathbb{P})$ and $\mathfrak{P}(\mathbb{P})$ is a power set of $\mathbb{P}$.*

(b) a fuzzy soft set ($\mathcal{FSS}$) [6] *over $\mathbb{P}$, where $\xi : \Upsilon \to \mathcal{FS}(\mathbb{P})$.*

(c) *an intuitionistic fuzzy soft set ($\mathcal{IFSS}$) [7] over $\mathbb{P}$, where $\xi : \Upsilon \to \mathcal{IFS}(\mathbb{P})$.*

(d) *a Pythagorean fuzzy soft set ($\mathcal{PFSS}$) [12] over $\mathbb{P}$, where $\xi : \Upsilon \to \mathcal{PFS}(\mathbb{P})$.*

(e) *a Fermatean fuzzy soft set ($\mathcal{FFSS}$) [15] over $\mathbb{P}$, where $\xi : \Upsilon \to \mathcal{FFS}(\mathbb{P})$.*

The collection of all $\mathcal{SS}$s (resp. $\mathcal{FSS}$s, $\mathcal{IFSS}$s, $\mathcal{PFSS}$s, $\mathcal{FFSS}$s) over $(\mathbb{P}, \Sigma)$ will be denoted by $\mathcal{SS}(\mathbb{P}, \Sigma)$ (resp. $\mathcal{FSS}(\mathbb{P}, \Sigma)$, $\mathcal{IFSS}(\mathbb{P}, \Sigma)$, $\mathcal{PFSS}(\mathbb{P}, \Sigma)$, $\mathcal{FFSS}$ $(\mathbb{P}, \Sigma)$).

A $\mathcal{FFSS}$ on $\mathbb{P}$ is a family of parameters formed by some $\mathcal{FSS}$ on $\mathbb{P}$. For any parameter $\epsilon \in \Upsilon$, $\xi(\epsilon)$ is a $\mathcal{FFS}$ associated with ϵ of $\mathbb{P}$. Then it is called the Fermatean fuzzy value set of parameter ϵ. $\xi(\epsilon)$ can be written as an $\mathcal{FFS}$ such that $\xi(\epsilon) = \{< p, m_{\xi(\epsilon)}(p), n_{\xi(\epsilon)}(p) >: p \in \mathbb{P}\}$, where $m_{\xi(\epsilon)}(p)$ and $n_{\xi(\epsilon)}(p)$ are the membership and non-membership functions, respectively satisfying the condition $0 \leq m^3_{\xi(\epsilon)}(p) + n^3_{\xi(\epsilon)}(p) \leq 1, \ \forall \ p \in \mathbb{P}$. Hence,

$$(\xi, \Upsilon) = \{(\epsilon, \{< p, m_{\xi(\epsilon)}(p), n_{\xi(\epsilon)}(p) >\}) : \epsilon \in \Upsilon, \ p \in \mathbb{P}\}.$$

Remark 2.5. [15] *Every $\mathcal{IFSS}$ is a $\mathcal{PFSS}$ and every $\mathcal{PFSS}$ is a $\mathcal{FFSS}$. But the converse may not be true.*

Definition 2.6. [15] *A $\mathcal{FFSS}$ (ν, Σ) over $\mathbb{P}$ is known as a null $\mathcal{FFSS}$ represented as $\tilde{\Phi}$ if for all $\epsilon \in \Sigma$, $\tilde{\Phi} = \bar{0}$ where $\bar{0}$ denote the null $\mathcal{FFS}$. Hence, $\tilde{\Phi} = \{(\epsilon, < p, 0, 1 >) : \epsilon \in \Sigma, \ p \in \mathbb{P}\}$.*

Definition 2.7. [15] *A $\mathcal{FFSS}$ (ν, Σ) over $\mathbb{P}$ is known as an absolute $\mathcal{FFSS}$ represented as $\tilde{\mathbb{P}}$ if $\forall \ \epsilon \in \Sigma$, $\tilde{\mathbb{P}} = \bar{1}$ where $\bar{1}$ denote the absolute FFS. Hence, $\tilde{\mathbb{P}} = \{(\epsilon, < p, 1, 0 >) : \epsilon \in \Sigma, \ p \in \mathbb{P}\}$.*

Definition 2.8. [15] *Let $\Sigma_1, \Sigma_2 \subset \Sigma$ and $(\xi_1, \Sigma_1), (\xi_2, \Sigma_2) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$. Then (ξ_1, Σ_1) is called a subset of (ξ_2, Σ_2) denoted by $(\xi_1, \Sigma_1) \subset (\xi_2, \Sigma_2)$ if:*

(a) *$\Sigma_1 \subset \Sigma_2$ and*

(b) *For all $\epsilon \in \Sigma_1$, $\xi_1(\epsilon) \subset \xi_2(\epsilon)$ that is, for all $p \in \mathbb{P}$ and $\epsilon \in \Sigma_1$, $m_{\xi_1}(p) \leq m_{\xi_2}(p)$ and $n_{\xi_1}(p) \geq n_{\xi_2}(p)$.*

Definition 2.9. [15] *Two $\mathcal{FFSS}$s (ξ_1, Σ_1) and (ξ_2, Σ_2) are said to be equal (written as $(\xi_1, \Sigma_1) = (\xi_2, \Sigma_2)$) if $(\xi_1, \Sigma_1) \subset (\xi_2, \Sigma_2)$ and $(\xi_2, \Sigma_2) \subset (\xi_1, \Sigma_1)$.*

Definition 2.10. [15] *Let $(\xi, \Sigma) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$. The complement of (ξ, Σ), denoted by $(\xi, \Sigma)^c$ defined by $(\xi, \Sigma)^c = (\xi^c, \Sigma)$, where $\xi^c : \Sigma \to \mathcal{FFS}(\mathbb{P})$ is a mapping given by $\xi^c(\epsilon) = (\xi(\epsilon))^c$ for every $\epsilon \in \Sigma$.*

Definition 2.11. [15] *Let* $(\xi_1, \Sigma_1), (\xi_2, \Sigma_2) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$. *The union of* (ξ_1, Σ_1) *and* (ξ_2, Σ_2) *is represented as* $(\xi_1, \Sigma_1) \cup (\xi_2, \Sigma_2)$ *is a* $\mathcal{FFSS}$ (ξ, Υ), *where* $\Upsilon = \Sigma_1 \cup \Sigma_2$ *and*

$$\xi(\epsilon) = \begin{cases} \xi_1(\epsilon), & \text{if } \epsilon \in \Sigma_1 - \Sigma_2, \\ \xi_2(\epsilon), & \text{if } \epsilon \in \Sigma_2 - \Sigma_1, \\ \xi_1(\epsilon) \cup \xi_2(\epsilon), & \text{if } \epsilon \in \Sigma_1 \cap \Sigma_2. \end{cases}$$

Definition 2.12. [15] *Let* $(\xi_1, \Sigma_1), (\xi_2, \Sigma_2) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$. *The intersection of* (ξ_1, Σ_1) *and* (ξ_2, Σ_2) *is represented as* $(\xi_1, \Sigma_1) \cap (\xi_2, \Sigma_2)$ *is a* $\mathcal{FFSS}$ (ξ, Υ), *where* $\Upsilon = \Sigma_1 \cap \Sigma_2$ *and*

$$\xi(\epsilon) = \xi_1(\epsilon) \cap \xi_2(\epsilon),\ \forall\ \epsilon \in \Upsilon.$$

Definition 2.13. [11] *A subfamily* Γ *of* $\mathcal{FFSS}(\mathbb{P}, \Sigma)$ *is called a Fermatean fuzzy soft topology* ($\mathcal{FFST}$) *on* $\mathbb{P}$ *if:*

(a) $\tilde{\Phi}, \tilde{\mathbb{P}} \in \Gamma$.

(b) $(\nu_i, \Sigma) \in \Gamma,\ \forall\ i \in \Lambda \Rightarrow \cup_{i \in \Lambda}(\nu_i, \Sigma) \in \Gamma$.

(c) $(\nu_1, \Sigma), (\nu_2, \Sigma) \in \Gamma \Rightarrow (\nu_1, \Sigma) \cap (\nu_2, \Sigma) \in \Gamma$.

If Γ *is a* $\mathcal{FFST}$ *on* $\mathbb{P}$ *then the structure* $(\mathbb{P}, \Gamma, \Sigma)$ *is called a Fermatean fuzzy soft topological space* ($\mathcal{FFSTS}$) *over* $\mathbb{P}$ *and the members of* Γ *are called Fermatean fuzzy soft open* ($\mathcal{FFSO}$) *sets and their complements are called Fermatean fuzzy soft closed* ($\mathcal{FFSC}$). *The family of all* $\mathcal{FFSC}$*s of* $(\mathbb{P}, \Sigma)$ *is denoted by* $\mathcal{FFSC}(\mathbb{P}, \Sigma)$.

Definition 2.14. [11] *Let* $(\mathbb{P}, \Gamma, \Sigma)$ *be a* $\mathcal{FFSTS}$ *and* $(\xi, \Sigma) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$. *Then the interior and closure of* (ξ, Σ) *denoted respectively by* $Int(\xi, \Sigma)$ *and* $Cl(\xi, \Sigma)$ *are defined as follows:*

(a) $Int(\xi, \Sigma) = \cup\{(\nu, \Sigma) \in \Gamma : (\nu, \Sigma) \subset (\xi, \Sigma)\}$.

(b) $Cl(\xi, \Sigma) = \cap\{(\nu, \Sigma) \in \mathcal{FFSC}(\mathbb{P}, \Sigma) : (\xi, \Sigma) \subset (\nu, \Sigma)\}$.

3. Fermatean Fuzzy Soft Mappings

Definition 3.1. *Let* $\mathcal{FFSS}(\mathbb{P}, \Sigma)$ *and* $\mathcal{FFSS}(\mathbb{Q}, \Omega)$ *be families of* $\mathcal{FFSS}$*s over* $\mathbb{P}$ *and* $\mathbb{Q}$ *respectively. Then* $f_{\psi\varphi} : \mathcal{FFSS}(\mathbb{P}, \Sigma) \to \mathcal{FFSS}(\mathbb{Q}, \Omega)$ *is called a Fermatean fuzzy soft mapping, where* $\psi : \mathbb{P} \to \mathbb{Q}$ *and* $\varphi : \Sigma \to \Upsilon$.

(a) *Let* $(\xi, \Sigma) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$. *The image of* (ξ, Σ) *under* $f_{\psi\varphi}$ *is written as* $f_{\psi\varphi}(\xi, \Sigma) = (\psi(\xi), \varphi(\Sigma))$ *is a* $\mathcal{FFSS}$ *in* $(\mathbb{Q}, \Upsilon)$ *such that*

$$m_{\psi(\xi)}(\iota)(q) = \begin{cases} \sup\limits_{\epsilon \in \varphi^{-1}(\iota) \cap \Upsilon,\ p \in \psi^{-1}(q)} m_{\xi(\epsilon)}(p), & \psi^{-1}(q) \neq \phi, \\ 0 & otherwise \end{cases}$$

and

$$n_{\psi(\xi)}(\iota)(q) = \begin{cases} \inf\limits_{\epsilon \in \varphi^{-1}(\iota) \cap \Upsilon,\ p \in \psi^{-1}(q)} n_{\xi(\epsilon)}(p), & \psi^{-1}(q) \neq \phi, \\ 1 & otherwise \end{cases}$$

$\forall\ \epsilon \in \Sigma,\ p \in \mathbb{P}, \iota \in \Omega$ *and* $q \in \mathbb{Q}$.

(b) *Let* $(\delta, \Omega) \in \mathcal{FFSS}(\mathbb{Q}, \Omega)$. *The inverse image of* (δ, Ω) *under* $f_{\psi\varphi}$, *denoted by* $f^{-1}_{\psi\varphi}((\delta, \Omega))$ *is a* $\mathcal{FFSS}$ *in* $(\mathbb{P}, \Sigma)$ *given by:*

$$m_{\psi^{-1}(\delta)}(\epsilon)(p) = m_{\delta(\varphi(\epsilon))}(\psi(p))$$

and

$$n_{\psi^{-1}(\delta)}(\epsilon)(p) = n_{\delta(\varphi(\epsilon))}(\psi(p))$$

$\forall\ \epsilon \in \Sigma$ *and* $p \in \mathbb{P}$

Theorem 3.1. *Let* $f_{\psi\varphi} : \mathcal{FFSS}(\mathbb{P}, \Sigma) \to \mathcal{FFSS}(\mathbb{Q}, \Omega)$ *is a Fermatean fuzzy soft mapping. Then:*

(a) $f_{\psi\varphi}((\alpha, \Upsilon)) \in \mathcal{FFSS}(\mathbb{Q}, \Omega),\ \forall\ (\alpha, \Upsilon) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$.

(b) $f^{-1}_{\Psi\varphi}((\delta, \Omega)) \in \mathcal{FFSS}(\mathbb{P}, \Sigma),\ \forall\ (\delta, \Omega) \in \mathcal{FFSS}(\mathbb{Q}, \Omega)$.

Proof.

(a) By definition 3.1(a) we have $f_{\psi\varphi}(\alpha, \Sigma) = (\psi(\alpha), \varphi(\Sigma))$ and

$$\begin{aligned} m^3_{\psi(\alpha)(\iota)}(q) + n^3_{\psi(\alpha)(\iota)}(q) &= \Big(\sup_{\epsilon \in \varphi^{-1}(\iota) \cap \Upsilon, p \in \psi^{-1}(q)} m_{\alpha(\epsilon)}(p) \Big)^3 \\ &\quad + \Big(\inf_{\epsilon \in \varphi^{-1}(\iota) \cap \Upsilon, p \in \psi^{-1}(q)} n_{\alpha(\epsilon)}(p) \Big)^3 \\ &= \sup_{\epsilon \in \varphi^{-1}(\iota) \cap \Upsilon, p \in \psi^{-1}(q)} m^3_{\alpha(\epsilon)}(p) \\ &\quad + \inf_{\epsilon \in \varphi^{-1}(\iota) \cap \Upsilon, p \in \psi^{-1}(q)} n^3_{\alpha(\epsilon)}(p) \\ &\leq \sup_{\epsilon \in \varphi^{-1}(\iota) \cap \Upsilon, p \in \psi^{-1}(q)} (1 - n^3_{\alpha(\epsilon)}(p)) \end{aligned}$$

$$+\inf_{\epsilon\in\varphi^{-1}(\iota)\cap\Upsilon,p\in\psi^{-1}(q)} n^3_{\alpha(\epsilon)}(p)$$
$$=1.$$

Whenever $\psi^{-1}(q)\neq\phi$. On the other hand if $\psi^{-1}(q)=\phi$, then we have $m^3_{\psi(\alpha)(\iota)}(q)+n^3_{\psi(\alpha)(\iota)}(q)=1$. Thus, we have shown that the Fermatean fuzzy membership grade condition is provided for Fermatean fuzzy soft image. Hence $f_{\psi\varphi}(\alpha,\Upsilon)\in FFS(\mathbb{Q},\Omega)$.

(b) Similar to the proof of (a).

Theorem 3.3. *Let $f_{\psi\varphi}:\mathcal{FFSS}(\mathbb{P},\Sigma)\to\mathcal{FFSS}(\mathbb{Q},\Omega)$ be a Fermatean fuzzy soft mapping. Then for $(\lambda,\Upsilon),(\mu,\Psi)\in\Gamma$, we have,*

(a) $f_{\psi\varphi}(\tilde{\Phi})=\tilde{\Phi}$.

(b) $f_{\psi\varphi}(\tilde{\mathbb{P}})=\tilde{\mathbb{Q}}$.

(c) $f_{\psi\varphi}((\lambda,\Upsilon)\cup((\mu,\Psi))=f_{\psi\varphi}((\lambda,\Upsilon))\cup f_{\psi\varphi}((\mu,\Psi))$.

(d) $f_{\psi\varphi}((\lambda,\Upsilon)\cap((\mu,\Psi))\subset f_{\psi\varphi}((\lambda,\Upsilon))\cap f_{\psi\varphi}((\mu,\Psi))$.

(e) If $(\lambda,\Upsilon)\subset(\mu,\Psi)$ *then* $f_{\psi\varphi}(\lambda,\Upsilon)\subset f_{\psi\varphi}(\mu,\Psi)$.

Proof. The proofs of (a) and (b) are obvious.

(c) We consider $(\lambda,\Upsilon)\cup(\mu,\Psi)=(\Theta,\Upsilon\cup\Psi)$ and $f_{\psi\varphi}((\lambda,\Upsilon)\cup((\mu,\Psi))=(\psi(\lambda),\varphi(\Upsilon))\cup(\psi(\mu),\varphi(\Psi))=(\Delta,\varphi(\Upsilon)\cup\varphi(\Psi))$. Then $f_{\psi\varphi}((\lambda,\Upsilon)\cup((\mu,\Psi))=(\psi(\Theta),\varphi(\Upsilon\cup\Psi))=(\psi(\Theta),\varphi()\cup\varphi(\Psi))$. For any $q\in\mathbb{Q}$ and $\iota\in\varphi(\Upsilon)\cup\varphi(\Psi)$, if $\psi^{-1}(q)\neq\phi$, then $m_\Delta(\iota)(q)=m_{\psi(\Theta)}(\iota)(q)=0$ and $n_\Delta(\iota)(q)=n_{\psi(\Theta)}(\iota)(q)=1$. Otherwise,

(i) If $\iota\in\varphi(\Upsilon)-\varphi(\Psi)$, then $\Delta(\iota)=\psi(\lambda)(\iota)$. On the other hand, $\iota\in\varphi(\Upsilon)-\varphi(\Psi)$ implies there does not exists $\epsilon\in\Psi$ such that $\varphi(\epsilon)=\iota$. That is, for any $\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon\cup\Psi)$, we have $\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon-\Psi)$. Hence by definition 19, we have

$$\begin{aligned}m_{\psi(\Theta)}(\iota)(q)&=\sup_{\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon\cup\Psi),\ p\in\psi^{-1}(q)} m_{\Theta(\epsilon)}(p)\\&=\sup_{\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon-\Psi),\ p\in\psi^{-1}(q)} m_{\Theta(\epsilon)}(p)\\&=\sup_{\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon\cup\Psi),\ p\in\psi^{-1}(q)} m_{\lambda(\epsilon)}(p)\\&=m_{\Delta(\iota)}(q)\end{aligned}$$

Similarly, we obtain $n_{\psi(\Theta)}(\iota)(q) = n_{\Delta(\iota)}(q)$.

(ii) If $\iota \in \varphi(\Psi) - \varphi(\Upsilon)$, analogous to (i) we have $m_{\psi(\Theta)}(\iota)(q) = m_{\Delta(\iota)}(q)$, $n_{\psi(\Theta)}(\iota)(q) = n_{\Delta(\iota)}(q)$.

(iii) If $\iota \in \varphi(\Upsilon) \cap \varphi(\Psi)$, then

$$\begin{aligned}
m_{\psi(\Theta)}(\iota)(q) &= \sup_{\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon\cup\Psi),\ p\in\psi^{-1}(q)} m_{\Theta(\epsilon)}(p) \\
&= \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Upsilon)\cup(\varphi^{-1}(\iota)\cap\Psi),\ p\in\psi^{-1}(q)} m_{\Theta(\epsilon)}(p) \\
&= (\sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Upsilon)-(\varphi^{-1}(\iota)\cap\Psi),\ p\in\psi^{-1}(q)} m_{\lambda(\epsilon)}(p)) \\
&\bigvee(\sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Upsilon)-(\varphi^{-1}(\iota)\cap\Psi),\ p\in\psi^{-1}(q)} max\{m_{\lambda(\epsilon)}(p), m_{\mu(\epsilon)}(p)\}) \\
&\bigvee(\sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Upsilon)-(\varphi^{-1}(\iota)\cap\Psi),\ p\in\psi^{-1}(q)} m_{\mu(\epsilon)}(p)) \\
&= max\left\{\begin{array}{ll} \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Upsilon),\ p\in\psi^{-1}(q)} & m_{\lambda(\epsilon)}(p), \\ \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Psi),\ p\in\psi^{-1}(q)} & m_{\mu(\epsilon)}(p) \end{array}\right\} \\
&= max\Big\{m_{\psi(\lambda)}(\iota)(q), m_{\psi(\mu)}(\iota)(q)\Big\} = m_{\Delta(\iota)}(q).
\end{aligned}$$

Similarly, we obtain $n_{\psi(\Theta)}(\iota)(q) = n_{\Delta(\iota)}(q)$. Therefore,

$$f_{\psi\varphi}((\lambda, \Upsilon) \cup (\mu, \Psi)) = f_{\psi\varphi}((\lambda, \Upsilon)) \cup f_{\psi\varphi}((\mu, \Psi)).$$

(d) Suppose that $(\lambda, \Upsilon)\cap(\mu, \Psi) = (\Theta, \Upsilon\cup\Psi)$ and $f_{\psi\varphi}((\lambda, \Upsilon)\cap(\mu, \Psi)) = (\psi(\lambda), \varphi(\Upsilon))$ $\cap(\psi(\mu), \varphi(\Psi)) = (\Delta, \varphi(\Upsilon)\cup\varphi(\Psi))$. Then $f_{\psi\varphi}((\lambda, \Upsilon)\cap((\mu, \Psi)) = (\psi(\Theta), \varphi(\Upsilon\cup\Psi)) = (\psi(\Theta), \varphi(\Upsilon) \cup \varphi(\Psi))$. For any $q \in \mathbb{Q}$ and $\iota \in \varphi(\Upsilon) \cup \varphi(\Psi)$, if $\psi^{-1}(q) \neq \phi$, then $m_\Delta(\iota)(q) = m_{\psi(\Theta)}(\iota)(q) = 0$ and $n_\Delta(\iota)(q) = n_{\psi(\Theta)}(\iota)(q) = 1$. Otherwise,

(i) If $\iota \in \varphi(\Upsilon) - \varphi(\Psi)$, then $\Delta(\iota) = \psi(\lambda)(\iota)$. On the other hand, $\iota \in \varphi(\Upsilon) - \varphi(\Psi)$ implies there does not exists $\epsilon \in \Psi$ such that $\varphi(\epsilon) = \iota$. That is, for any $\epsilon \in \varphi^{-1}(\iota) \cap (\Upsilon \cup \Psi)$, we have $\epsilon \in \varphi^{-1}(\iota) \cap (\Upsilon - \Psi)$. Hence by definition 19, we have

$$\begin{aligned}
m_{\psi(\Theta)}(\iota)(q) &= \sup_{\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon\cup\Psi),\ p\in\psi^{-1}(q)} m_{\Theta(\epsilon)}(p) \\
&= \sup_{\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon-\Psi),\ p\in\psi^{-1}(q)} m_{\Theta(\epsilon)}(p)
\end{aligned}$$

$$
\begin{aligned}
&= \sup_{\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon-\Psi),\ p\in\psi^{-1}(q)} m_{\lambda(\epsilon)}(p)\\
&\leq min\left\{\begin{matrix} \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Upsilon),\ p\in\psi^{-1}(q)} m_{\lambda(\epsilon)}(p), \\ \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Psi),\ p\in\psi^{-1}(q)} m_{\mu(\epsilon)}(p)\end{matrix}\right\}\\
&= min\{m_{\psi(\lambda)}(\iota)(q), m_{\psi(\mu)}(\iota)(q)\}\\
&= m_{\Delta(\iota)}(q).
\end{aligned}
$$

Similarity, we obtain $n_{\psi(\Theta)}(\iota)(q) \geq n_{\Delta(\iota)}(q)$.

(ii) If $\iota \in \varphi(\Upsilon) - \varphi(\Psi)$, analogous to (i) we have $m_{\psi(\Theta)}(\iota)(q) = m_{\Delta(\iota)}(q)$, $n_{\psi(\Theta)}(\iota)(q) = n_{\Delta(\iota)}(q)$.

(iii) If $\iota \in \varphi(\Upsilon) \cap \varphi(\Psi)$, then

$$
\begin{aligned}
m_{\psi(\Theta)}(\iota)(q) &= \sup_{\epsilon\in\varphi^{-1}(\iota)\cap(\Upsilon\cup\Psi),\ p\in\psi^{-1}(q)=} m_{\Theta(\epsilon)}(p)\\
&= \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap(\Upsilon\cup\Psi),\ p\in\psi^{-1}(q)} max\{m_{\lambda(\epsilon)}(p), m_{\mu(\epsilon)}(p)\}\\
&\leq max\left\{\begin{matrix} \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Upsilon),\ p\in\psi^{-1}(q)} m_{\lambda(\epsilon)}(p), \\ \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Psi),\ p\in\psi^{-1}(q)} m_{\mu(\epsilon)}(p)\end{matrix}\right\}\\
&= max\{m_{\psi(\lambda)}(\iota)(q), m_{\psi(\mu)}(\iota)(q)\}\\
&= m_{\Delta(\iota)}(q).
\end{aligned}
$$

Similarly, we obtain $n_{\psi(\Theta)}(\iota)(q) = n_{\Delta(\iota)}(q)$. Therefore,

$$f_{\psi\varphi}((\lambda, \Upsilon) \cap ((\mu, \Psi)) \subset f_{\psi\varphi}((\lambda, \Upsilon)) \cap f_{\psi\varphi}((\mu, \Psi)).$$

(e) Suppose that $(\lambda, \Upsilon) \subset (\mu, \Psi)$. Then $\Upsilon \subset \Psi$ and for any $\epsilon \in \Upsilon$ and $p \in \mathbb{P}$ we have $\psi(\Upsilon) \subset \psi(\Psi)$. we have,

$$
\begin{aligned}
m_{\psi(\lambda)}(\iota)(q) &= \begin{cases} \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Upsilon),\ p\in\psi^{-1}(q)} m_{\lambda(\epsilon)}(p), & \psi^{-1}(q) \neq \phi,\\ 0 & \text{otherwise}\end{cases}\\
&\leq \begin{cases} \sup_{\epsilon\in(\varphi^{-1}(\iota)\cap\Psi),\ p\in\psi^{-1}(q)} m_{\mu(\epsilon)}(p), & \psi^{-1}(q) \neq \phi,\\ 0 & \text{otherwise}\end{cases}\\
&= m_{\psi(\mu)}(\iota)(q)
\end{aligned}
$$

Similarly, we obtain $n_{\psi(\lambda)}(\iota)(q) \geq n_{\psi(\mu)}(\iota)(q)$. Therefore,

$$f_{\psi\varphi}((\lambda, \Upsilon)) \subset f_{\psi\varphi}((\mu, \Psi)).$$

Theorem 3.4. *Let $f_{\psi\varphi} : \mathcal{FFSS}(\mathbb{P}, \Sigma) \to \mathcal{FFSS}(\mathbb{Q}, \Omega)$ be a Fermatean fuzzy soft mapping. Then for $(\xi, \chi), (\zeta, \kappa) \in \mathcal{FFSS}(\mathbb{Q}, \Omega)$, we have,*

(a) $f_{\psi\varphi}^{-1}(\tilde{\Phi}) = \tilde{\Phi}$.

(b) $f_{\psi\varphi}^{-1}(\tilde{\mathbb{Q}}) = \tilde{\mathbb{P}}$.

(c) $f_{\psi\varphi}^{-1}((\xi, \chi) \cup ((\zeta, \kappa)) = f_{\psi\varphi}^{-1}((\xi, \chi)) \cup f_{\psi\varphi}^{-1}((\zeta, \kappa))$.

(d) $f_{\psi\varphi}^{-1}((\xi, \chi) \cap ((\zeta, \kappa)) \subset f_{\psi\varphi}^{-1}((\xi, \chi)) \cap f_{\psi\varphi}^{-1}((\zeta, \kappa))$.

(e) If $(\xi, \chi) \subset (\zeta, \kappa)$ *then* $f_{\psi\varphi}^{-1}(\xi, \chi) \subset f_{\psi\varphi}^{-1}(\zeta, \kappa)$.

(f) $f_{\psi\varphi}^{-1}((\xi, \chi)^c) = (f_{\psi\varphi}^{-1}((\xi, \chi)))^c$

Proof. Straightforward.

4. Fermatean Fuzzy Soft Continuous Mappings

Definition 4.1. *Let $(\mathbb{P}, \Gamma_1, \Sigma)$ and $(\mathbb{Q}, \Gamma_2, \Omega)$ be two $\mathcal{FFSTS}$s, a Fermatean fuzzy soft mapping $f_{\psi\varphi} : \mathcal{FFSS}(\mathbb{P}, \Sigma) \to \mathcal{FFSS}(\mathbb{Q}, \Omega)$ is Fermatean fuzzy soft continuous if $f_{\psi\varphi}^{-1}((\mu, \Omega)) \in \Gamma_1, \ \forall \ (\mu, \Omega) \in \Gamma_2$.*

Example 4.2. Let $\mathbb{P} = \{p_1, p_2\}, \mathbb{Q} = \{q_1, q_2\}, \Sigma = \{\epsilon_1, \epsilon_2\}, \Omega = \{\iota_1, \iota_2\}$ and $(\xi_1, \Sigma), (\xi_2, \Sigma)$ be $\mathcal{FFSS}$s over $\mathbb{P}$, defined as follows:

$$(\xi_1, \Sigma) = \begin{Bmatrix} (\epsilon_1, \{< p_1, 0.85, 0.65 >, < p_2, 0.9, 0.5 >\}), \\ (\epsilon_2, \{< p_1, 0.9, 0.4 >, < p_2, 0.75, 0.85 >\} \end{Bmatrix}$$
$$(\xi_2, \Sigma) = \begin{Bmatrix} (\epsilon_1, \{< p_1, 0.65, 0.75 >, < p_2, 0.6, 0.7 >\}), \\ (\epsilon_2, \{< p_1, 0.7, 0.85 >, < p_2, 0.6, 0.95 >\} \end{Bmatrix}.$$

Then $\Gamma_1 = \{\Phi, \mathbb{P}, (\xi_1, \Sigma), (\xi_2, \Sigma)\}$ is a $\mathcal{FFST}$ over $\mathbb{P}$ and hence $(\mathbb{P}, \Gamma_1, \Sigma)$ is a $\mathcal{FFST}$S. Let $\Gamma_2 = \{\Phi, \mathbb{Q}, (\delta_1, \Omega), (\delta_2, \Omega)\}$ where $(\delta_1, \Omega), (\delta_2, \Omega)$ are $\mathcal{FFSS}$ over $\mathbb{Q}$, defined as follows:

$$(\delta_1, \Omega) = \begin{Bmatrix} (\iota_1, \{< q_1, 0.85, 0.65 >, < q_2, 0.9, 0.5 >\}), \\ (\iota_2, \{< q_1, 0.9, 0.4 >, < q_2, 0.75, 0.85 >\} \end{Bmatrix}$$
$$(\delta_2, \Omega) = \begin{Bmatrix} (\iota_2, \{< q_1, 0.65, 0.75 >, < q_2, 0.6, 0.7 >\}), \\ (\iota_2, \{< q_1, 0.7, 0.85 >, < q_2, 0.6, 0.95 >\} \end{Bmatrix}.$$

Then Γ_2 is a $\mathcal{FFST}$ over $\mathbb{Q}$ and hence $(\mathbb{Q}, \Gamma_2, \Omega)$ is a $\mathcal{FFSTS}$. Let $f_{\psi\varphi}$: $\mathcal{FFSS}(\mathbb{P}, \Gamma_1, \Sigma) \rightarrow \mathcal{FFSS}(\mathbb{Q}, \Gamma_2, \Omega)$ be a Fermatean fuzzy soft mapping defined as follows:

$$\psi(p_1) = q_1 \quad \varphi(\epsilon_1) = \iota_1$$
$$\psi(p_2) = q_2 \quad \varphi(\epsilon_2) = \iota_2$$

It can be easily verified that $f^{-1}_{\psi\varphi}(\delta, \Omega) \in \Gamma_1, \forall\, (\delta, \Omega) \in \Gamma_2$. Thus $f_{\psi\varphi}$ is a Fermatean fuzzy soft continuous mapping.

Theorem 4.3. *A Fermatean fuzzy soft mapping* $f_{\psi\varphi} : (\mathbb{P}, \Gamma_1, \Sigma) \rightarrow (\mathbb{Q}, \Gamma_2, \Omega)$ *is Fermatean fuzzy soft continuous if and only if* $f^{-1}_{\psi\varphi}((\delta, \Omega)) \in \mathcal{FFSC}(\mathbb{P}, \Sigma)$, $\forall\, (\delta, \Omega) \in \mathcal{FFSC}(\mathbb{Q}, \Omega)$.

Proof. Let $(\delta, \Omega) \in \mathcal{FFSC}(\mathbb{Q}, \Omega)$. Then $(\delta, \Omega)^c \in \Gamma_2$. Since $f_{\psi\varphi}$ is continuous $(f^{-1}_{\psi\varphi}((\delta, \Omega)))^c = f^{-1}_{\psi\varphi}((\delta, \Omega)^c) \in \Gamma_1$. Hence $f^{-1}_{\psi\varphi}((\delta, \Omega)) \in \mathcal{FFSC}(\mathbb{P}, \Sigma)$.

Conversely, suppose that $f^{-1}_{\psi\varphi}((\delta, \Omega)) \in \mathcal{FFSC}(\mathbb{P}, \Sigma)$ whenever $(\delta, \Omega) \in \mathcal{FFSC}$ $(\mathbb{Q}, \Omega)$. Let $(\xi, \Omega) \in \Gamma_2$. Then $(\xi, \Omega)^c \in \mathcal{FFSC}(\mathbb{Q}, \Omega)$. Therefore by hypothesis $f^{-1}_{\psi\varphi}((\xi, \Omega)^c) \in \mathcal{FFSC}(\mathbb{P}, \Sigma)$. But $f^{-1}_{\psi\varphi}((\xi, \Omega)^c) = (f^{-1}_{\psi\varphi}((\xi, \Omega)))^c$, we have $(f^{-1}_{\psi\varphi}((\xi, \Omega)))^c \in \mathcal{FFSC}(\mathbb{P}, \Sigma)$. This implies that $f^{-1}_{\psi\varphi}((\xi, \Omega)) \in \Gamma_1$. Hence $f_{\psi\varphi}$ is Fermatean fuzzy soft continuous.

Theorem 4.4. *A Fermatean fuzzy soft mapping* $f_{\psi\varphi} : (\mathbb{P}, \Gamma_1, \Sigma) \rightarrow (\mathbb{Q}, \Gamma_2, \Omega)$ *is Fermatean fuzzy soft continuous if and only if* $f^{-1}_{\psi\varphi}(Int(\delta, \Omega)) \subset Int(f^{-1}_{\psi\varphi}((\delta, \Omega))$, $\forall\, (\delta, \Omega) \in \mathcal{FFSS}(\mathbb{Q}, \Omega)$.

Proof. Suppose that $f_{\psi\varphi}$ is Fermatean fuzzy soft continuous and $(\delta, \Omega) \in \mathcal{FFSS}$ $(\mathbb{Q}, \Omega)$. Then $f^{-1}_{\psi\varphi}(Int(\delta, \Omega)) \in \Gamma_1$ and from $Int(\delta, \Omega) \subset (\delta, \Omega)$ we obtain that $f^{-1}_{\psi\varphi}(Int(\delta, \Omega)) \subset f^{-1}_{\psi\varphi}((\delta, \Omega))$, because $Int(f^{-1}_{\psi\varphi}((\delta, \Omega)) \in \Gamma$ is largest such that $Int(f^{-1}_{\psi\varphi}((\delta, \Omega)) \subset f^{-1}_{\psi\varphi}((\delta, \Omega))$. Hence $f^{-1}_{\psi\varphi}(Int(\delta, \Omega)) \subset Int(f^{-1}_{\psi\varphi}((\delta, \Omega)))$.

Conversely, suppose that $f^{-1}_{\psi\varphi}(Int(\delta, \Omega)) \subset Int(f^{-1}_{\psi\varphi}((\delta, \Omega)))$, $\forall\, (\delta, \Omega) \in \mathcal{FFSS}$ $(\mathbb{Q}, \Omega)$. If $(\delta, \Omega) \in \Gamma_2$, then we have, $f^{-1}_{\psi\varphi}((\delta, \Omega)) \subset f^{-1}_{\psi\varphi}(Int(\delta, \Omega)) \subset Int(f^{-1}_{\psi\varphi}((\delta, \Omega)))$ $\subset f^{-1}_{\psi\varphi}((\delta, \Omega))$. So, $f^{-1}_{\psi\varphi}((\delta, \Omega)) \in \Gamma_1$. Hence $f_{\psi\varphi}$ is Fermatean fuzzy soft continuous.

Theorem 4.5. *A Fermatean fuzzy soft mapping* $f_{\psi\varphi} : (\mathbb{P}, \Gamma_1, \Sigma) \rightarrow (\mathbb{Q}, \Gamma_2, \Omega)$ *is Fermatean fuzzy soft continuous if and only if* $f_{\psi\varphi}(Cl(\xi, \Omega)) \subset Cl(f_{\psi\varphi}((\xi, \Omega))$, $\forall\, (\xi, \Omega) \in \mathcal{FFSS}(\mathbb{Q}, \Omega)$.

Proof. Suppose that $f_{\psi\varphi}$ is Fermatean fuzzy soft continuous and $(\xi, \Omega) \in \mathcal{FFSS}$ $(\mathbb{P}, \Sigma)$. Since $Cl(f_{\psi\varphi}((\xi, \Omega))) \in \mathcal{FFSC}(\mathbb{Q}, \Omega)$, $f^{-1}_{\psi\varphi}(Cl(f_{\psi\varphi}(\xi, \Omega))) \in \mathcal{FFSC}(\mathbb{P}, \Sigma)$. Therefor, $Cl(f^{-1}_{\psi\varphi}(Cl(f_{\psi\varphi}((\xi, \Omega))))) = f^{-1}_{\psi\varphi}(Cl(f_{\psi\varphi}((\xi, \Omega))))$ and $f_{\psi\varphi}((\xi, \Omega)) \subset Cl$ $(f_{\psi\varphi}((\xi, \Omega)))$. Thus we have $(\xi, \Omega) \subset f^{-1}_{\psi\varphi}(f_{\psi\varphi}((\xi, \Omega))) \subset f^{-1}_{\psi\varphi}(Cl(f_{\psi\varphi}((\xi, \Omega))))$.

It follows that $(\xi,\Omega) \subset Cl(f^{-1}_{\psi\varphi}(Cl(f_{\psi\varphi}((\xi,\Omega))))) \subset f^{-1}_{\psi\varphi}(Cl(f_{\psi\varphi}((\xi,\Omega))))$. Hence $f_{\psi\varphi}(Cl(\xi,\Omega)) \subset Cl(f_{\psi\varphi}((\xi,\Omega)))$.

Conversely, suppose that $f_{\psi\varphi}(Cl(\xi,\Omega)) \subset Cl(f_{\psi\varphi}((\xi,\Omega)))$, $\forall\ (\xi,\Omega) \in \mathcal{FFSS}(\mathbb{Q},\Omega)$ and $(\delta,\Omega) \in \mathcal{FFSS}(\mathbb{Q},\Omega)$. So $Cl(\delta,\Omega) = (\delta,\Omega)$. By hypothesis, $f_{\psi\varphi}(Cl(f^{-1}_{\psi\varphi}((\delta,\Omega)))) \subset Cl(f_{\psi\varphi}(f^{-1}_{\psi\varphi}((\delta,\Omega)))) \subset Cl(\delta,\Omega) = (\delta,\Omega)$ is obtained. Hence $Cl(f^{-1}_{\psi\varphi}((\delta,\Omega))) = f^{-1}_{\psi\varphi}((\delta,\Omega))$ and $f^{-1}_{\psi\varphi}((\delta,\Omega)) \subset Cl(f^{-1}_{\psi\varphi}((\delta,\Omega)))$. That is $Cl(f^{-1}_{\psi\varphi}((\delta,\Omega))) = f^{-1}_{\psi\varphi}((\delta,\Omega))$. This implies that $f^{-1}_{\psi\varphi}((\delta,\Omega)) \in \mathcal{FFSC}(\mathbb{P},\Sigma)$. Hence $f_{\psi\varphi}$ is Fermatean fuzzy soft continuous.

5. Fermatean Fuzzy Soft Open and Closed Mappings

Definition 5.1. *Let $(\mathbb{P},\Gamma_1,\Sigma)$ and $(\mathbb{Q},\Gamma_2,\Omega)$ be two $\mathcal{FFSTS}$s. Then the Fermatean fuzzy soft mapping $f_{\psi\varphi} : (\mathbb{P},\Gamma_1,\Sigma) \to (\mathbb{Q},\Gamma_1,\Omega)$ is said to be:*

(a) Fermatean fuzzy soft open if $f_{\psi\varphi}(\xi,\Sigma) \in \Gamma_2,\ \forall\ (\xi,\Sigma) \in \Gamma_1$.

(b) Fermatean fuzzy soft closed if $f_{\psi\varphi}(\xi,\Sigma) \in \mathcal{FFSC}(\mathbb{Q},\Omega), \forall(\xi,\Sigma) \in \mathcal{FFSC}(\mathbb{P},\Sigma)$.

Theorem 5.2. *Let $f_{\psi\varphi} : (\mathbb{P},\Gamma_1,\Sigma) \to (\mathbb{Q},\Gamma_2,\Omega)$ be a Fermatean fuzzy soft mapping. Then the next statements are equivalent:*

(a) $f_{\psi\varphi}$ is Fermatean fuzzy soft open.

(b) $f_{\psi\varphi}(Int(\lambda,\Sigma)) \subset Int(f_{\psi\varphi}(\lambda,\Sigma)),\ \forall\ (\lambda,\Sigma) \in \mathcal{FFSS}(\mathbb{P},\Sigma)$.

(c) $Int(f^{-1}_{\psi\varphi}(\mu,\Sigma)) \subset f^{-1}_{\psi\varphi}(Int(\mu,\Sigma)),\ \forall\ (\mu,\Sigma) \in \mathcal{FFSS}(\mathbb{Q},\Omega)$.

Proof. (a) $\Rightarrow$ (b) Let $(\lambda,\Sigma) \in \mathcal{FFSS}(\mathbb{P},\Sigma)$. Clearly $Int(\lambda,\Sigma) \in \Gamma$. Since $f_{\psi\varphi}$ is Fermatean fuzzy soft open, $f_{\psi\varphi}(Int(\lambda,\Sigma)) \in \Gamma$. Thus, $f_{\psi\varphi}(Int(\lambda,\Sigma)) = Int(f_{\psi\varphi}(\lambda,\Sigma)) \subset f_{\psi\varphi}(Int(\lambda,\Sigma))$.

(b) $\Rightarrow$ (c) Let $(\lambda,\Sigma) \in \mathcal{FFSS}(\mathbb{Q},\Omega)$. Then $f^{-1}_{\psi\varphi}(\lambda,\Sigma) \in FFS(\mathbb{P},\Sigma)$. Then By (b) $f_{\psi\varphi}(Int(f^{-1}_{\psi\varphi}(\lambda,\Sigma))) \subset Int(f_{\psi\varphi}(f^{-1}_{\psi\varphi}(\lambda,\Sigma))) \subset Int(\lambda,\Sigma)$. Thus we have $Int(f^{-1}_{\psi\varphi}(\lambda,\Sigma)) \subset f^{-1}_{\psi\varphi}(f_{\psi\varphi}(Int(f^{-1}_{\psi\varphi}(\lambda,\Sigma)))) \subset f_{\psi\varphi}(Int(\lambda,\Sigma)$.

(c) $\Rightarrow$ (a) Let $(\lambda,\Sigma) \in \mathcal{FFSS}(\mathbb{P},\Sigma)$. Then $Int(\lambda,\Sigma) = (\lambda,\Sigma)$ and $f_{\psi\varphi}(\lambda,\Sigma) \in \mathcal{FFSS}(\mathbb{P},\Sigma)$. By (c) $(\lambda,\Sigma) = Int(\lambda,\Sigma) \subset Int(f^{-1}_{\psi\varphi}(f_{\psi\varphi}(\lambda,\Sigma))) \subset f^{-1}_{\psi\varphi}(Int(f_{\psi\varphi}(\lambda,\Sigma)))$. Hence we have $f_{\psi\varphi}(\lambda,\Sigma) \subset f_{\psi\varphi}(f^{-1}_{\psi\varphi}(Int(f_{\psi\varphi}(\lambda,\Sigma)))) \subset Int(f_{\psi\varphi}(\lambda,\Sigma)) \subset f_{\psi\varphi}(\lambda,\Sigma)$. Thus $f_{\psi\varphi}(\lambda,\Sigma) = Int(f_{\psi\varphi}(\lambda,\Sigma))$ and hence $f_{\psi\varphi}(\lambda,\Sigma) \in \mathcal{FFSS}(\mathbb{P},\Sigma)$. Therefore $f_{\psi\varphi}$ is Fermatean fuzzy soft open.

Theorem 5.3. *Let $f_{\psi\varphi} : (\mathbb{P},\Gamma_1,\Sigma) \to (\mathbb{Q},\Gamma_2,\Omega)$ be a Fermatean fuzzy soft mapping. Then the next statements are equivalent:*

(a) $f_{\psi\varphi}$ is Fermatean fuzzy soft closed.

(b) $Cl(f_{\psi\varphi}(\lambda, \Sigma)) \subset f_{\psi\varphi}(Cl(\lambda, \Sigma)),\ \forall\ (\lambda, \Sigma) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$.

Proof. (a) $\Rightarrow$ (b) Let $(\lambda, \Sigma) \in FFS(\mathbb{P}, \Sigma)$. Clearly $Cl(\lambda, \Sigma) \in \mathcal{FFSC}(\mathbb{P}, \Sigma)$. Since $f_{\psi\varphi}$ is Fermatean fuzzy soft closed, $f_{\psi\varphi}(Cl(\lambda, \Sigma)) \in \mathcal{FFSC}(\mathbb{Q}, \Omega)$. Then we have $Cl(f_{\psi\varphi}(\lambda, \Sigma)) = Cl(f_{\psi\varphi}(Cl(\lambda, \Sigma))) \subset f_{\psi\varphi}(Cl(\lambda, \Sigma))$.

(b) $\Rightarrow$ (c) Let $(\lambda, \Sigma) \in \mathcal{FFSC}(\mathbb{P}, \Sigma)$. Then $Cl(\lambda, \Sigma) = (\lambda, \Sigma)$. By (b) $Cl(f_{\psi\varphi}(\lambda, \Sigma)) \subset f_{\psi\varphi}(Cl((\lambda, \Sigma)) = f_{\psi\varphi}(\lambda, \Sigma) \subset Cl(f_{\psi\varphi}(\lambda, \Sigma))$. Thus $f_{\psi\varphi}(\lambda, \Sigma) = Cl(f_{\psi\varphi}(\lambda, \Sigma))$ and hence $f_{\psi\varphi}(\lambda, \Sigma) \in FFSC(\mathbb{Q}, \Omega)$. Therefore $f_{\psi\varphi}$ is Fermatean fuzzy soft closed.

Theorem 5.4. *Let* $f_{\psi\varphi} : (\mathbb{P}, \Gamma_1, \Sigma) \to (\mathbb{Q}, \Gamma_2, \Omega)$ *be a bijective Fermatean fuzzy soft mapping. Then the next statements are equivalent:*

(a) $f_{\psi\varphi}$ *is Fermatean fuzzy soft closed.*

(b) $Cl(f_{\psi\varphi}(Int(\lambda, \Sigma))) \subset Cl(f_{\psi\varphi}(\lambda, \Sigma)),\ \forall\ (\lambda, \Sigma) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$.

(c) $f^{-1}_{\psi\varphi}(Cl(\mu, \Sigma)) \subset f^{-1}_{\psi\varphi}(Cl(\mu, \Sigma)),\ \forall\ (\mu, \Sigma) \in \mathcal{FFSS}(\mathbb{Q}, \Omega)$.

Proof. Obvious.

Theorem 5.5. *Let* $(\mathbb{P}, \Gamma_1, \Sigma)$ *and* $(\mathbb{Q}, \Gamma_2, \Omega)$ *be two* $\mathcal{FFSTS}$*s and let* $f_{\psi\varphi} : \mathcal{FFSS}(\mathbb{P}, \Sigma) \to \mathcal{FFSS}(\mathbb{Q}, \Omega)$ *be a bijective frematean fuzzy soft mapping. Then the next statements are equivalent:*

(a) $f_{\psi\varphi}$ *is Fermatean fuzzy soft continuous and frematean fuzzy soft open.*

(b) $f_{\psi\varphi}$ *is Fermatean fuzzy soft continuous and frematean fuzzy soft closed.*

(c) $f_{\psi\varphi}(Cl(\lambda, \Sigma)) = Cl(f_{\psi\varphi}(\lambda, \Sigma))$ *for every* $(\lambda, \Sigma) \in FFS(\mathbb{P}, \Sigma)$.

(d) $Cl(f^{-1}_{\psi\varphi}(\mu, \Sigma)) = f^{-1}_{\psi\varphi}(Cl(\mu, \Sigma))$ *for every* $(\mu, \Sigma) \in \mathcal{FFSS}(\mathbb{Q}, \Omega)$.

(e) $f^{-1}_{\psi\varphi}(Int(\mu, \Sigma)) = Int(f^{-1}_{\psi\varphi}(\mu, \Sigma))$ *for every* $(\mu, \Sigma) \in \mathcal{FFSS}(\mathbb{Q}, \Omega)$.

(f) $Int(f_{\psi\varphi}(\lambda, \Sigma)) = f_{\psi\varphi}(Int(\lambda, \Sigma))$ *for every* $(\lambda, \Sigma) \in \mathcal{FFSS}(\mathbb{P}, \Sigma)$.

Proof. Easy and left to the readers.

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