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MATHEMATICAL ANALYSIS OF COSMOLOGICAL MODELS WITH LINEARLY VARYING DECELERATION PARAMETER

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Abstract: Bianchi type-V is investigated using a decaying cosmological constant with perfect fluid. The study solves Einstein field equations by assuming that the deceleration parameter q is functionally connected to the Hubble parameter H, which yields the scale factor a. As cosmic time t increases, the cosmological model experiences exponential inflation. The physical characteristics and behavior of the cosmological model are also covered.

Keywords and Phrases: Deceleration parameter, cosmological term, perfect fluid, Bianchi type-V.

2020 Mathematics Subject Classification: 83C15.

1. Introduction

The theory of the deceleration parameter plays a vital role in the study of the universe's physical phenomenon, whether it is accelerating or decelerating cosmological model in Bianchi type-V. The frame of mind of the cosmologists is to believe that the universe is in the mode of accelerating expansion at present. Bianchi type-V cosmological models are engaging the authors to research because these models are homogeneous and anisotropic, giving a physically and geometrically well-shaped structure than the isotropic model of FRW (Friedmann Robert-Walker) models. This also draws the attention of researchers in the explanation of the early universe. Einstein's field equations explain the universe's evolution in the view of the equation of state for the matter content.

There is weighty ground for the belief that the expansion of the observable universe is meeting with acceleration at late time [16, 19-23]. The current results of supernovae and the distribution of the galaxies on a large scale have astonishing research in cosmology that the universe seems to be dominated by dark energy [1-7, 14, 17, 30] that resembles negative gravity and speeds up the expansion of the cosmos than matter as the universe expands and clusters more feebly than matter. The cosmological term is the simplest form of dark energy since it is constant throughout space and time. This leads to the Lambda Cold Dark Matter (ΛCDM) model, which is the currently accepted model of cosmology. It accurate to many cosmological observations [8, 15, 18].

The solutions of Einstein field equations are given in view of the law of variation for Hubble's parameter which was suggested by Berman [4] that gives a constant value for the deceleration parameter in the simplest case. It is important to note that the majority of well-known models of inflationary theories and theories by Einstein and Brans-Disk with curvature parameters of $k = 0$ have constant deceleration parameters. Several writers [5, 12, 13, 25] have studied cosmological models with a constant deceleration value in previous scenarios. The deceleration parameter q is regarded in this article as an appropriate linear function of Hubble's parameter H i.e. $q = -1 + \beta H$. It provides the scale factor $a = e^{\frac{1}{\beta}\sqrt{2\beta t+k}}$ (β and k are constants). Further, in order to solve cosmological problems, we have assumed $\Lambda = \frac{\alpha}{a^2}$. This paper is discussed in five sections. Sec. 2 describes metric and field equations, Sec. 3 is involved in the solution of field equations and the physical and geometrical aspects of the models are discussed in Sect. 4. Finally, Sec. 5 stands for results and concluding remarks for the model.

2. Metric and Einstein field equations

Bianchi type-V Universe is given by the metric

$$ds^2 = -dt^2 + A^2(t)dx_1^2 + e^{2mx}\{B^2(t)dx_2^2 + C^2(t)dx_3^2\}, \tag{2.1}$$

where, m is a constant.

The energy-momentum tensor in the existence of cosmic matter with bulk viscous fluid is

$$T_i^j = (\rho + \bar{p})u_i u^j + \bar{p}\delta_i^j, \tag{2.2}$$

$$\text{where} \qquad \bar{p} = p - \zeta u^i_{;i}. \tag{2.3}$$

The energy density, isotropic pressure, effective pressure, and bulk viscous coefficient are represented here by the variables ρ, p, $\bar{p}$, and ζ respectively. The fluid's four-velocity vector is denoted by u_i such that

$$u_i u^i = -1 \tag{2.4}$$

We confirm that the equation of state is satisfied by the non-vacuum component of matter as

$$p = \omega\rho \quad \text{where} \quad 0 \leq \omega \leq 1 \tag{2.5}$$

Under $8\pi G = 1$, the Einstein field equations are

$$R_i^j - \frac{1}{2}R\delta_i^j = -T_i^j + \Lambda(t)\delta_i^j, \tag{2.6}$$

where, G is the gravitational constant.
In the comoving coordinate system, for the line element (2.1) and matter distribution (2.2), the field equation (2.6) are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = -\bar{p} + \Lambda \tag{2.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -\bar{p} + \Lambda \tag{2.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -\bar{p} + \Lambda \tag{2.9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3m^2}{A^2} = \rho + \Lambda \tag{2.10}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{2A}{A} = 0 \tag{2.11}$$

The average scale factor a is given by

$$a = (ABC)^{1/3} \tag{2.12}$$

In the similarity with Friedmann Robertson Walker universe, a generalised deceleration parameter (q) and a generalised Hubble's parameter (H) are introduced as

$$q = \left(\frac{1}{H}\right) - 1 \tag{2.13}$$

$$H = \frac{\dot{a}}{a} \tag{2.14}$$

The anisotropy parameter A is formulated as

$$\bar{A} = \frac{2}{3}\left(\frac{\sigma}{H}\right)^2 \tag{2.15}$$

We define two scalar quantities: shear scalar σ and volume expansion θ as

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \tag{2.16}$$

$$\theta = 3H \tag{2.17}$$

For the Bianchi type-V, θ and σ_i^j can be re-written as

$$\theta = \frac{3\dot{a}}{a} = H \tag{2.18}$$

$$\sigma^2 = \frac{k_1^2}{a^6} \tag{2.19}$$

Eqns. (2.8) and (2.9) can be expressed using H, σ and q as

$$\bar{p} = \Lambda = \frac{m^2}{a^2} + H^2(2q-1) - \sigma^2 \tag{2.20}$$

$$\rho = \Lambda = \frac{-3m^2}{a^2} + 3H^2 - \sigma^2 \tag{2.21}$$

3. Solution of the field equations

Using equation (2.12) in eqns (2.7)-(2.11), we have

$$\frac{\dot{A}}{A} = \frac{\dot{a}}{a} \tag{3.1}$$

$$\frac{\dot{B}}{B} = \frac{\dot{a}}{a} - \frac{k_1}{a^3} \tag{3.2}$$

$$\frac{\dot{C}}{C} = \frac{\dot{a}}{a} + \frac{k_1}{a^3} \tag{3.3}$$

where k_1 is a constant.

Integrating (3.1), (3.2) and (3.3), we obtain

$$A = c_1 a, B = c_2 a \exp\left[-k_1 \int \frac{dt}{a^3}\right] \quad \text{and} \quad C = c_3 a \exp\left[k_1 \int \frac{dt}{a^3}\right] \tag{3.4}$$

where c_1, c_2, c_3 are constants, satisfying the relation $c_1c_2c_3 = 1$ for $c_1 = 1$, $c_3 = c_2 - 1$.

The eqn. (3.4) can be simplified to:

$$A = a, \quad B = c_2 a \exp\left[-k_1 \int \frac{dt}{a^3}\right], \quad C = c_2^1 a \exp\left[k_1 \int \frac{dt}{a^3}\right] \tag{3.5}$$

We take that deceleration parameter q is functionally related with Hubble parameter H. Recent observations [9, 10, 24] have indicated that q is in the range of 0 to -1 and the universe is now speeding.

$$q = -1 + \beta H \tag{3.6}$$

where β is adjustable constant.
Simplifying equation (3.5), we get

$$a = e^{\frac{1}{\beta}\sqrt{2\beta t+k}} \tag{3.7}$$

where k is integration constant.

a $\underrightarrow{yields}$ constant, at $t = -\frac{k_1}{2\beta}$,
Using eqn. (3.7) in eqn. (3.5), we have

$$A = e^{\frac{1}{\beta}\sqrt{2\beta t+k}} \tag{3.8}$$

$$B = e^{\frac{1}{\beta}\sqrt{2\beta t+k}} \exp\left\{k_1\left(\frac{\sqrt{2\beta t+k}}{3} + \frac{\beta}{9}\right) e^{-\frac{1}{\beta}\sqrt{2\beta t+k}}\right\} \tag{3.9}$$

$$C = e^{\frac{1}{\beta}\sqrt{2\beta t+k}} \exp\left\{-k_1\left(\frac{\sqrt{2\beta t+k}}{3} + \frac{\beta}{9}\right) e^{-\frac{1}{\beta}\sqrt{2\beta t+k}}\right\} \tag{3.10}$$

Hence the model (2.1) reduces to

$$ds^2 = -dt^2 + e^{\frac{2}{\beta}\sqrt{2\beta t+k}}\left[dx_1^2 + \left\{\begin{array}{l} \exp\left(2mx + 2k_1\left(\frac{\sqrt{2\beta t+k}}{3} + \frac{\beta}{3}\right) e^{\frac{-3\sqrt{2\beta t+k}}{\beta}}\right) dx_2^2 \\ + \exp\left(2mx - 2k_1\left(\frac{\sqrt{2\beta t+k}}{3} + \frac{\beta}{3}\right) e^{\frac{3\sqrt{2\beta t+k}}{\beta}}\right) dx_3^2 \end{array}\right\}\right] \tag{3.11}$$

4. Discussion

H, V, θ, σ, $\bar{A}$ and q are calculated as

$$H = \frac{1}{\sqrt{2\beta t+k}} \tag{4.1}$$

$$V = e^{\frac{3}{\beta}\sqrt{2\beta t+k}} \tag{4.2}$$

$$\theta = \frac{3}{\sqrt{2\beta t+k}} \tag{4.3}$$

$$\sigma^2 = \frac{k_1^2}{e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} \tag{4.4}$$

$$\bar{A} = \frac{2}{3}.\frac{(2\beta t+k)k_1^2}{e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} \tag{4.5}$$

$$q = \frac{1}{\sqrt{2\beta t+k}} - 1 \tag{4.6}$$

Let us consider

$$\Lambda = \alpha/a^2, \qquad \alpha \text{ is constant} \tag{4.7}$$

Using eqn (3.7) in eqn (4.7), we have

$$\Lambda = \frac{\alpha}{e^{\frac{2}{\beta}\sqrt{2\beta t+k}}} \tag{4.8}$$

Using eqn (4.8) in eqns (2.20)-(2.21), isotropic pressure $\bar{p}$ energy density ρ and effective pressure p are given by

$$\bar{p} = \frac{m^2+\alpha}{e^{\frac{2}{\beta}\sqrt{2\beta t+k}}} - \frac{k_1^2}{e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} - \frac{(2\beta - 3\sqrt{2\beta t+k})}{(2\beta t+k)^{3/2}} \tag{4.9}$$

$$\rho = \frac{-(\alpha+3m^2)}{e^{\frac{2}{\beta}\sqrt{2\beta t+k}}} + \frac{3}{2\beta t+k} - \frac{k_1^2}{e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} \tag{4.10}$$

$$p = \frac{-(\alpha+3m^2)\omega}{e^{\frac{2}{\beta}\sqrt{2\beta t+k}}} + \frac{3\omega}{2\beta t+k} - \frac{k_1^2\omega}{e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} \tag{4.11}$$

ζ is also obtained as

$$\zeta = -\frac{\alpha(1+\omega) - m^2(1+3\omega)}{e^{\frac{2}{\beta}\sqrt{2\beta t+k}}} - \frac{2\beta - 3(1-\omega)\sqrt{2\beta t+k}}{(2\beta t+k)^{3/2}} + \frac{k_1^2(1-\omega)}{e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} \tag{4.12}$$

We observe that at $t = t_1$, isotropic pressure $\bar{p}$, energy density ρ, effective pressure p and bulk viscous coefficient ζ are finite. As $t \to \infty$, isotropic pressure $\bar{p}$, energy density ρ, effective pressure p and bulk viscous coefficient ζ are all zero. The cosmological term $\Lambda \to 0$ as $t \to \infty$ and Λ is finite at $t = t_1$.

5. Conclusion

The Bianchi-V cosmological model with a variable cosmological term Λ has been examined in this article. In the outline of the conditions that the expansion scalar θ and the shear scalar σ are precisely proportional and a linear function of the Hubble parameter H is the deceleration parameter, q as $q = -1 + \beta H$ and provides $a = e^{\frac{1}{\beta}\sqrt{2\beta t + k}}$. At $t = t_1$, H, θ, and σ incline to diverge. As $t \to \infty$, H, θ, ρ, p, Λ, ζ and σ incline to zero. Additionally, we see that as $t \to \infty$, $\bar{A} \to 0$. This indicates that the model becomes more isotropic as the values of t become larger. The deceleration parameter q is positive for $t < \frac{\beta^2 - k_1}{2\beta}$, which reflects that universe is in decelerating phase, and q is negative for $t > \frac{\beta^2 - k_1}{2\beta}$, which indicates that universe is in accelerating phase. The solutions obtained are consistent with the recent observations and articles [26-29].

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