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α-PRECOMPACT SPACE IN FUZZY TOPOLOGICAL SPACE

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Abstract: In this paper we introduce and study a new type of compactness, viz., fuzzy α-precompactness by using fuzzy α-preopen set [1] as a basic tool. Here we also characterize this space by fuzzy net and prefilterbase. We have shown that fuzzy α-precompactness implies fuzzy almost compactness [4] and the converse is true only on fuzzy α-preregular space [1].

Keywords and Phrases: Fuzzy α-preopen set, fuzzy α-preregular space, fuzzy regularly α-preclosed set, fuzzy α-precompact set (space), α-preadherent point of a prefilterbase, α-precluster point of a fuzzy net.

2020 Mathematics Subject Classification: 54A40, 03E72.

1. Introduction

In [4], fuzzy almost compactness is introduced. In this paper we introduce fuzzy α-precompactness which is weaker than fuzzy almost compactness. Here we use fuzzy net [8] and prefilterbase [6] to characterize fuzzy α-precompactness.

2. Preliminary

Throughout this paper, (X, τ) or simply by X we shall mean an fts. In 1965, L.A. Zadeh introduced fuzzy set [9] A which is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support [9] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [9] of a fuzzy set A in an fts X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [9] while AqB means A is quasi-coincident (q-coincident, for short) [8] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [3] and fuzzy interior [3] of A respectively. A fuzzy set A in X is called a fuzzy neighbourhood (fuzzy nbd, for short) [8] of a fuzzy point x_t if there exists a fuzzy open set G in X such that $x_t \in G \leq A$. If, in addition, A is fuzzy open, then A is called fuzzy open nbd of x_t. A fuzzy set A is said to be a fuzzy q-nbd of a fuzzy point x_t in an fts X if there is a fuzzy open set U in X such that $x_t q U \leq A$. If, in addition, A is fuzzy open, then A is called a fuzzy open q-nbd [8] of x_t.

A fuzzy set A in an fts (X, τ) is called fuzzy α-open [2] if $A \leq int(cl(intA))$. The complement of a fuzzy α-open set is called fuzzy α-closed [2]. The union (intersection) of all fuzzy α-open (resp., fuzzy α-closed) sets contained in (resp., containing) a fuzzy set A is called fuzzy α-interior [2] (resp., fuzzy α-closure [2]) of A, denoted by $\alpha intA$ (resp., αclA).

Let $(D, \geq)$ be a directed set and X be an ordinary set. Let J denote the collection of all fuzzy points in X. A function $S : D \to J$ is called a fuzzy net in X [8]. It is denoted by $\{S_n : n \in (D, \geq)\}$. A non empty family $\mathcal{F}$ of fuzzy sets in X is called a prefilterbase on X if (i) $0_X \notin \mathcal{F}$ and (ii) for any $U, V \in \mathcal{F}$, there exists $W \in \mathcal{F}$ such that $W \leq U \bigcap V$ [6].

3. Fuzzy α-Preopen Sets : Some Prerequisites

In this section we recall some definitions and results from [1, 3, 4, 5, 7] for ready references. Also some new results are established here.

Definition 3.1. [1] *A fuzzy set A in an fts (X, τ) is called fuzzy α-preopen if $A \leq \alpha int(clA)$. The complement of this set is called fuzzy α-preclosed set.*

The union (resp., intersection) of all fuzzy α-preopen (resp., fuzzy α-preclosed) sets contained in (containing) a fuzzy set A is called fuzzy α-preinterior (resp., fuzzy α-preclosure) of A, denoted by $\alpha pintA$ (resp., $\alpha pclA$).

Definition 3.2. [1] *A fuzzy set A in an fts (X, τ) is called fuzzy α-prenbd of a fuzzy point x_t in X if there exists a fuzzy α-preopen set U in X such that $x_t \in U \leq A$. If, in addition, A is fuzzy α-preopen, then A is called fuzzy α-preopen nbd of x_t.*

Definition 3.3. [1] *A fuzzy set A in an fts (X, τ) is called fuzzy α-pre q-nbd of a fuzzy point x_t in X if there exists a fuzzy α-preopen set U in X such that $x_t q U \leq A$.*

If, in addition, A is fuzzy α-preopen, then A is called fuzzy α-preopen q-nbd of x_t.

Result 3.4. [1] *Union (resp., intersection) of any two fuzzy α-preopen (resp., fuzzy α-preclosed) sets is also so.*

Result 3.5. [1] *$x_t \in \alpha pclA$ if and only if every fuzzy α-preopen q-nbd U of x_t, UqA.*

Result 3.6. [1] *$\alpha pcl(\alpha pclA) = \alpha pclA$ for any fuzzy set A in an fts (X, τ).*

Result 3.7. *$\alpha pcl(A \bigvee B) = \alpha pclA \bigvee \alpha pclB$, for any two fuzzy sets A, B in X.*
Proof. It is clear that

$$\alpha pclA \bigvee \alpha pclB \subseteq \alpha pcl(A \bigvee B) \tag{1}$$

Conversely, let $x_t \in \alpha pcl(A \bigvee B)$. Then for any fuzzy α-preopen q-nbd U of x_t, $Uq(A \bigvee B) \Rightarrow$ there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1 \Rightarrow$ either $U(y) + A(y) > 1 \Rightarrow UqA$ or $U(y) + B(y) > 1 \Rightarrow UqB \Rightarrow$ either $x_t \in \alpha pclA$ or $x_t \in \alpha pclB \Rightarrow x_t \in \alpha pclA \bigvee \alpha pclB$.

Result 3.8. *For any fuzzy set A in an fts (X, τ),*
(i) $\alpha pcl(1_X \setminus A) = 1_X \setminus \alpha pintA$,
(ii) $\alpha pint(1_X \setminus A) = 1_X \setminus \alpha pclA$.
Proof. (i). Let $x_t \in \alpha pcl(1_X \setminus A)$ for any $A \in I^X$. If possible, let $x_t \notin 1_X \setminus \alpha pintA$. Then $x_t q(\alpha pintA)$. Then there exists a fuzzy α-preopen set B in X with $B \leq A$ such that $x_t qB$. Then B is a fuzzy α-preopen q-nbd of x_t. By assumption, $Bq(1_X \setminus A) \Rightarrow Aq(1_X \setminus A)$, which is absurd.

Conversely, let $x_t \in 1_X \setminus \alpha pintA$ for any $A \in I^X$. Then $x_t \not q(\alpha pintA)$ and so $x_t \not qU$ for any fuzzy α-preopen set U in X with $U \leq A \Rightarrow x_t \in 1_X \setminus U$ which is fuzzy α-preclosed set in X with $1_X \setminus A \leq 1_X \setminus U$. So $x_t \in \alpha pcl(1_X \setminus A)$.
(ii) Writing $1_X \setminus A$ for A in (i), we get the result.

Definition 3.9. *Let A be a fuzzy set in an fts (X, τ). A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $sup\{U(x) : U \in \mathcal{U}\} = 1$, for each $x \in suppA$* [5]. *If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy α-preopen), we call $\mathcal{U}$ is fuzzy open* [5] *(resp., fuzzy α-preopen) cover of A. In particular, if $A = 1_X$, we get the definition of fuzzy cover of X* [3].

Definition 3.10. *A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts (X, τ) is said to have a finite (resp., finite proximate) subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigvee \mathcal{U}_0 \geq A$* [5] *(resp., $\bigvee\{clU : U \in \mathcal{U}_0\} \geq A$* [7]*). In particular, if $A = 1_X$, we get $\bigvee \mathcal{U}_0 = 1_X$ (resp., $\bigvee\{clU : U \in \mathcal{U}_0\} = 1_X$* [4]*).*

Definition 3.11. [4] *An fts (X, τ) is called fuzzy almost compact space if every* *fuzzy open cover has a finite proximate subcover.*

4. Fuzzy α-Precompact Space

In this section we first introduce fuzzy α-precompactness and then characterize this space via fuzzy net and prefilterbase.

Definition 4.1. *A fuzzy set A in an fts (X, τ) is said to be a fuzzy α-precompact set if every fuzzy α-preopen cover $\mathcal{U}$ of A has a finite αp-proximate subcover, i.e., there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{\alpha pclU : U \in \mathcal{U}_0\} \geq A$. If, in addition, $A = 1_X$, we say that the fts X is fuzzy α-precompact space.*

Definition 4.2. *Let x_t be a fuzzy point in an fts (X, τ). A prefilterbase $\mathcal{F}$ on X is called*
(a) αp-adhere at x_t, written as $x_t \in \alpha p\text{-}ad\mathcal{F}$, if for each fuzzy α-preopen q-nbd U of x_t and each $F \in \mathcal{F}$, $Fq(\alpha pclU)$, i.e., $x_t \in \alpha pclF$, for each $F \in \mathcal{F}$;
(b) αp-converge to x_t, written as $\mathcal{F}\overrightarrow{\alpha p}x_t$, if to each fuzzy α-preopen q-nbd U of x_t, there corresponds some $F \in \mathcal{F}$ such that $F \leq \alpha pclU$.

Definition 4.3. *Let x_t be a fuzzy point in an fts (X, τ). A fuzzy net $\{S_n : n \in (D, \geq)\}$ is said to*
(a) αp-adhere at x_t, denoted by $x_t \in \alpha p\text{-}ad(S_n)$, if for each fuzzy α-preopen q-nbd U of x_t and each $n \in D$, there exists $m \in D$ with $m \geq n$ such that $S_m q(\alpha pclU)$;
(b) αp-converge to x_t, denoted by $S_n\overrightarrow{\alpha p}x_t$, if for each fuzzy α-preopen q-nbd U of x_t, there exists $m \in D$ such that $S_n q(\alpha pclU)$, for all $n \geq m (n \in D)$.

Theorem 4.4. *For a fuzzy set A in an fts X, the following statements are equivalent:*
(a) A is a fuzzy α-precompact set,
(b) for every prefilterbase $\mathcal{B}$ in X, $[\bigwedge\{\alpha pclB : B \in \mathcal{B}\}] \bigwedge A = 0_X \Rightarrow$ there exists a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge\{\alpha pintB : B \in \mathcal{B}_0\} \not{q} A$,
(c) for any family $\mathcal{F}$ of fuzzy α-preclosed sets in X with $\bigwedge\{F : F \in \mathcal{F}\} \bigwedge A = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge\{\alpha pintF : F \in \mathcal{F}_0\} \not{q} A$,
(d) every prefilterbase on X, each member of which is q-coincident with A, αp-adheres at some fuzzy point in A.

Proof. (a) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{\alpha pclB : B \in \mathcal{B}\}] \bigwedge A = 0_X$. Then for any $x \in suppA$, $[\bigwedge\{\alpha pclB : B \in \mathcal{B}\}](x) = 0 \Rightarrow 1 - [\bigwedge\{\alpha pclB(x) : B \in \mathcal{B}\}] = 1 \Rightarrow \bigvee[(1_X \setminus \alpha pclB)(x) : B \in \mathcal{B}] = 1 \Rightarrow sup\{\alpha pint(1_X \setminus B)(x) : B \in \mathcal{B}\} = 1 \Rightarrow \{\alpha pint(1_X \setminus B) : B \in \mathcal{B}\}$ is a fuzzy α-preopen cover of A. By (a), there exists a finite αp-proximate subcover $\{\alpha pint(1_X \setminus B_1), \alpha pint(1_X \setminus B_2), ..., \alpha pint(1_X \setminus B_n)\}$ (say) of it for A. Thus $A \leq \bigvee_{i=1}^{n} \alpha pcl(\alpha pint(1_X \setminus B_i))$

$= \bigvee_{i=1}^{n}[1_X \setminus \alpha pint(\alpha pclB_i)] = 1_X \setminus \bigwedge_{i=1}^{n} \alpha pint(\alpha pclB_i) \Rightarrow \bigwedge_{i=1}^{n} \alpha pint(\alpha pclB_i) \leq 1_X \setminus A \Rightarrow$ $A \not q \bigwedge_{i=1}^{n} \alpha pint(\alpha pclB_i) \Rightarrow A \not q \bigwedge_{i=1}^{n} \alpha pintB_i$.

(b) $\Rightarrow$ (a). Let the condition (b) hold, and suppose that there exists a fuzzy α-preopen cover $\mathcal{U}$ of A having no finite αp-proximate subcover for A. Then for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in suppA$ such that $sup\{\alpha pclU(x) : U \in \mathcal{U}_0\} < A(x)$, i.e., $1 - sup\{(\alpha pclU)(x) : U \in \mathcal{U}_0\} > 1 - A(x) \geq 0 \Rightarrow inf\{(1_X \setminus \alpha pclU)(x) : U \in \mathcal{U}_0\} > 0$. Thus $\{\bigwedge_{U \in \mathcal{U}_0} (1_X \setminus \alpha pclU) : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ (=$\mathcal{B}$, say) is a prefilterbase in X. If there exists a finite subcollection $\{U_1, U_2, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigwedge_{i=1}^{n} \alpha pint(1_X \setminus \alpha pclU_i) \not q A$, then $A \leq 1_X \setminus \bigwedge_{i=1}^{n} \alpha pint(1_X \setminus \alpha pclU_i) = \bigvee_{i=1}^{n}[1_X \setminus \alpha pint(1_X \setminus \alpha pclU_i)] = \bigvee_{i=1}^{n} \alpha pcl(\alpha pclU_i) = \bigvee_{i=1}^{n} \alpha pclU_i$ (by Result 3.6). Thus $\mathcal{U}$ has a finite αp-proximate subcover for A, contradicts our hypothesis. Hence for every finite subcollection $\{\bigwedge_{U \in \mathcal{U}_1} (1_X \setminus \alpha pclU), ..., \bigwedge_{U \in \mathcal{U}_k} (1_X \setminus \alpha pclU)\}$ of $\mathcal{B}$, where $\mathcal{U}_1, ..., \mathcal{U}_k$ are finite subset of $\mathcal{U}$, we have $[\bigwedge_{U \in \mathcal{U}_1 \bigvee ... \bigvee \mathcal{U}_k} \alpha pint(1_X \setminus \alpha pclU)] q A$. By (b), $[\bigwedge_{U \in \mathcal{U}} \alpha pcl(1_X \setminus \alpha pclU)] \bigwedge A \neq 0_X$. Then there exists $x \in suppA$, such that $\inf_{U \in \mathcal{U}}[\alpha pcl(1_X \setminus \alpha pclU)](x) > 0 \Rightarrow 1 - \inf_{U \in \mathcal{U}}[\alpha pcl(1_X \setminus \alpha pclU)](x) < 1 \Rightarrow \sup_{U \in \mathcal{U}}[1_X \setminus \alpha pcl(1_X \setminus \alpha pclU)](x) < 1 \Rightarrow \sup_{U \in \mathcal{U}} U(x) \leq \sup_{U \in \mathcal{U}} \alpha pint(\alpha pclU)(x) < 1$ which contradicts that $\mathcal{U}$ is a fuzzy α-preopen cover of A.

(a) $\Rightarrow$ (c). Let $\mathcal{F}$ be a family of fuzzy α-preclosed sets in X such that $\bigwedge\{F : F \in \mathcal{F}\} \bigwedge A = 0_X$. Then for each $x \in suppA$ and for each positive integer n, there exists some $F_n \in \mathcal{F}$ such that $F_n(x) < 1/n \Rightarrow 1 - F_n(x) > 1 - 1/n \Rightarrow \sup_{F \in \mathcal{F}}[(1_X \setminus F)(x)] = 1$ and so $\{1_X \setminus F : F \in \mathcal{F}\}$ is a fuzzy α-preopen cover of A. By (a), there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $A \leq \bigvee_{F \in \mathcal{F}_0} \alpha pcl(1_X \setminus F) \Rightarrow$ $1_X \setminus A \geq 1_X \setminus \bigvee_{F \in \mathcal{F}_0} \alpha pcl(1_X \setminus F) = \bigwedge_{F \in \mathcal{F}_0} (1_X \setminus \alpha pcl(1_X \setminus F)) = \bigwedge_{F \in \mathcal{F}_0} \alpha pintF$. Hence $A \not q (\bigwedge_{F \in \mathcal{F}_0} \alpha pintF)$, where $\mathcal{F}_0$ is a finite subcollection of $\mathcal{F}$.

(c) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{\alpha pclB : B \in \mathcal{B}\}] \bigwedge A = 0_X$. Then the family $\mathcal{F} = \{\alpha pclB : B \in \mathcal{B}\}$ is a family of fuzzy α-preclosed sets in X with $(\bigwedge F) \bigwedge A = 0_X$. By (c), there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $[\bigwedge\{\alpha pint(\alpha pclB) : B \in \mathcal{B}_0\}] \not{q} A \Rightarrow (\bigwedge_{B\in\mathcal{B}_0} \alpha pintB) \not{q} A$.

(a) $\Rightarrow$ (d). Let $\mathcal{F}$ be a prefilterbase in X, each member of which is q-coincident with A. If possible, let $\mathcal{F}$ do not αp-adhere at any fuzzy point in A. Then for each $x \in suppA$, there exists $n_x \in \mathcal{N}$ (the set of all natural numbers) such that $x_{1/n_x} \in A$. Then there are a fuzzy α-preopen set $U^x_{n_x}$ and a member $F^x_{n_x}$ of $\mathcal{F}$ such that $x_{1/n_x} q U^x_{n_x}$ and $\alpha pclU^x_{n_x} \not{q} F^x_{n_x}$. Thus $U^x_{n_x}(x) > 1 - 1/n_x$ so that $sup\{U^x_n(x) : n \in \mathcal{N}, n \geq n_x\} = 1$. Thus $\{U^x_n : n \in \mathcal{N}, n \geq n_x, x \in suppA\}$ forms a fuzzy α-preopen cover of A. By (a), there exist finitely many points $x_1, x_2, ..., x_k \in suppA$ and $n_1, n_2, ..., n_k \in \mathcal{N}$ such that $A \leq \bigvee_{i=1}^{k} \alpha pclU^{x_i}_{n_{x_i}}$. Choose $F \in \mathcal{F}$ such that $F \leq \bigwedge_{i=1}^{k} F^{x_i}_{n_i}$. Then $F \not{q} [\bigvee_{i=1}^{k} \alpha pclU^{x_i}_{n_{x_i}}]$, i.e., $F \not{q} A$, a contradiction.

(d) $\Rightarrow$ (a). If possible, let there exist a fuzzy α-preopen cover $\mathcal{U}$ of A such that for every finite subset $\mathcal{U}_0$ of $\mathcal{U}$, $\bigvee\{\alpha pclU : U \in \mathcal{U}_0\} \not\geq A$. Then $\mathcal{F} = \{1_X \setminus \bigvee_{U\in\mathcal{U}_0} \alpha pclU : \mathcal{U}_0 \text{ is a finite subset of } \mathcal{U}\}$ is a prefilterbase on X such that FqA, for each $F \in \mathcal{F}$. By (d), $\mathcal{F}$ αp-adheres at some fuzzy point $x_t \in A$. As $\mathcal{U}$ is a fuzzy cover of A, $\sup_{U\in\mathcal{U}} U(x) = 1 \Rightarrow$ there exists $U_0 \in \mathcal{U}$ such that $U_0(x) > 1 - t \Rightarrow x_t q U_0$. As $x_t \in \alpha p\text{-ad}\mathcal{F}$ and $1_X \setminus \alpha pclU_0 \in \mathcal{F}$, we have $\alpha pclU_0 q(1_X \setminus \alpha pclU_0)$, a contradiction.

Theorem 4.5. *For a fuzzy set A in an fts X, the following implications hold :*
(a) every fuzzy net in A αp-adheres at some fuzzy point in A,
$\Leftrightarrow$(b) every fuzzy net in A has a αp-convergent fuzzy subnet,
$\Leftrightarrow$ (c) every prefilterbase in A αp-adheres at some fuzzy point in A,
$\Rightarrow$ (d) for every family $\{B_\gamma : \gamma \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\gamma\in\Lambda} \alpha pclB_\gamma] \bigwedge A = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\gamma\in\Lambda_0} B_\gamma) \bigwedge A = 0_X$,
$\Rightarrow$ (e) A is fuzzy α-precompact set.

Proof. (a) $\Rightarrow$ (b). Let a fuzzy net $\{S_n : n \in (D, \geq)\}$ in A where $(D, \geq)$ is a directed set, αp-adhere at a fuzzy point $x_t \in A$. Let Q_{x_t} denote the set of the fuzzy α-preclosures of all fuzzy α-preopen q-nbds of x_t. For any $B \in Q_{x_t}$, we can choose some $n \in D$ such that $S_n q B$. Let E denote the set of all ordered pairs (n, B) with the property that $n \in D$, $B \in Q_{x_t}$ and $S_n q B$. Then $(E, \gg)$ is a directed set where $(m, C) \gg (n, B)$ if and only if $m \geq n$ in D and $C \leq B$. Then $T : (E, \gg) \to (X, \tau)$ given by $T(n, B) = S_n$, is a fuzzy subnet of $\{S_n : n \in (D, \geq)\}$. Let V be any fuzzy α-preopen q-nbd of x_t. Then there is $n \in D$ such that that $(n, \alpha pclV) \in E$ and hence $S_n q(\alpha pclV)$. Now, for any $(m, U) \gg (n, \alpha pclV)$, $T(m, U) = S_m q U \leq \alpha pclV \Rightarrow T(m, U) q(\alpha pclV)$. Hence $T \overrightarrow{\alpha p} x_t$.

(b) $\Rightarrow$ (a). If a fuzzy net $\{S_n : n \in (D, \geq)\}$ does not αp-adhere at a fuzzy point x_t, then there is a fuzzy α-preopen q-nbd U of x_t and an $n \in D$ such that $S_m \not{q}(\alpha pclU)$, for all $m \geq n$. Then obviously no fuzzy subnet of the fuzzy net can αp-converge to x_t.

(a) $\Rightarrow$ (c). Let $\mathcal{F} = \{F_\gamma : \gamma \in \Lambda\}$ be a prefilterbase in A. For each $\gamma \in \Lambda$, choose a fuzzy point $x_{F_\gamma} \in F_\gamma$ and construct the fuzzy net $S = \{x_{F_\gamma} : F_\gamma \in \mathcal{F}\}$ in A with $(\mathcal{F}, \gg)$ as domain, where for two members $F_\gamma, F_\beta \in \mathcal{F}$, $F_\gamma \gg F_\beta$ if and only if $F_\gamma \leq F_\beta$. By (a), the fuzzy net S αp-adheres at some fuzzy point x_s $(0 < s \leq 1) \in A$. Then for any fuzzy α-preopen q-nbd U of x_s and any $F_\gamma \in \mathcal{F}$, there exists $F_\beta \in \mathcal{F}$ such that $F_\beta \gg F_\gamma$ and $x_{F_\beta} q(\alpha pclU)$. Then $F_\beta q(\alpha pclU)$ and hence $F_\gamma q(\alpha pclU)$. Thus $\mathcal{F}$ αp-adheres at x_s.

(c) $\Rightarrow$ (a). Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A. Consider the prefilterbase $\mathcal{F} = \{T_n : n \in D\}$ generated by the net, where $T_n = \{S_m : m \in D, m \geq n\}$. By (c), there exists a fuzzy point $a_\gamma \in A$ such that $\mathcal{F}$ αp-adheres at a_γ. Then for each fuzzy α-preopen q-nbd U of a_γ and each $F \in \mathcal{F}$, $Fq(\alpha pclU)$, i.e., $(\alpha pclU) q T_n$, for all $n \in D$. Hence the given fuzzy net αp-adheres at a_γ.

(c) $\Rightarrow$ (d). Let $\mathcal{B} = \{B_\gamma : \gamma \in \Lambda\}$ be a family of fuzzy sets in X such that for every finite subset Λ_0 of Λ, $(\bigwedge_{\gamma \in \Lambda_0} B_\gamma) \bigwedge A \neq 0_X$. Then $\mathcal{F} = \{(\bigwedge_{\gamma \in \Lambda_0} B_\gamma) \bigwedge A : \Lambda_0$ is a finite subset of $\Lambda\}$ is a prefilterbase in A. By (c), $\mathcal{F}$ αp-adheres at some fuzzy point $a_t \in A$ $(0 < t \leq 1)$. Then for each $\gamma \in \Lambda$ and each fuzzy α-preopen q-nbd U of a_t, $B_\gamma q(\alpha pclU)$, i.e., $a_t \in \alpha pclB_\gamma$, for each $\gamma \in \Lambda$. Consequently, $(\bigwedge_{\gamma \in \Lambda} \alpha pclB_\gamma) \bigwedge A \neq 0_X$.

(d) $\Rightarrow$ (e). Let $\mathcal{U} = \{U_\gamma : \gamma \in \Lambda\}$ be a fuzzy α-preopen cover of a fuzzy set A. Then by (d), $A \bigwedge [\bigwedge_{\gamma \in \Lambda} (1_X \setminus U_\gamma)] = A \bigwedge [1_X \setminus \bigvee_{\gamma \in \Lambda} U_\gamma] = 0_X$. If for some $\gamma \in \Lambda$, $1_X \setminus \alpha pclU_\gamma = 0_X$, then we are done. If $1_X \setminus \alpha pclU_\gamma$ $(=B_\gamma$, say$) \neq 0_X$, then for each $\gamma \in \Lambda$, $\mathcal{B} = \{B_\gamma : \gamma \in \Lambda\}$ is a family of non-null fuzzy sets. We show that

$\bigwedge_{\gamma\in\Lambda} \alpha pclB_\gamma \leq \bigwedge_{\gamma\in\Lambda}(1_X \setminus U_\gamma)$. In fact, let x_t $(0 < t \leq 1)$ be a fuzzy point such that $x_t \in \alpha pclB_\gamma = \alpha pcl(1_X \setminus \alpha pclU_\gamma)$. If $x_t q U_\gamma$, then $\alpha pclU_\gamma q(1_X \setminus \alpha pclU_\gamma)$, which is absurd. Hence $x_t \not{q} U_\gamma \Rightarrow x_t \in 1_X \setminus U_\gamma$. Then $[\bigwedge_{\gamma\in\Lambda} \alpha pclB_\gamma] \bigwedge A \leq A \bigwedge [\bigwedge_{\gamma\in\Lambda}(1_X \setminus U_\gamma)] = 0_X$. By (d), there exists a finite subset Λ_0 of Λ such that $[\bigwedge_{\gamma\in\Lambda_0} B_\gamma] \bigwedge A = 0_X$, i.e., $A \leq 1_X \setminus \bigwedge_{\gamma\in\Lambda_0} B_\gamma = \bigvee_{\gamma\in\Lambda_0}(1_X \setminus B_\gamma) = \bigvee_{\gamma\in\Lambda_0} \alpha pclU_\gamma$ and (e) follows.

Definition 4.6. *A fuzzy set A in an fts (X, τ) is said to be fuzzy regularly α-preopen if $A = \alpha pint(\alpha pclA)$. The complement of such a set is called fuzzy regularly α-preclosed.*

Definition 4.7. *A fuzzy point x_t in X is said to be a fuzzy αp-cluster point of a prefilterbase $\mathcal{B}$ if $x_t \in \alpha pclB$, for all $B \in \mathcal{B}$. If, in addition, $x_t \in A$, for a fuzzy set A, then $\mathcal{B}$ is said to have a fuzzy αp-cluster point in A.*

Theorem 4.8. *A fuzzy set A in an fts (X, τ) is fuzzy α-precompact if and only if for each prefilterbase $\mathcal{F}$ in X which is such that for each set of finitely many members $F_1, F_2, ..., F_n$ from $\mathcal{F}$ and for any fuzzy regularly α-preclosed set C containing A, one has $(F_1 \bigwedge ... \bigwedge F_n)qC$, $\mathcal{F}$ has a fuzzy αp-cluster point in A.*

Proof. Let A be fuzzy α-precompact set and suppose $\mathcal{F}$ be a prefilterbase in X such that $[\bigwedge\{\alpha pclF : F \in \mathcal{F}\}] \bigwedge A = 0_X ...(1)$. Let $x \in suppA$. Consider any $n \in \mathcal{N}$ (the set of all natural numbers) such that $1/n < A(x)$, i.e., $x_{1/n} \in A$. By (1), $x_{1/n} \notin \alpha pclF_x^n$, for some $F_x^n \in \mathcal{F}$. Then there exists a fuzzy α-preopen q-nbd U_x^n of $x_{1/n}$ such that $\alpha pclU_x^n \not{q} F_x^n$. Now $U_x^n(x) > 1 - 1/n \Rightarrow sup\{U_x^n(x) : 1/n < A(x), n \in \mathcal{N}\} = 1 \Rightarrow \mathcal{U} = \{U_x^n : x \in suppA, n \in \mathcal{N}\}$ forms a fuzzy α-preopen cover of A such that for U_x^n, there exists $F_x^n \in \mathcal{F}$ with $U_x^n \not{q} F_x^n$. Since A is fuzzy α-precompact, there exist finitely many members $U_{x_1}^{n_1}, , , , , U_{x_k}^{n_k}$ of $\mathcal{U}$ such that $A \leq \bigvee_{i=1}^{k} \alpha pclU_{x_i}^{n_i} = \alpha pcl(\bigvee_{i=1}^{k} U_{x_i}^{n_i})$ (by Result 3.7) (=U, say). Now $F_{x_1}^{n_1}, ..., F_{x_k}^{n_k} \in \mathcal{F}$ such that $U_{x_i}^{n_i} \not{q} F_{x_i}^{n_i}$ for $i = 1, 2, ..., k$. Now U is a fuzzy regularly α-preclosed set containing A such that $\alpha pclU \not{q}(F_{x_1}^{n_1} \bigwedge .. \bigwedge F_{x_k}^{n_k}) \Rightarrow U \not{q}(F_{x_1}^{n_1} \bigwedge .. \bigwedge F_{x_k}^{n_k})$.
Conversely, let $\mathcal{B}$ be a prefilterbase in X having no fuzzy αp-cluster point in A. Then by hypothesis, there is a fuzzy regularly α-preclosed set C containing A such that for some finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$, $(\bigwedge \mathcal{B}_0) \not{q}C$. Then $(\bigwedge \mathcal{B}_0) \not{q}A$. By Theorem 4.4 (b) $\Rightarrow$ (a), A is fuzzy α-precompact set.

From Theorem 4.4, Theorem 4.5 and Theorem 4.8, we have the characterizations of fuzzy α-precompact space as follows.

Theorem 4.9. *For an fts X, the following statements are equivalent :*
(a) X is fuzzy α-precompact,
(b) every fuzzy net in X αp-adheres at some fuzzy point in X,
(c) every fuzzy net in X has a αp-convergent fuzzy subnet,
(d) every prefilterbase in X αp-adheres at some fuzzy point in X,
(e) for every family $\{B_\gamma : \gamma \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\gamma\in\Lambda} \alpha pcl B_\gamma] = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\gamma\in\Lambda_0} B_\gamma) = 0_X$,
(f) for every prefilterbase $\mathcal{B}$ in X with $\bigwedge\{\alpha pcl B : B \in \mathcal{B}\} = 0_X$, there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge\{\alpha pint B : B \in \mathcal{B}_0\} = 0_X$,
(g) for any family $\mathcal{F}$ of fuzzy α-preclosed sets in X with $\bigwedge \mathcal{F} = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge\{\alpha pint F : F \in \mathcal{F}_0\} = 0_X$.

Theorem 4.10. *An fts X is fuzzy α-precompact if and only if for any collection $\{F_\gamma : \gamma \in \Lambda\}$ of fuzzy α-preopen sets in X having finite intersection property $\bigwedge\{\alpha pcl F_\gamma : \gamma \in \Lambda\} \neq 0_X$.*

Proof. Let X be fuzzy α-precompact space and $\mathcal{F} = \{F_\gamma : \gamma \in \Lambda\}$ be a collection of fuzzy α-preopen sets in X with finite intersection property. Suppose $\bigwedge\{\alpha pcl F_\gamma : \gamma \in \Lambda\} = 0_X$. Then $\{1_X \setminus \alpha pcl F_\gamma : \gamma \in \Lambda\}$ is a fuzzy α-preopen cover of X. By hypothesis, there exists a finite subset Λ_0 of Λ such that $1_X = \bigvee\{\alpha pcl(1_X \setminus \alpha pcl F_\gamma) : \gamma \in \Lambda_0\} = \bigvee\{1_X \setminus \alpha pint(\alpha pcl F_\gamma) : \gamma \in \Lambda_0\} \leq \bigvee\{1_X \setminus F_\gamma : \gamma \in \Lambda_0\} = 1_X \setminus \bigwedge_{\gamma\in\Lambda_0} F_\gamma \Rightarrow \bigwedge_{\gamma\in\Lambda_0} F_\gamma = 0_X$ which contradicts the fact that $\mathcal{F}$ has finite intersection property.
Conversely, suppose that X is not fuzzy α-precompact space. Then there is a fuzzy α-preopen cover $\mathcal{F} = \{F_\gamma : \gamma \in \Lambda\}$ of X such that for every finite subset Λ_0 of Λ, $\bigvee\{\alpha pcl F_\gamma : \gamma \in \Lambda_0\} \neq 1_X$. Then $1_X \setminus \bigvee\{\alpha pcl F_\gamma : \gamma \in \Lambda_0\} \neq 0_X \Rightarrow \bigwedge_{\gamma\in\Lambda_0} (1_X \setminus \alpha pcl F_\gamma) \neq 0_X$, for every finite subset Λ_0 of Λ. Thus $\{1_X \setminus \alpha pcl F_\gamma : \gamma \in \Lambda\}$ is a collection of fuzzy α-preopen sets with finite intersection property. By hypothesis, $\bigwedge_{\gamma\in\Lambda} \alpha pcl(1_X \setminus \alpha pcl F_\gamma) \neq 0_X$, i.e., $1_X \setminus \bigvee_{\gamma\in\Lambda} \alpha pint(\alpha pcl F_\gamma) \neq 0_X \Rightarrow \bigvee_{\gamma\in\Lambda} \alpha pint(\alpha pcl F_\gamma) \neq 1_X$. Hence $\bigvee_{\gamma\in\Lambda} F_\gamma \neq 1_X$, a contradiction as $\mathcal{F}$ is a fuzzy α-preopen cover of X.

Definition 4.11. *Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy α-preopen sets in X, i.e., for each member n of a directed set $(D, \geq)$, S_n is a fuzzy α-preopen set in*

X. A fuzzy point x_t in X is said to be a fuzzy αp-cluster point of the fuzzy net if for every $n \in D$ and every fuzzy α-preopen q-nbd V of x_t, there exists $m \in D$ with $m \geq n$ such that $S_m q V$.

Theorem 4.12. *An fts X is fuzzy α-precompact if and only if every fuzzy net of fuzzy α-preopen sets in X has a fuzzy αp-cluster point in X.*

Proof. Let $\mathcal{U} = \{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy α-preopen sets in a fuzzy α-precompact space X. For each $n \in D$, let us put $F_n = \alpha pcl[\bigvee\{S_m : m \in D$ and $m \geq n\}]$. Then $\mathcal{F} = \{F_n : n \in D\}$ is a family of fuzzy α-preclosed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{\alpha pintF : F \in \mathcal{F}_0\} \neq 0_X$. By Theorem 4.9 (a)$\Rightarrow$ (g), $\bigwedge\limits_{n\in D} F_n \neq 0_X$. Let $x_t \in \bigwedge\limits_{n\in D} F_n$. Then $x_t \in F_n$, for all $n \in D$. Thus for any fuzzy α-preopen q-nbd A of x_t and any $n \in D$, $Aq[\bigvee\{S_m : m \geq n\}]$ and so there exists some $m \in D$ with $m \geq n$ and $AqS_m \Rightarrow x_t$ is a fuzzy αp-cluster point of $\mathcal{U}$.

Conversely, let $\mathcal{F}$ be a collection of fuzzy α-preclosed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{\alpha pintF : F \in \mathcal{F}_0\} \neq 0_X$. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of $\mathcal{F}$ directed by the relation '$\gg$' such that for $F_1, F_2 \in \mathcal{F}^*$, $F_1 \gg F_2$ if and only if $F_1 \leq F_2$. Let $F^* = \alpha pintF$, for each $F \in \mathcal{F}^*$. Then $F^* \neq 0_X$. Consider the fuzzy net $\mathcal{U} = \{F^* : F \in (\mathcal{F}^*, \gg)\}$ of non-null fuzzy α-preopen sets of X. By hypothesis, $\mathcal{U}$ has a fuzzy αp-cluster point, say x_t. We claim that $x_t \in \bigwedge \mathcal{F}$. In fact, let $F \in \mathcal{F}$ be arbitrary and A be any fuzzy α-preopen q-nbd of x_t. Since $F \in \mathcal{F}^*$ and x_t is a fuzzy αp-cluster point of $\mathcal{U}$, there exists $G \in \mathcal{F}^*$ such that $G \gg F$ (i.e., $G \leq F$) and $G^*qA \Rightarrow GqA \Rightarrow FqA \Rightarrow x_t \in \alpha pclF = F$, for each $F \in \mathcal{F} \Rightarrow x_t \in \bigwedge \mathcal{F} \Rightarrow \bigwedge \mathcal{F} \neq 0_X$. By Theorem 4.9 (g) $\Rightarrow$ (a), X is fuzzy α-precompact space.

Definition 4.13. *A fuzzy cover $\mathcal{U}$ by fuzzy α-preclosed sets of an fts (X, τ) will be called a fuzzy αp-cover of X if for each fuzzy point x_t $(0 < t < 1)$ in X, there exists $U \in \mathcal{U}$ such that U is a fuzzy α-preopen nbd of x_t.*

Theorem 4.14. *An fts (X, τ) is fuzzy α-precompact if and only if every fuzzy αp-cover of X has a finite subcover.*

Proof. Let X be fuzzy α-precompact space and $\mathcal{U}$ be any fuzzy αp-cover of X. Then for each $n \in \mathcal{N}$ (the set of all natural numbers) with $n > 1$, there exist $U_x^n \in \mathcal{U}$ and a fuzzy α-preopen set V_x^n in X such that $x_{1-1/n} \leq V_x^n \leq U_x^n$. Then $V_x^n(x) \geq 1 - 1/n \Rightarrow sup\{V_x^n(x) : n \in \mathcal{N}\} = 1 \Rightarrow \mathcal{V} = \{V_x^n : x \in X, n \in \mathcal{N}, n > 1\}$ is a fuzzy α-preopen cover of X. As X is fuzzy α-precompact, there exist finitely many points $x_1, x_2, ..., x_m \in X$ and $n_1, n_2, ..., n_m \in N \setminus \{1\}$ such that

$$1_X = \bigvee_{k=1}^{m} \alpha pclV_{x_k}^{n_k} \leq \bigvee_{k=1}^{m} \alpha pclU_{x_k}^{n_k} = \bigvee_{k=1}^{m} U_{x_k}^{n_k}.$$

Conversely, let $\mathcal{U}$ be fuzzy α-preopen cover of X. For any fuzzy point x_γ $(0 < \gamma < 1)$ in X, as $\sup_{U \in \mathcal{U}} U(x) = 1$, there exists $U_{x_\gamma} \in \mathcal{U}$ such that $U_{x_\gamma}(x) \geq \gamma$ $(0 < \gamma < 1)$. Then $\mathcal{V} = \{\alpha pclU : U \in \mathcal{U}\}$ is a fuzzy αp-cover of X and the rest is clear.

The following theorem gives a necessary condition for an fts to be fuzzy α-precompact.

Theorem 4.15. *If an fts X is fuzzy α-precompact, then every prefilterbase on X with at most one αp-adherent point is αp-convergent.*

Proof. Let $\mathcal{F}$ be a prefilterbase with at most one αp-adherent point in a fuzzy α-precompact fts X. Then by Theorem 4.9, $\mathcal{F}$ has at least one αp-adherent point in X. Let x_t be the unique αp-adherent point of $\mathcal{F}$ and if possible, let $\mathcal{F}$ do not αp-converge to x_t. Then for some fuzzy α-preopen q-nbd U of x_t and for each $F \in \mathcal{F}$, $F \not\leq \alpha pclU$, so that $F \bigwedge \{1_X \setminus \alpha pclU\} \neq 0_X$. Then $\mathcal{G} = \{F \bigwedge (1_X \setminus \alpha pclU) : F \in \mathcal{F}\}$ is a prefilterbase in X and hence has a αp-adherent point y_s (say) in X. Now $\alpha pclU \not{q} G$, for all $G \in \mathcal{G}$ so that $x_t \neq y_s$. Again, for each fuzzy α-preopen q-nbd V of y_s and each $F \in \mathcal{F}$, $\alpha pclVq(F \bigwedge (1_X \setminus \alpha pclU)) \Rightarrow \alpha pclVqF \Rightarrow y_s$ is a fuzzy αp-adherent point of $\mathcal{F}$, where $x_t \neq y_s$. This contradicts the fact that x_t is the only fuzzy αp-adherent point of $\mathcal{F}$.

Some results on fuzzy α-precompactness of an fts are given by the following theorem.

Theorem 4.16. *Let (X, τ) be an fts and $A \in I^X$. Then the following statements are true :*
(a) If A is fuzzy α-precompact, then so is $\alpha pclA$,
(b) Union of two fuzzy α-precompact sets is also so,
(c) If X is fuzzy α-precompact, then every fuzzy regularly α-preclosed set A in X is fuzzy α-precompact.

Proof. (a). Let $\mathcal{U}$ be a fuzzy α-preopen cover of $\alpha pclA$. Then $\mathcal{U}$ is also a fuzzy α-preopen cover of A. As A is fuzzy α-precompact, there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $A \leq \bigvee\{\alpha pclU : U \in \mathcal{U}_0\} = \alpha pcl\{\bigvee U : U \in \mathcal{U}_0\} \Rightarrow \alpha pclA \leq \alpha pcl\{\alpha pcl[\bigvee\{U : U \in \mathcal{U}_0\}]\} = \alpha pcl\{\bigvee U : U \in \mathcal{U}_0\} = \bigvee\{\alpha pclU : U \in \mathcal{U}_0\}$. Hence the proof.

(b). Obvious.

(c). Let $\mathcal{U} = \{U_\gamma : \gamma \in \Lambda\}$ be a fuzzy α-preopen cover of a fuzzy regularly α-preclosed set A in X. Then for each $x \notin suppA$, $A(x) = 0 \Rightarrow (1_X \setminus A)(x) = 1 \Rightarrow \mathcal{U} \bigvee \{(1_X \setminus A)\}$ is a fuzzy α-preopen cover of X. Since X is fuzzy α-

precompact, there are finitely many members $U_1, U_2, ..., U_n$ in $\mathcal{U}$ such that $1_X = (\alpha pclU_1 \bigvee ... \bigvee \alpha pclU_n) \bigvee \alpha pcl(1_X \setminus A)$. We claim that $\alpha pintA \leq \alpha pclU_1 \bigvee ... \bigvee \alpha pclU_n$. If not, there exists a fuzzy point $x_t \in \alpha pintA$, but $x_t \notin (\alpha pclU_1 \bigvee ... \bigvee \alpha pclU_n)$, i.e., $t > max\{(\alpha pclU_1)(x), ..., (\alpha pclU_n)(x)\}$. As $1_X = (\alpha pclU_1 \bigvee ... \bigvee \alpha pclU_n) \bigvee \alpha pcl(1_X \setminus A)$, $[\alpha pcl(1_X \setminus A)](x) = 1 \Rightarrow 1 - \alpha pintA(x) = 1 \Rightarrow \alpha pintA(x) = 0 \Rightarrow x_t \notin \alpha pintA$, a contradiction. Hence $A = \alpha pcl(\alpha pintA) \leq \alpha pcl(\alpha pclU_1 \bigvee .... \bigvee \alpha pclU_n) = \alpha pclU_1 \bigvee ... \bigvee \alpha pclU_n$ (by Result 3.6 and Result 3.7) $\Rightarrow A$ is fuzzy α-precompact set.

5. Applications

In this section we establish the mutual relationship between fuzzy almost compactness [4] and fuzzy α-precompactness. Here it is shown that fuzzy α- precompactness implies fuzzy almost compactness, but converse is true in fuzzy α-preregular space [1]. Again it is established that fuzzy α- precompactness remains invariant under fuzzy α-preirresolute function [1].

Since for any fuzzy set A in an fts X, $\alpha pclA \leq clA$ (as every fuzzy closed set is fuzzy α-preclosed [1], we can state the following theorem easily.

Theorem 5.1. *Every fuzzy α-precompact space is fuzzy almost compact.*

To get the converse we have to recall the following definition and theorem for ready references.

Definition 5.2. [1] *An fts (X, τ) is said to be fuzzy α-preregular if for each fuzzy α-preclosed set F in X and each fuzzy point x_t in X with $x_t q(1_X \setminus F)$, there exists a fuzzy open set U in X and a fuzzy α-preopen set V in X such that $x_t qU$, $F \leq V$ and $U \not{q} V$.*

Theorem 5.3. [1] *An fts (X, τ) is fuzzy α-preregular iff every fuzzy α-preclosed set is fuzzy closed.*

Theorem 5.4. *A fuzzy α-preregular, fuzzy almost compact space X is fuzzy α-precompact.*

Proof. Let $\mathcal{U}$ be a fuzzy α-preopen cover of a fuzzy α-preregular, fuzzy almost compact space X. Then by Theorem 5.3, $\mathcal{U}$ is a fuzzy open cover of X. As X is fuzzy almost compact, there is a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{clU : U \in \mathcal{U}_0\} = \bigvee\{\alpha pclU : U \in \mathcal{U}_0\}$ (by Theorem 5.3) $= 1_X \Rightarrow X$ is fuzzy α-precompact.

Next we recall the following definition and theorem for ready references.

Definition 5.5. [1] *A function $f : X \to Y$ is said to be fuzzy α-preirresolute if the inverse image of every fuzzy α-preopen set in Y is fuzzy α-preopen in X.*

Theorem 5.6. [1] *For a function $f : X \to Y$, the following statements are equivalent :*
(i) f is fuzzy α-preirresolute,
(ii) $f(\alpha pclA) \leq \alpha pcl(f(A))$, for all $A \in I^X$,
(iii) for each fuzzy point x_t in X and each fuzzy α-preopen q-nbd V of $f(x_t)$ in Y, there exists a fuzzy α-preopen q-nbd U of x_t in X such that $f(U) \leq V$.

Theorem 5.7. *Fuzzy α-preirresolute image of a fuzzy α-precompact space is fuzzy α-precompact.*

Proof. Let $f : X \to Y$ be fuzzy α-preirresolute surjection from a fuzzy α-precompact space X to an fts Y, and let $\mathcal{V}$ be a fuzzy α-preopen cover of Y. Let $x \in X$ and $f(x) = y$. Since $sup\{V(y) : V \in \mathcal{V}\} = 1$, for each $n \in \mathcal{N}$ (the set of all natural numbers), there exists some $V_x^n \in \mathcal{V}$ with $V_x^n(y) > 1 - 1/n$ and so $y_{1/n} q V_x^n$. By fuzzy α-preirresoluteness of f, by Theorem 5.6 (i)$\Rightarrow$ (iii), $f(U_x^n) \leq V_x^n$, for some fuzzy α-preopen set U_x^n in X q-coincident with $x_{1/n}$. Since $U_x^n(x) > 1 - 1/n$, $sup\{U_x^n(x) : n \in \mathcal{N}\} = 1$. Then $\mathcal{U} = \{U_x^n : n \in \mathcal{N}, x \in X\}$ is a fuzzy α-preopen cover of X. By fuzzy α-precompactness of X, $\bigvee_{i=1}^{k} \alpha pclU_{x_i}^{n_i} = 1_X$, for some finite subcollection $\{U_{x_1}^{n_1}, ..., U_{x_k}^{n_k}\}$ of $\mathcal{U}$. Then $1_Y = f(\bigvee_{i=1}^{k} \alpha pclU_{x_i}^{n_i}) = \bigvee_{i=1}^{k} f(\alpha pclU_{x_i}^{n_i}) \leq \bigvee_{i=1}^{k} \alpha pcl(f(U_{x_i}^{n_i}))$ (by Theorem 5.6 (i) $\Rightarrow$ (ii)) $\leq \bigvee_{i=1}^{k} \alpha pclV_{x_i}^{n_i} \Rightarrow Y$ is fuzzy α-precompact space.

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