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SOFT SUPER-SPACE OF A SOFT METRIC SPACE WITH SOFT POINT

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Abstract: In the study of Guzide Senel [14] introduce the soft distance function defined in soft topological space. In the present paper, we introduced soft super-space of soft metric spaces with soft points to contribute to the progress of soft distance function and some of its examples are also present here. We hope that the result findings in this paper will help researcher enhance and promote the further study on soft super-space of soft metric spaces to carry out a general framework for their applications in practical life.

Keywords and Phrases: Soft set, soft null set, soft universal set, soft metric space, soft point, soft distance soft subspace and so on.

2020 Mathematics Subject Classification: 03E72, 03E75.

1. Introduction and Definitions

The complexity of uncertainty data is available in many fields. Like environment, engineering, economics, science, social science etc. In 1999, Molodtsov [13] introduced the most important concept of soft set as a new mathematical tool for dealing with uncertainties. In (2003) Maji et al., [12] studied the theory of soft sets initiated by Molodtsov, they study equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. Also defined Soft binary operations like AND, OR and operations of union, intersection with examples.

In (2011) Shabir M. and Naz M. [15] gave soft topological spaces, which are defined over an initial universe with a fixed set of parameters. The notion of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms are introduced and their basic properties are investigating. It is also show that a soft topological space gives a parameterized family of topological spaces with the help of an examples.

In (2014), Hac J. Akta G. and Serif Ozlu [1] introduces order of the soft groups, power of the soft sets, power of the soft groups, and cyclic soft group on a group. They also investigate the relationship between cyclic soft groups and classical groups. In (2017), Güzide Şenel [13] The problem of not to analyses the distance between two soft points encountered in many practical applications such as soft distance that cannot be defined in soft topological spaces. They generate the soft metric spaces whose structure is represented by soft distance function shown by $\tilde{d}$. The description of d provides an exact method to analyse the distance between two soft points. By using this method, they studied new classes of soft mappings. They also present different definitions of $\tilde{d}$, in the sense that they provided soft metric spaces that were not describe before. In 2019 Izzettin Demir and Resime Bozyikit [7] introduced a new soft topology related to a self-soft mapping and studied some of its basic properties and also introduced the concept of a soft orbit and explained it with examples. Furthermore, the notion of a soft b-metric space was also introduced.

In the field of mathematics, matrices are most important tool in lots of branches in mathematics, engineering, medical sciences and so on. Its properties are also very useful tool in such a branch of mathematics. In this paper, we introduced soft super space of a soft metric space with soft point. Moreover, we define some examples related to soft super space of soft metric space.

2. Preliminaries

Definition 2.1. Molodtsov [13] *Soft Set Let U be an initial universe set and let E be a set of parameters. A pair (F, E) is called a soft set (over U) if and only if F is a mapping or E into the set of aft subsets of the set U.*

Definitions 2.2. [12] *A soft set (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$ In other words, a soft set over U is a parameterized family of subsets of the universal $U, e \in A$. $F(e)$ may be considered as the* *set of* $\in$*- approximate elements of the set* (F, A)*. Clearly a soft set is not a set. Inspired by Molodtsov we have given one new example of soft set.*

Example 2.1. [12] Suppose the following
U - is the set of kid's care under consideration. E - is the set of parameters. Each parameter is a word or a sentence. $E =$ Minimum fee; project and activities; Music and Movement; Safe Outdoor Activities; Clean and safe Environment; Smart Classes; Expert Faculties
In this case to define a soft set means to point out Minimum fee, Project and activity and so on. The soft set describe the "Attractiveness of the Kid's Care" which Mr. X is going to joined. We consider below the same example in more detail for our next discussion.
Suppose that there are six Coaching Centers in the universe U given by
$U = \{c_1, c_2, c_3, c_4, c_5, c_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$
e_1 stands for the parameter Minimum fee
e_2 stands for the parameter Project and Activities
e_3 stands for the parameter Safe Outdoor Activities
e_4 stands for the parameter Safe and Clean Environment
e_5 stands for the parameter Smart Classes
Suppose that

$$\begin{aligned}
F(e_1) &= c_2, c_4 \\
F(e_2) &= c_1, c_3 \\
F(e_3) &= c_4, c_5 \\
F(e_4) &= c_1, c_4, c_5 \\
F(e_5) &= c_3
\end{aligned}$$

The soft set (F, E) is a parameterized family $\{F(e_i), i = 1, 2, 3, ...\}$ of subsets of the set U and gives us a collection of approximate descriptions of an object. Therefore, $F(e_1)$ means "Kid's Care (Minimum Fee)" whose functional-value is the set $\{c_2, c_4\}$. Thus, we can view the soft set (F, E) as a collection of approximations as below: $(F, E) = \{$Minimum Fee $= \{c_2, c_4\}$, Project and Activities $= \{c_1, c_3\}$, Safe Outdoor Activities $= \{c_4, c_5\}$, Clean and Safe Environment $= \{c_1, c_3, c_5\}$, Smart Classes $= \{c_3\}$, For example, for the approximation = "Minimum Fee $\{c_2, c_4\}$", we have the following:
(i) the predicate name is expensive Kid's Care; and
(ii) the approximate value set or value set is $\{c_2, c_4\}$

Definitions 2.3. [12] *Null Soft Set - A soft set* (F, A) *over* U *is said to be a NULL* *soft set denoted by* $\emptyset$, *if* $\forall e \in A, F(e) = \emptyset$.

Example 2.2. [12] Suppose that,
U is the set of wooden houses under consideration;
A is the set of parameters. Let there be five houses in the universe U given by $U = \{h_1, h_2, h_3, h_4, h_5\}$ and $A = \{$brick; muddy; steel; stone$\}$. The soft set (F, A) describes the "construction of the houses". The soft sets (F, A) is defined as
F (brick) means the brick-built houses,
F (muddy) means the muddy houses,
F (steel) means the steel-built houses,
F (stone) means the stone-built house
The soft set (F, A) is the collection of approximations as below: $(F, A) = \{$brick-built houses = 4, muddy houses = 4, steel built houses = 4, stone built houses = 4$\}$. Here, (F, A) is Null soft set.

Definitions 2.4. [12] *Absolute soft set – The soft set* F *is called an Absolute Soft set, denoted by* $\tilde{X}$ *if* $F(e) = X$ *for every* $e \in E$.

Definitions 2.5. [14] *Soft point - The soft set* f *is called a soft point in* S, *if for the parameter* $e_i \in E$ *such that* $f(e_i) \neq \emptyset$ *and* $f(ek) = \emptyset$, *for all* $ek \in E/\{e_i\}$ *is denoted by* $(e_{i_f})j$ *for all* $ijk \neq N_+$. *(Note that the set of all soft points of* f *will be denoted by* $SP(f)$*)*.

Definitions 2.6. [14] *Soft metric spaces - Let* $\emptyset \neq X \subseteq E, f \in Sx(U)$ *and* $f : X \to P(U)$ *be one to one function* fi, fj, $fs \in Sx(U)$ *and* $(e_{i_f})i$, $(e_{j_f})j$, $(e_s)s \in f$. *A mapping*

$$\tilde{d}SP(f) \times SP(f) \to \tilde{R}(E)$$

is said to be a soft metric on the soft set f *if* $\tilde{d}$ *satisfies the following condition:*
(i) $\tilde{d}((e_{i_f})i, (e_{j_f})j) \geq 0$.
(ii) $\tilde{d}((e_{i_f})i, (e_{j_f})j) = 0 \Leftrightarrow \tilde{d}(e_{i_f})i = (e_{j_f})j$
(iii) $\tilde{d}((e_{i_f})i, (e_{j_f})j) = \tilde{d}((e_{j_f})j, (e_{i_f})i)$.
(iv) $\tilde{d}((e_{i_f})i, (e_{j_f})j) \leq \tilde{d}((e_{i_f})i, (e_s)s) + \tilde{d}((e_{s_f})s, (e_{j_f})j)$
If $\tilde{d}$ *is a soft metric on the soft set then,* f *is called soft metric space and denoted by* $(f, \tilde{d})$

Definitions 2.7. [14] *Soft distance - Let* $(e_{i_f})i, (e_{j_f})j$ *be soft points of a soft metric space. The value of* $\tilde{d}((e_{i_f})i, (e_{j_f})j)$ *is called as the soft distance between the soft points* $(e_{i_f})i$ *and* $(e_{j_f})j$.

3. Main Theorems

In this section we presented the concept of soft super-space of metric space with soft point.

Definitions 3.1. *If $(f,\tilde{d})$ is a soft metric space and $(g,\tilde{d})$ is a soft subset of $(f,\tilde{d})$ we can induce soft metric on $(f,\tilde{d})$ by restricting the metric of $(g,\tilde{d})$ on $(f,\tilde{d})$. Can we do the reverse thin?*
Suppose $(f,\tilde{d})$ is a soft metric space and $(g,\tilde{d})$ is a proper superset of $(f,\tilde{d})$. we define a soft metric on $(f,\tilde{d})$ that is an extension of $\tilde{d}$? for these.
Consider a soft metric space $(g,\tilde{d})$ and $(f,\tilde{d}) \supsetneq (g,\tilde{d})$. Since $(f,\tilde{d})\backslash(g,\tilde{d})$ is non-empty, take any soft metric on $(g,\tilde{d})\backslash(f,\tilde{d})$.
Choose and fix two soft points $(e_{i_f})i \in (f,\tilde{d})$ and $(e_{j_f})j \in (f,\tilde{d})\backslash(g,\tilde{d})$.
Now define $\tilde{D} : (f,\tilde{d}) \times (f,\tilde{d}) \to \mathbb{R}$ as

$$\tilde{D}((e_{x_f})x,(e_{y_f})y) = \begin{cases} \tilde{d}((e_{x_f})x,(e_{y_f})y) \quad \text{if} \quad (e_{x_f})x,(e_{y_f})y \in (g,\tilde{d}) \\ \tilde{e}((e_{x_f})x,(e_{y_f})y) \quad \text{if} \quad (e_{x_f})x,(e_{y_f})y \in ((f,\tilde{d})(g,\tilde{d}) \\ \tilde{d}((e_{x_f})x,((e_{i_f})i) + 1 + e((e_{j_f})j,(e_{y_f})y \\ \text{if} \quad (ex_f)x \in (g,d) \text{ and } (ey_f)y \in (f,\tilde{d})\backslash(g,\tilde{d}) \\ \tilde{e}((ex_f)x(ej_f)j) + 1 + \tilde{d}((ei_f)i,(ey_f)y) \\ \text{if } (ex_f)x \in (f,\tilde{d})\backslash(g,\tilde{d}) \quad \text{and} \quad (ey_f)y \in (g,\tilde{d}) \end{cases}$$

Then $\tilde{D}$ is a soft metric on $(f,\tilde{d})$ such that $d = \tilde{D}|(g,\tilde{d}) \times (g,\tilde{d})$.
First observe that $\tilde{D}((e_{x_f})x,(e_{y_f})y) \geq 0 \forall (e_{x_f})x(e_{y_f})y \in (f,\tilde{d})$. Thus, P1 holds.
For P2, consider any $(e_{x_f})x,(e_{y_f})y \in (f,\tilde{d})$.
Case (1) $(e_{x_f})x,(e_{y_f})y \in (g,\tilde{d})$ Then

$$\begin{aligned} \tilde{D}((e_{x_f})x,(e_{y_f})y) = 0 &\Leftrightarrow \tilde{d}((e_{x_f})x,(e_{y_f})y) = 0 \\ [\because (e_{x_f})x,(e_{y_f})y \in (f,\tilde{d}) &\Rightarrow \tilde{d}((e_{x_f})x,(e_{y_f})y) = \tilde{d}((e_{x_f})x,(e_{y_f})y)] \\ &\Leftrightarrow (e_{x_f})x = (e_{y_f})y \end{aligned}$$

Case (2) $((e_{x_f})x,(e_{y_f})y) \in (f,\tilde{d})\backslash(g,\tilde{d})$ Then

$$\begin{aligned} \tilde{D}((e_{x_f})x,(e_{y_f})y) = 0 &\Leftrightarrow \tilde{e}((e_{x_f})x,(e_{y_f})y) = 0 \\ [\because ((e_{x_f})x,(e_{y_f})y) \in (f,\tilde{d})\backslash(g,\tilde{d}) &\Rightarrow \tilde{e}((e_{x_f})x,(e_{y_f})y) = \tilde{e}((e_{x_f})x,(e_{y_f})y)] \\ &\Leftrightarrow ((e_{x_f})x = (e_{y_f})y \end{aligned}$$

Case (3) $(e_{x_f})x \in (g,\tilde{d})$ and $(e_{y_f})y \in \backslash(f,\tilde{d})\backslash(g,\tilde{d})$
In this case $(e_{x_f})x \neq (e_{y_f})y$ and $\tilde{D}((e_{x_f})x,(e_{y_f})y \neq 0$

Case (4) $(e_{x_f})x \in (f^{\wedge}, d^*)\backslash(g, \tilde{d})$ and $(e_{y_f})y \in (g, \tilde{d})$
In this case $(e_{x_f})x \neq (e_{y_f})y$ and $D^*((e_{x_f})x, (e_{y_f})y \neq 0$
From all the above cases it follows that
$\tilde{D}(e_{x_f})x, (e_{y_f})y = 0 \Leftrightarrow (e_{x_f})x = (e_{y_f})y \forall (e_{x_f})x, (e_{y_f})y \in (f, \tilde{d})$. Thus, P2 holds.
To see that P3 holds, consider any $(e_{x_f})x, (e_{y_f})y \in (f, \tilde{d})$
Case (1) $(e_{x_f})x, (e_{y_f})y \in (g, \tilde{d})$ Then
$\tilde{D}((e_{x_f})x, (e_{y_f})y) = d^{\wedge}((e_{x_f})x, (e_{y_f})y) = \tilde{d}((e_{x_f})x, (e_{y_f})y) = \tilde{D}((e_{x_f})x, (e_{y_f})y)$.
Case (2) $(e_{x_f})x, (e_{y_f})y \in (f, \tilde{d})\backslash(g, \tilde{d})$ Then
$\tilde{D}((e_{x_f})x, (e_{y_f})y) = \tilde{e}((e_{x_f})x, (e_{y_f})y) = \tilde{e}((e_{x_f})x, (e_{y_f})y) = \tilde{D}((e_{x_f})x, (e_{y_f})y)$.
Case (3) $(e_{x_f})x \in (g, \tilde{d})$ and $(e_{y_f})y) \in (f, \tilde{d})\backslash(g, \tilde{D})$ Then

$$\begin{aligned}\tilde{D}(e_{x_f})x, (e_{y_f})y) &= \tilde{d}(e_{x_f})x, (e_{i_f})i)) + 1 + \tilde{e}((e_{j_f})j), (e_{y_f})y) \\ &= \tilde{e}((e_{j_f})j), e_{x_f})x) + 1 + \tilde{d}(e_{x_f})x, ((e_{i_f})i) \\ &= \tilde{e}(e_{x_f})x, ((e_{j_f})j) + 1 + \tilde{d}((e_{i_f})i), (e_{x_f})x \\ &= \tilde{D}((e_{y_f})y, (e_{x_f})x)\end{aligned}$$

Case (4) $(e_{x_f})x \in (f, \tilde{d})\backslash(g, \tilde{d})$ and $(e_{y_f})y \in (g, \tilde{d})$ Similar to Case (3).
Hence P3 is verified.
Now for triangle Inequality consider any $(e_{x_f})x, (e_{y_f})y, (e_{z_f})z \in (f, \tilde{d})$.
Case (1) $(e_{x_f})x, (e_{y_f})y \in (g, \tilde{d})$. If $(e_{z_f})z \in (g, \tilde{d})$. then

$$\begin{aligned}\tilde{D}((e_{x_f})x, (e_{y_f})y) = \tilde{d}((e_{x_f})x, (e_{y_f})y) &\leq \tilde{d}((e_{x_f})x, (e_{z_f})z) + \tilde{d}((e_{z_f})z, (e_{y_f})y) \\ &= \tilde{D}((e_{x_f})x, (e_{z_f})z) + \tilde{D}((e_{z_f})z, (e_{y_f})y\end{aligned}$$

If $((e_{z_f})z \in (f, \tilde{d})\backslash(g, \tilde{d})$, then

$$\begin{aligned}\tilde{D}((e_{x_f})x, (e_{y_f})y) &= \tilde{d}((e_{x_f})x, (e_{y_f})y) \\ &\leq \tilde{d}((e_{x_f})x, (e_{i_f})i) + \tilde{d}((e_{i_f})i, (e_{y_f})y) \quad [\because (e_{i_f})i \in (g, \tilde{d})] \\ &\leq [\tilde{d}((e_{x_f})x, (e_{i_f})i) + 1 + \tilde{e}((e_{j_f})j, (e_{z_f})z)] + [\tilde{e}((e_{z_f})z, (e_{j_f})j) \\ &\quad + 1 + \tilde{e}((e_{i_f})i, (e_{y_f})y)] \\ &= \tilde{D}((e_{x_f})x, (e_{z_f})z) + \tilde{D}((e_{z_f})z, (e_{y_f})y)\end{aligned}$$

Case (2) $(e_{x_f})x, (e_{y_f})y) \in (f, \tilde{d})\backslash(g, \tilde{d})$. If $(e_{z_f})z \in (g, \tilde{d})$, then

$$\begin{aligned}\tilde{D}((e_{x_f})x, (e_{y_f})y) &= \tilde{e}((e_{x_f})x, (e_{y_f})y)\\ &\leq \tilde{e}((e_{x_f})x, (e_{j_f})j) + \tilde{e}((e_{j_f})j, (e_{y_f})y) \quad [\because (e_{i_f})i \in (g, \tilde{d})]\\ &\leq [\tilde{e}((e_{x_f})x, (e_{j_f})j) + 1 + \tilde{d}(e_{i_f})i, (e_{z_f})z]\\ &\quad + [\tilde{d}(e_z)z, (e_{i_f})i + 1 + \tilde{e}(e_{j_f})j, (e_{y_f})y]\\ &= \tilde{D}(e_{x_f})x, (e_{z_f})z) + \tilde{D}(e_{z_f})z, (e_{y_f})y\end{aligned}$$

If $(e_{z_f})z \in (f, \tilde{d})\backslash(g, \tilde{d})$ then

$$\begin{aligned}\tilde{D}(e_{x_f})x, (e_{z_f})z &= \tilde{e}(e_{x_f})x, (e_{y_f})y \leq \tilde{e}(e_{x_f})x, (e_{z_f})z + \tilde{e}(e_{z_f})z, (e_{y_f})y\\ &= \tilde{D}(e_{x_f})x, (e_{z_f})z + \tilde{e}(e_{z_f})z, (e_{y_f})y\end{aligned}$$

Case (3) $(e_{x_f})x \in (g, \tilde{d})$ and $(e_{y_f})y \in (f, \tilde{d})\backslash(g, \tilde{d})$. If $(e_{z_f})z \in (g, \tilde{d})$, then

$$\begin{aligned}\tilde{D}(e_{x_f})x, (e_{y_f})y &= \tilde{d}(e_{x_f})x, (e_{i_f})i + 1 + \tilde{e}(e_{j_f})j, (e_{y_f})y\\ &\leq \tilde{d}(e_{x_f})x, (e_{z_f})z + \tilde{d}(e_{z_f})z, (e_{i_f})i + 1 + \tilde{e}(e_{j_f})j, (e_{y_f})y\\ &\leq \tilde{d}(e_{x_f})x, (e_{z_f})z + \{\tilde{d}(e_{z_f})z, (e_{i_f})i + 1 + \tilde{e}(e_{j_f})j, (e_{y_f})y\}\\ &= \tilde{D}(e_{x_f})x, (e_{z_f})z + \tilde{D}(e_{z_f})z, (e_{y_f})y\end{aligned}$$

If $(e_{z_f})z \in (f, \tilde{d})\backslash(g, \tilde{d})$, then

$$\begin{aligned}\tilde{D}(e_{x_f})x, (e_{y_f})y &= \tilde{d}(e_{x_f})x, (e_{i_f})i + 1 + \tilde{e}(e_{j_f})j, (e_{y_f})y\\ &\leq \tilde{d}(e_{x_f})x, (e_{i_f})i + 1 + \tilde{e}(e_{j_f})j, (e_{z_f})z + \tilde{e}(e_{z_f})z, (e_{y_f})y\\ &\leq \{\tilde{d}(e_{x_f})x, (e_{i_f})i + 1 + \tilde{e}(e_{j_f})j, (e_{z_f})z\} + \tilde{e}(e_{z_f})z, (e_{y_f})y\\ &= \tilde{D}(e_{x_f})x, (e_{z_f})z + \tilde{D}(e_{z_f})z, (e_{y_f})y\end{aligned}$$

Case (4) $(e_{x_f})x \in (f, \tilde{d})\backslash(g, \tilde{d})$, and $(e_{y_f})y \in (g, \tilde{d})$.
Similar to case III
It is important to note that instead of 1, we could have used any positive real number α. Also, we have an arbitrary choice of metric e on $(f, \tilde{d})\backslash(g, \tilde{d})$. Thus given super space of a soft metric space $(f, \tilde{d})$ and a proper superset $(g, \tilde{d})$ of $(f, \tilde{d})$. We can have uncountable number of metrics on $(g, \tilde{d})$ extending the metric $\tilde{d}$ on $(f, \tilde{d})$.

4. Examples of soft super spaces

Example 4.1. Let $(f, \tilde{d})$ be any non-empty soft metric space and $\tilde{D} : SP(f) \times SP(f) \to \tilde{R}(E)$ be any real valued function. Then $\tilde{D}$ is a super space of soft metric on $\tilde{d}$ if and only if for all $(e_{a_f})a, (e_{b_f})b, (e_{z_f})z \in \tilde{d}$, follow the condition $\tilde{D}((e_{a_f})a, (e_{b_f})b) \leq \tilde{D}((e_{z_f})z, (e_{a_f})a) + \tilde{D}((e_{z_f})z, (e_{b_f})b)$.

Solution. Let $\tilde{D}$ is a super space of soft metric on $\tilde{d}$. Then $\tilde{D}$ satisfies all the four axioms of a soft super metric. Now for triangle Inequality consider any $(e_{a_f})a, (e_{b_f})b, (e_{c_f})c \in (f, \tilde{d})$.

Case (1) $(e_{a_f})a, (e_{b_f})b \in (g, \tilde{d})$. If $(e_{c_f})c \in (g, \tilde{d})$. Then

$$\begin{aligned}\tilde{D}((e_{a_f})a, (e_{b_f})b) &= \tilde{d}((e_{a_f})a, (e_{b_f})b) \leq \tilde{d}((e_{a_f})a, (e_{c_f})c) + \tilde{d}((e_{c_f})c, (e_{b_f})b) \\ &= \tilde{D}((e_{a_f})a, (e_{c_f})c) + \tilde{D}((e_{c_f})c, (e_{b_f})b)\end{aligned}$$

If $((e_{c_f})c) \in (f, \tilde{d}) \backslash (g, \tilde{d})$, then

$$\begin{aligned}\tilde{D}((e_{a_f})a, (e_{b_f})b) &= \tilde{d}((e_{a_f})a, (e_{b_f})b) \\ &\leq \tilde{d}((e_{a_f})a, (e_{p_f})p) + \tilde{d}((e_{p_f})p, (e_{y_f})y) \quad [\because (e_{p_f})p \in (g, \tilde{d})] \\ &\leq [\tilde{d}((e_{a_f})a, (e_{p_f})p) + 1 + \tilde{e}((e_{q_f})q, (e_{c_f})c)] \\ &\quad + [\tilde{e}((e_{c_f})c, (e_{q_f})q) + 1 + \tilde{e}((e_{p_f})p, (e_{b_f})b)] \\ &= \tilde{D}((e_{a_f})a, (e_{c_f})c) + \tilde{D}((e_{c_f})c, (e_{b_f})b)\end{aligned}$$

Case (2) $((e_{a_f})a, (e_{b_f})b) \in (f, \tilde{d}) \backslash (g, \tilde{d})$, If $(e_{c_f})c \in (g, \tilde{d})$, then

$$\begin{aligned}\tilde{D}((e_{a_f})a, (e_{b_f})b) &= \tilde{e}((e_{a_f})a, (e_{b_f})b) \\ &\leq \tilde{e}((e_{a_f})a, (e_{q_f})q) + \tilde{e}((e_{q_f})q, (e_{b_f})b) \quad [\because (e_{p_f})p \in (g, \tilde{d})] \\ &\leq [\tilde{e}(e_{a_f})a, (e_{q_f})q + 1 + \tilde{d}(e_{a_f})a, (e_{c_f})c] \\ &\quad + [\tilde{d}(e_{c_f})c, (e_{p_f})p + 1 + \tilde{e}(e_{q_f})q, (e_{b_f})b] \\ &= \tilde{D}(e_{a_f})a, (e_{c_f})c + \tilde{D}(e_{c_f})c, (e_{b_f})b\end{aligned}$$

If $(e_{c_f})c \in (f, \tilde{d}) \backslash (g, \tilde{d})$ then

$$\begin{aligned}\tilde{D}(e_{a_f})a, (e_{c_f})c &= \tilde{e}(e_{a_f})a, (e_{b_f})b \leq \tilde{e}(e_{a_f})a, (e_{c_f})c + \tilde{e}(e_{c_f})c, (e_{b_f})b \\ &= \tilde{D}(e_{a_f})a, (e_{c_f})c + \tilde{e}(e_{c_f})c, (e_{b_f})b\end{aligned}$$

Case (3) $(e_{a_f})a \in (g,\tilde{d})$ and $(e_{b_f})b \in (f,\tilde{d})\backslash(g,\tilde{d})$, If $(e_{c_f})c \in (g,\tilde{d})$, then

$$\begin{aligned}\tilde{D}(e_{a_f})a,(e_{b_f})b) &= \tilde{d}((e_{a_f})a,(e_{p_f})p) + 1 + \tilde{e}((e_{q_f})q,(e_{b_f})b)\\ &\leq \tilde{d}((e_{a_f})a,(e_{c_f})c) + \tilde{d}((e_{c_f})c,(e_{p_f})p) + 1 + \tilde{e}((e_{q_f})q,(e_{b_f})b)\\ &\leq \tilde{d}(e_{a_f})a,(e_{c_f})c + \{\tilde{d}(e_{c_f})c,(e_{p_f})p + 1 + \tilde{e}(e_{q_f})q,(e_{b_f})b\}\\ &= \tilde{D}(e_{a_f})a,(e_{c_f})c + \tilde{D}(e_{c_f})c,(e_{b_f})b\end{aligned}$$

If $(e_{c_f})c \in (f,\tilde{d})\backslash(g,\tilde{d})$, then

$$\begin{aligned}\tilde{D}(e_{a_f})a,(e_{b_f})b &= \tilde{d}(e_{a_f})a,(e_{p_f})p + 1 + \tilde{e}(e_{q_f})q,(e_{b_f})b\\ &\leq \tilde{d}(e_{a_f})a,(e_{p_f})p + 1 + \tilde{e}(e_{q_f})q,(e_{c_f})c + \tilde{e}(e_{c_f})c,(e_{b_f})b\\ &\leq \{\tilde{d}(e_{a_f})a,(e_{p_f})p + 1 + \tilde{e}(e_{q_f})q,(e_{c_f})c\} + \tilde{e}(e_{c_f})c,(e_{b_f})b\\ &= \tilde{D}(e_{a_f})a,(e_{c_f})c + \tilde{D}(e_{c_f})c,(e_{b_f})b\end{aligned}$$

Thus $\tilde{D}$ satisfies all the axioms and hence is a super space of soft metric space on $\tilde{d}$.

Example 4.2. Suppose $\tilde{D}$ is a super space of soft metric on $\tilde{d}$. Then prove that for the any soft point $(e_{x_f})x,(e_{z_f})z,(e_{y_f})y,(e_{w_f})w \in \tilde{d}$ prove that

$$|\tilde{D}((e_{x_f})x,(e_{y_f})y) - \tilde{D}((e_{z_f})z,(e_{w_f})w)| \leq \tilde{D}((e_{x_f})x,(e_{z_f})z) + \tilde{D}((e_{y_f})y,(e_{w_f})w).$$

Proof. Consider for any $(e_{x_f})x,(e_{z_f})z,(e_{y_f})y \in \tilde{d}$

$$\tilde{D}((e_{x_f})x,(e_{y_f})y) \leq \tilde{D}((e_{x_f})x,(e_{Z_f})Z) + \tilde{D}((e_{z_f})z,(e_{y_f})y).$$

[by Triangular Inequality]

$$\leq \tilde{D}((e_{x_f})x,(e_{Z_f})Z) + [\tilde{D}((e_{z_f})z,(e_{w_f})w) + \tilde{D}((e_{w_f})w,(e_{y_f})y)]$$

[by Triangular Inequality]

$$\leq \tilde{D}((e_{x_f})x,(e_{Z_f})Z) + \tilde{D}((e_{z_f})z,(e_{w_f})w) + \tilde{D}((e_{y_f})y,(e_{w_f})w$$

[By symmetric]

$$\tilde{D}((e_{x_f})x,(e_{y_f})y) - \tilde{D}((e_{z_f})z,(e_{w_f})w) \leq \tilde{D}((e_{x_f})x,(e_{Z_f})Z) + \tilde{D}((e_{y_f})y,(e_{w_f})w \quad (1)$$

Again,

$$\begin{aligned}
\tilde{D}((e_{z_f})z,(e_{w_f})w) &\leq \tilde{D}((e_{z_f})z,(e_{x_f})x)+\tilde{D}((e_{x_f})x,(e_{w_f})w).\\
&\leq \tilde{D}((e_{x_f})x,(e_{Z_f})Z)+[\tilde{D}((e_{x_f})x,(e_{y_f})y)+\tilde{D}((e_{y_f})y,(e_{w_f})w)]\\
&= \tilde{D}((e_{z_f})z,(e_{w_f})w)-\tilde{D}((e_{x_f})x,(e_{y_f})y)\\
&\leq \tilde{D}((e_{x_f})x,(e_{Z_f})Z)+\tilde{D}((e_{y_f})y,(e_{w_f})w) \qquad (2)
\end{aligned}$$

From equations (1) and (2) it follow that

$$|\tilde{D}((e_{x_f})x,(e_{y_f})y)-\tilde{D}((e_{z_f})z,(e_{w_f})w)| \leq \tilde{D}((e_{x_f})x,(e_{z_f})z)+\tilde{D}((e_{y_f})y,(e_{w_f})w).$$

for all $(e_{x_f})x,(e_{z_f})z,(e_{y_f})y,(e_{w_f})w \in \tilde{d}$.

5. Conclusion

In this work, we introduced the concept of soft super-space of soft metric space discussed the motivation behind studying this concept with soft points and introduced the notions of soft super spaces and how they are defined for a given a method for finding soft super spaces from soft metric spaces is discussed and some examples of soft super spaces are also presented. We hope that the Content presented in the paper would be of immense help to the students who are Beginners in the subject.

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