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INDUCED REGULAR PERFECT GRAPHS

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Abstract: A graph G is said to be $\mathcal{R}$-perfect if, for all induced subgraphs H of G, the induced regular independence number of each induced subgraph H is equal to its corresponding induced regular cover. Here, the induced regular independence number is the maximum number of vertices in H such that no two belong to the same induced regular subgraph in H, and the induced regular cover of H is the minimum number of induced regular subgraphs in H required to cover the vertex set of H. This article introduces the notion of induced regular perfect graphs or $\mathcal{R}$-perfect graphs through which we study the structural properties of $\mathcal{R}$-perfect graphs and identify a forbidden class of graphs for the same. This further leads to the characterization of $\mathcal{R}$-perfect biconnected graphs. With these results, we derive and prove a general characterization for $\mathcal{R}$-perfect graphs.

Keywords and Phrases: Perfect graphs, $\mathcal{F}$-perfect graphs, Regular graphs, $\mathcal{R}$-perfect graphs, Graph minors.

2020 Mathematics Subject Classification: 05C17, 05C10, 05C60, 05C83.

1. Introduction

The graphs considered in this paper are finite, simple and undirected unless stated otherwise. All terminologies not defined in this paper are followed as in [1], [2] and [7]. Berge [1] defined the concept of perfect graphs in the year 1973. He defined two types of perfection:

(i) G is γ-perfect, if $\chi(H) = \omega(H)$ for every induced subgraph H of G.

(ii) G is α-perfect, if $\alpha(H) = \theta(H)$ for every induced subgraph H of G.

Here, $\chi(H)$ represents the chromatic number of the graph H, $\omega(H)$ denotes the clique number of the graph H, $\alpha(H)$ is the independence number of H and $\theta(H)$ represents the clique covering number of H. Further, in [7] it was proved that *G is γ-perfect if and only if G is α-perfect* (or) equivalently, *G is perfect if and only if G is α-perfect.* Ravindra [6] introduced the concept of $\mathcal{F}$-perfect graphs in the year 2011, as an extension of perfect graphs by Berge. He defined $\mathcal{F}$-perfect graphs as follows: *Assuming that $\mathcal{F}_\mathcal{G}$ is the class of all induced subgraphs in G and $\mathcal{F}$ is any subclass of $\mathcal{F}_\mathcal{G}$ such as complete graphs or their complements, stars, complete bipartite graphs, cycles etc. A graph G is said to be $\mathcal{F}$-perfect if $\alpha_\mathcal{F}(H) = \theta_\mathcal{F}(H)$ for all induced subgraph $H \subseteq G$, where $\alpha_\mathcal{F}(H)$ denotes the maximum number vertices in H such that no two of them lie in the same element of $\mathcal{F}$ and $\theta_\mathcal{F}(H)$ denotes the minimum number of elements in $\mathcal{F}$ required to cover the vertex set of H.* It was also obtained in the paper that $\alpha_\mathcal{F}(G) \leq \theta_\mathcal{F}(G)$, for any graph G. Gokul et. al. [3] introduced the concept of induced cycle perfect graphs and characterised them as series parallel graphs that contains no subdivisions of $K_{2,3}$ as an induced subgraph graph. The concept of graph minor was introduced and studied by L. Lovász in [5]. *"An undirected graph H is called a minor of the graph G if H can be formed from G by deleting edges and vertices and by contracting edges."*

2. Preliminaries

The definitions and results mentioned in this section will be used to provide more clarity and also to prove certain results presented in the article.

Definition 2.1. [4] *An $ISK4$ is a graph that is an induced subdivision of the complete graph on four vertices, K_4.*

Definition 2.2. [3] *A graph is said to be $ISK_{2,3}$-free if it does not contain any subdivision of $K_{2,3}$ as an induced subgraph.*

Theorem 2.3. [4] *If G is $ISK4$-free, then either G is a series parallel graph or G contains prism, a wheel or a $K_{3,3}$ as an induced subgraph.*

Considering $\mathcal{C}$ to be the class of induced cycles in $\mathcal{F}_\mathcal{G}$, the concept of induced cycle perfect graphs or $\mathcal{C}$-perfect graphs has been defined, as follows.

Definition 2.4. [3] *A graph G is said to be $\mathcal{C}$-perfect if $\alpha_\mathcal{C}(H) = \theta_\mathcal{C}(H)$ for all induced subgraphs H of G, where every vertex in H belongs to at least one cycle in H.*

Property $\mathcal{P}$ [3] *Every graph G (inclusive of its subgraphs H) considered for $\mathcal{C}$-*

perfection is in such a way that every vertex in G (or H) belongs to at least one cycle in G (or H).

Theorem 2.5. [3] *If a graph G is $\mathcal{C}$-perfect, then it is wounded wheel free.*

Lemma 2.6. [3] *If G is $\mathcal{C}$-perfect, then G is $ISK_{2,3}$-free.*

Lemma 2.7. [3] *If G is $\mathcal{C}$-perfect, then G is $ISK4$-free.*

Lemma 2.8. [3] *If G is $\mathcal{C}$-perfect, then G is K_4 minor free.*

Theorem 2.9. [3] *Let G be a Hamiltonian graph satisfying property $\mathcal{P}$. Then, G is $\mathcal{C}$-perfect if and only if G is outerplanar.*

Theorem 2.10. [3] *A graph G satisfying property $\mathcal{P}$ is $\mathcal{C}$-perfect if and only if G is a series parallel graph having no subdivision of $K_{2,3}$ as an induced subgraph.*

OR

A graph G is $\mathcal{C}$-perfect if and only if G is K_4 minor free and has no subdivision of $K_{2,3}$ as an induced subgraph.

3. Results on $\mathcal{R}$-perfect graphs

In this article we consider $\mathcal{R}$ to be a subclass of $\mathcal{F}_{\mathcal{G}}$ that contains all induced regular subgraphs of G, and define the concept of induced regular perfect graphs or $\mathcal{R}$-perfect graphs.

A min-max equality link between the invariants α_R(max) and θ_R(min) is seen while investigating $\mathcal{R}$-perfect graphs. We have groups $G_1, G_2, \ldots, G_r$, $r \geq 2$, in a society, such that there is a person with the same number of acquaintances (regular relation) in each of the groups G_i. Our goal is to identify the ideal group of people, referred to as leaders, who can best represent each group. are a few reasons to investigate $\mathcal{R}$-perfect graphs. Of these motives, the primary one is the desire to comprehend the structural properties of $\mathcal{R}$-perfect graphs.

(i) An ***induced regular-independent set or $\mathcal{R}$-independent set*** *of G is a collection of vertices in G such that no two of them belong to the same induced regular subgraph and is denoted by* $\mathrm{I}_{\mathcal{R}}(G)$. (It must be noted that $\mathrm{I}_{\mathcal{R}}(G)$ is not unique for a graph.)

(ii) The ***induced regular independence number*** *of G, denoted by $\alpha_{\mathcal{R}}(G)$ is the cardinality of the largest induced regular-independent set of G.*

(iii) An ***induced regular-cover*** *or* ***$\mathcal{R}$-cover*** *of G is a collection of elements in $\mathcal{R}$ of a graph G whose union is G. Let* $\mathrm{T}_{\mathcal{R}}(G)$ *denote any smallest set of* *induced regular subgraphs in G that forms an $\mathcal{R}$-cover.* (It must be noted that $\mathrm{T}_{\mathcal{R}}(G)$ is not unique for a graph.)

(iv) The ***$\mathcal{R}$-covering number*** *of G, denoted by $\theta_{\mathcal{R}}(G)$, is the minimum number of elements in $\mathcal{R}$ required to cover the vertex set of G.*

Also, from [6] we deduce the relation between $\alpha_{\mathcal{R}}$ and $\theta_{\mathcal{R}}$ for any graph G, which is given by,

$$\alpha_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(G) \tag{1}$$

With the definitions of the parameters, $\alpha_{\mathcal{R}}$ and $\theta_{\mathcal{R}}$ in hand, we define induced regular perfect graphs or $\mathcal{R}$-perfect graphs as follows:

Definition 3.1. *A graph G is said to be $\mathcal{R}$-perfect if $\alpha_{\mathcal{R}}(H) = \theta_{\mathcal{R}}(H)$ for all induced subgraphs H of G.*

Propositions 3.2, 3.3 and 3.4 provides examples of some basic classes of $\mathcal{R}$-perfect graphs.

Proposition 3.2. *All trees are $\mathcal{R}$-perfect.*

Proof. The only regular graphs in a tree are K_2s. Therefore $\mathcal{R}$-perfection for trees is equivalent to K_2-perfection or Berge's perfection. It is known that all trees are perfect graphs and hence it is obtained that trees are $\mathcal{R}$-perfect.

Through Proposition 3.2 we can conclude that all acyclic graphs are $\mathcal{R}$-perfect.

Proposition 3.3. *Cycles and complete graphs and are $\mathcal{R}$-perfect.*

Proof. Initially considering cycles, they are 2-regular and all induced subgraphs of cycles are trees which are $\mathcal{R}$-perfect by Proposition 3.2. This implies that all cycles, C_n are $\mathcal{R}$-perfect with $\alpha_{\mathcal{R}}(C_n) = 1 = \theta_{\mathcal{R}}(C_n)$. Now, complete graphs are regular graphs and all induced subgraphs of complete graphs are also complete graphs. Hence it implies that complete graphs are $\mathcal{R}$-perfect, with $\alpha_{\mathcal{R}}(K_n) = 1 = \theta_{\mathcal{R}}(K_n)$.

Proposition 3.4. *$K_n - e$ is $\mathcal{R}$-perfect for all $n \geq 3 (n \in \mathbb{N})$, where e is an edge in the complete graph K_n.*

Proof. Let us consider the graph $K_n - e$, where e is an edge in the complete graph K_n that is incident to the vertices i and j. Considering the induced regular independence set $\mathrm{I}_{\mathcal{R}}(K_n - e)$, it can be observed that adding both the vertices i and j, each of degree $n-2$, in $\mathrm{I}_{\mathcal{R}}(K_n - e)$ results in a maximum regular independence set. Hence obtaining that $\alpha_{\mathcal{R}}(K_n - e) = 2$. Also the induced regular cover of $K_n - e$ can be obtained by considering a triangle containing the vertex i (or j) and the clique K_{n-1} where $i \notin V(K_{n-1})$ (or $j \notin V(K_{n-1})$). This implies that $\theta_{\mathcal{R}}(K_n - e) = 2$, since $K_n - e$ is not a regular graph. Therefore it is obtained that $\alpha_{\mathcal{R}}(K_n - e) = 2 = \theta_{\mathcal{R}}(K_n - e)$. Now considering all induced subgraphs H of $K_n - e$,

Case 1: If $i \in V(H)$ and $j \in V(H)$, then $H \cong K_m - e$, where m is the number of vertices in H. Therefore from the above paragraph, $\alpha_{\mathcal{R}}(H) = 2 = \theta_{\mathcal{R}}(H)$ for all such induced subgraphs of $K_n - e$.
Case 2: If $i \notin V(H)$ or $j \notin V(H)$ or both, then the obtained graph is a complete graph and therefore is $\mathcal{R}$-perfect. From Cases 1 and 2 it is obtained that $\alpha_{\mathcal{R}}(H) = 2 = \theta_{\mathcal{R}}(H)$ for all induced subgraphs of $K_n - e$. Therefore $K_n - e$ is $\mathcal{R}$-perfect.

In the following Proposition 3.5 and Corollary 3.6 we identify a couple of examples for graphs that are not $\mathcal{R}$-perfect for higher orders. To be specific both the classes of graphs mentioned in theses results are not $\mathcal{R}$-perfect for $n \geq 4$.

Proposition 3.5. *A wheel graph $W_{1,n}$ is $\mathcal{R}$-perfect if and only if $n = 3$.*
Proof. If $n = 3$, the wheel graph is a K_4. The result is a direct consequence. Conversely let $G = W_{1,n}$ be a wheel graph on $n + 1$ vertices, where $n \geq 4$, and let C_n be the induced cycle on n vertices in G. The regular induced subgraph of largest degree in G are cycles. Here, considering any vertex in G to $\mathrm{I}_{\mathcal{R}}(G)$ it can be identified that no other vertex in G can be added to $\mathrm{I}_{\mathcal{R}}(G)$ as every other vertex in G would share at least two induced cycles with the vertex v. Since the selection of v is arbitrary it can be observed that $\alpha_{\mathcal{R}}(G) = 1$. But since G is not a regular graph it is clear that $\theta_{\mathcal{R}}(G) > 1$. Hence $\alpha_{\mathcal{R}}(G) \neq \theta_{\mathcal{R}}(G)$ implying that $W_{1,n}$ is not $\mathcal{R}$-perfect for all $n \geq 4$. Hence the Proposition.

Corollary 3.6. *Helm graph H_n is $\mathcal{R}$-perfect if and only if $n = 3$.*
Proof. A helm graph H_n is formed by adjoining a pendant edge at each node of the cycle C_n in a wheel graph $W_{1,n}$. Therefore it is can be obtained from Proposition 3.5 and the definition of $\mathcal{R}$-perfect graphs that H_n is not $\mathcal{R}$-perfect for all $n \geq 4$ as it contains an induced wheel graph with $n \geq 4$. Now, the only remaining possibility is $n = 3$. The $\mathcal{R}$-perfection of H_3 can be easily identified by analysing $\mathrm{I}_{\mathcal{R}}(H_3)$ and $\mathrm{T}_{\mathcal{R}}(H_3)$. Since H_3 is a graph formed by adjoining a pendant edge to every vertex of a C_3 in K_4, we are forced to consider the three pendant vertices in H_3 to $\mathrm{I}_{\mathcal{R}}(H_3)$, this selection leaves us with a single vertex which does not share an induced regular graph with any of these pendant vertices, that is the central vertex. Therefore, we add the central vertex of H_3 to complete the largest $\mathrm{I}_{\mathcal{R}}(H_3)$. Hence, $\alpha_{\mathcal{R}}(H_3) = 4$. Similarly, $\mathrm{T}_{\mathcal{R}}(H_3)$ is formed by adding K_4 so as to cover the central vertex and all the three pendant edges to cover the pendant vertices. This results in $\theta_{\mathcal{R}}(H_3) = 4 = \alpha_{\mathcal{R}}(H_3)$. Now all induced subgraphs of H_3 are either trees or cycles with pendant vertices or cycles or K_4 or K_4 with pendant vertices, all of which are $\mathcal{R}$-perfect. Therefore it is concluded that H_3 is $\mathcal{R}$-perfect. This proves the Corollary.

As mentioned in Section 2, we use the notation $ISK4$ and $ISK_{2,3}$ to denote the induced subdivisions of K_4 and $K_{2,3}$ respectively. Now, let $\overline{ISK4}$ denote all non trivial induced subdivisions of K_4, that is, all induced subdivisions of K_4 other than K_4 itself.

The following couple of results involve these notations and proves necessary conditions for a graph to be $\mathcal{R}$-perfect.

Lemma 3.7. *A graph G is $\mathcal{R}$-perfect only if G is $\overline{ISK4}$-free.*
Proof. Considering any non trivial subdivision of K_4 it can be observed that the resulting graph would satisfy Property $\mathcal{P}$ and has no regular graphs of degree greater than 2, hence the induced regular subgraphs with highest degree are cycles. This implies that $\mathcal{R}$-perfection in $\overline{ISK4}$ is equivalent to $\mathcal{C}$-perfection. But it can be deduced from Lemma 2.7 that any graph containing $\overline{ISK4}$ is not $\mathcal{C}$-perfect, implying that $\overline{ISK4}$ is not $\mathcal{R}$-perfect and hence by the definition of $\mathcal{R}$-perfect graphs, any graph G containing $\overline{ISK4}$ is not $\mathcal{R}$-perfect.

Lemma 3.8. *A graph G is $\mathcal{R}$-perfect only if G is $ISK_{2,3}$-free.*
Proof. Similar to the approach in the previous lemma it is obtained that any subdivision of $K_{2,3}$ satisfies Property $\mathcal{P}$ and has no regular graphs of degree greater than 2, hence the regular subgraphs with highest degree are cycles. This implies that $\mathcal{R}$-perfection in $ISK_{2,3}$ is equivalent to $\mathcal{C}$-perfection. But it is proved in Lemma 2.6 that any graph containing $ISK_{2,3}$ is not $\mathcal{C}$-perfect, implying that it is not $\mathcal{R}$-perfect and hence by the definition of $\mathcal{R}$-perfect graphs, any graph G containing $ISK_{2,3}$ is not $\mathcal{R}$-perfect.

The converse of Lemma 3.8 is not true. The wounded wheel graph on 5 vertices, $W^{1,4}_{\{v_1\}}$ is an example of an $ISK4$-free graph which is not $\mathcal{R}$-perfect.

From Lemmas 3.8, and 3.7 it can be inferred that $\overline{ISK4}$ and $ISK_{2,3}$ are minimal $\mathcal{R}$-imperfect graphs, since all their induced subgraphs are paths, or cycles, or cycles connected to paths all of which are $\mathcal{R}$-perfect from Propositions 3.2 and 3.3. The method of proof in Lemmas 3.7 and 3.8 lead to the following theorem.

Proposition 3.9. *All $\mathcal{C}$-perfect graphs are $\mathcal{R}$-perfect.*
Proof. Initially considering G to be Hamiltonian $\mathcal{C}$-perfect graph, it is obtained from Theorem 2.9 that G is outerplanar as it is $\mathcal{C}$-perfect. Since G is $\mathcal{C}$-perfect it is K_4-minor free and $ISK_{2,3}$-free (Refer Theorems 2.6 and 2.8). Hence it can be deduced that the regular subgraph of largest degree are cycles. Therefore, $\mathcal{R}$-perfection here is equivalent to $\mathcal{C}$-perfection when considering all graphs and subgraphs satisfying property $\mathcal{P}$ (Refer Property 2 in Section 3). This implies that for G and all its induced subgraphs satisfying Property $\mathcal{P}$, G is $\mathcal{R}$-perfect.
Now let us consider all induced subgraphs H of G, not satisfying Property $\mathcal{P}$. We prove $\mathcal{R}$-perfection for all such graphs. This gives rise to two cases:

Case 1: If H is a tree, then by Proposition 3.2 H is $\mathcal{R}$-perfect.
Case 2: Suppose H is a cyclic graph. The $\mathcal{C}$-perfection of G implies that the biconnected components of H is also $\mathcal{C}$-perfect and hence is $\mathcal{R}$-perfect as it satisfies Property $\mathcal{P}$. Also by Proposition 3.2, trees are $\mathcal{R}$-perfect. Clearly, the biconnected components are connected to the tree by a cut vertices. Assuming H' to be a biconnected component of H and T to be a tree connected to it, we prove the $\mathcal{R}$-perfection of it by analysing $\mathrm{I}_{\mathcal{R}}(H)$ and $\mathrm{T}_{\mathcal{R}}(H)$. This further can be extended to the graph H. Assuming that $v \in V(H)$ is the common cut-vertex that connects H' to T, we observe the following conditions:
Subcase 1: If $v \in \mathrm{I}_{\mathcal{R}}(H')$ and $v \in \mathrm{I}_{\mathcal{R}}(T)$, then $\alpha_{\mathcal{R}}(H) = \alpha_{\mathcal{R}}(H') + \alpha_{\mathcal{R}}(T) - |\mathrm{I}_{\mathcal{R}}(H') \cap \mathrm{I}_{\mathcal{R}}(T)|$. This implies that $\alpha_{\mathcal{R}}(H) = \alpha_{\mathcal{R}}(H') + \alpha_{\mathcal{R}}(T) - 1$. Since $v \in \mathrm{I}_{\mathcal{R}}(T)$ and since H' and T are connected by cut vertices we can form $\mathrm{T}_{\mathcal{R}}(H)$ by adding all cycles from $\mathrm{T}_{\mathcal{R}}(H')$ and all K_2s from $\mathrm{T}_{\mathcal{R}}(T)$ except for the K_2 containing v as it is already covered by $\mathrm{T}_{\mathcal{R}}(H')$. Therefore it is obtained that $\theta_{\mathcal{R}}(H) = \theta_{\mathcal{R}}(H') + \theta_{\mathcal{R}}(T) - 1 = \alpha_{\mathcal{R}}(H') + \alpha_{\mathcal{R}}(T) - 1 = \alpha_{\mathcal{R}}(H)$.
Subcase 2: Let $v \in \mathrm{I}_{\mathcal{R}}(H')$ and $v \notin \mathrm{I}_{\mathcal{R}}(T)$, or vice versa, then $\alpha_{\mathcal{R}}(H) = \alpha_{\mathcal{R}}(H') + \alpha_{\mathcal{R}}(T)$, since $\mathrm{I}_{\mathcal{R}}(H')$ and $\mathrm{I}_{\mathcal{R}}(T)$ are disjoint sets. Now, forming the set $\mathrm{T}_{\mathcal{R}}(H)$, it can be observed that since $\mathrm{T}_{\mathcal{R}}(H')$ covers the vertex set of H' and $\mathrm{T}_{\mathcal{R}}(T)$ covers the vertex set of T, the union of the sets $\mathrm{T}_{\mathcal{R}}(H')$ and $\mathrm{T}_{\mathcal{R}}(T)$ forms a vertex cover for H. That is, $\mathrm{T}_{\mathcal{R}}(H) \subseteq \mathrm{T}_{\mathcal{R}}(H') \bigcup \mathrm{T}_{\mathcal{R}}(T)$. Therefore, $\theta_{\mathcal{R}}(H) \leq \theta_{\mathcal{R}}(H') + \theta_{\mathcal{R}}(T) = \alpha_{\mathcal{R}}(H') + \alpha_{\mathcal{R}}(T) = \alpha_{\mathcal{R}}(H)$ (Since H' and T are $\mathcal{R}$-perfect). That is, $\theta_{\mathcal{R}}(H) \leq \alpha_{\mathcal{R}}(H)$. But it is obtained from Equation 1, that $\alpha_{\mathcal{R}}(H) \leq \theta_{\mathcal{R}}(H)$ for all H. Hence, $\alpha_{\mathcal{R}}(H) = \theta_{\mathcal{R}}(H)$. In both the above cases the graph is $\mathcal{R}$-perfect. Now with a similar approach followed in Subcases 1 and 2 we can extend the same result to the graph H and hence all induced subgraphs not satisfying Property $\mathcal{P}$ is also $\mathcal{R}$-perfect. Hence the Theorem.

As a consequence of Theorem 3.9 we arrive at the Corollaries 3.10 and 3.11.

Corollary 3.10. *If G is a graph formed by adjoining trees to the vertices of a biconnected, $\mathcal{R}$-perfect graph, then G is $\mathcal{R}$-perfect.*

The proof to Corollary 3.10 is the direct consequence of Subcases 1 and 2 of Theorem 3.9.

Corollary 3.11. *If G is a graph satisfying property $\mathcal{P}$ and having no regular subgraphs of degree greater than 2, then G is $\mathcal{R}$-perfect if and only if G is $\mathcal{C}$-perfect.*

Wounded wheel graphs are derived graphs obtained from wheel graphs as defined in Paper [3]. It is defined as follows: Let $C_n : \{v_1, v_2, ..., v_n, v_1\}$ be a cycle of length n. Then a *wounded wheel graph, denoted by $W^{1,n}_{\{v_i | v_i \leftrightarrow v\}}$, is a graph on $n+1$ vertices obtained by adding a new vertex say v to C_n and making v adjacent to* *either a pair of non-adjacent vertices of C_n or to any k vertices lying on C_n, where $3 \leq k \leq n-1$. Here, $\{v_i | v_i \leftrightarrow v\}$ is the set of vertices in C_n which are adjacent to the central vertex v. Figure 1 illustrates a wounded wheel graph on 9 vertices obtained using C_8.*

The following lemma involves this class of graphs and also is a necessary condition for $\mathcal{R}$-perfection.

Lemma 3.12. *A graph G is $\mathcal{R}$-perfect only if it is wounded wheel-free.*

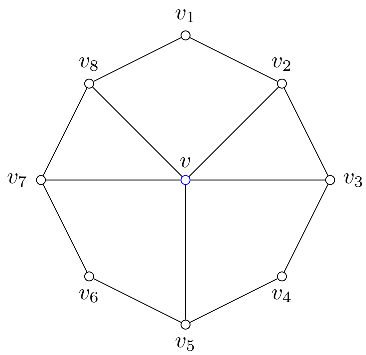

Figure 1: Wounded wheel graph, $W^{1,8}_{\{v_2,v_3,v_5,v_7,v_8\}}$

Proof. From the definition of wounded wheel graphs it can be observed that it satisfies Property $\mathcal{P}$ and has no regular subgraphs of degree greater that two. This implies from Corollary 3.11, that $\mathcal{R}$-perfection in wounded wheel graphs is equivalent to $\mathcal{C}$-perfection. But from Theorem 2.5, wounded wheel graphs are not $\mathcal{C}$-perfect and hence are not $\mathcal{R}$-perfect. Therefore by the definition of $\mathcal{R}$-perfect graphs, any graph G containing wounded wheel graphs are not $\mathcal{R}$-perfect.

Now we define a new class of minimal $\mathcal{R}$-imperfect graphs which will be proved to be a forbidden class for $\mathcal{R}$-perfect graphs in this article. Let C and C' be two cycles in a graph G, that shares a common path P of length at least 2. Then the graph G is said to belong to the class $ISK4^+$ if;

1. G has two edges e_1 and e_2, which are formed by connecting vertices from P to vertices from $C \setminus P$ or $C' \setminus P$ such that both e_1 and e_2 forms a K_3 with one or both end vertices of P.

2. G is a wounded wheel graph or a wheel graph on at least 5 vertices.

If both e_1 and e_2 have a common end vertex in $ISK4^+$, then the path P in the so formed $ISK4^+$ has length 2, (else the graph would contain an $ISK_{2,3}$ contradicting the minimality of it) and it is a wounded wheel graph.

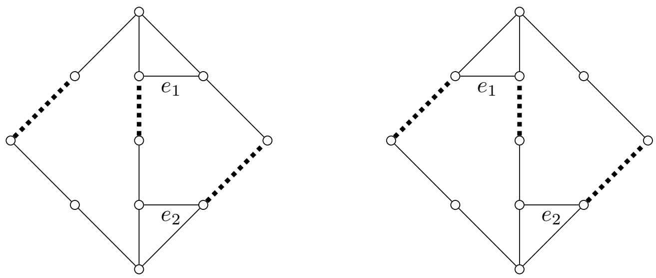

Figure 2: $ISK4^+$ type 1

Lemma 3.13. *$ISK4^+$ is minimal $\mathcal{R}$-imperfect.*
Proof. It can be easily observed that $\alpha_{\mathcal{R}}(ISK4^+) = 1$ and $\theta_{\mathcal{R}}(ISK4^+) = 2$ for all classes of graphs in $ISK4^+$. Now all proper subgraphs of any graphs in $ISK4^+$ are cycles, subdivision of a pan, subdivision of a bull, paths or $\mathcal{C}$-perfect graphs, all of which are $\mathcal{R}$-perfect by Propositions 3.2, 3.3, 3.9 and Corollary 3.10. Hence all graphs in $ISK4^+$ are minimal $\mathcal{R}$-imperfect.

Theorem 2.3 states that if a graph G is $ISK4$-free then G is series-parallel or contains $K_{3,3}$, prism or wheel as an induced subgraph. Now if G is $\overline{ISK4}$-free then G is $ISK4$-free or contains K_4 as an induced subgraph. This implies that G is series-parallel or contains $K_{3,3}$, prism or wheel as an induced subgraph or contains K_4. That is if G is $\overline{ISK4}$-free, then G is series-parallel or contains $K_{3,3}$, prism or wheel as an induced subgraph or contains K_4. Now, since K_4 is a wheel graph we modify the above statement as given in Corollary 3.14.

Corollary 3.14. *If G is $\overline{ISK4}$-free then either G is series-parallel or contains $K_{3,3}$, prism or wheel as an induced subgraph.*

The above result is a Corollary to the Theorem 2.3. The Theorem 3.15 given below is used to characterise $\mathcal{R}$-perfect graphs.

Theorem 3.15. *A graph G is $\mathcal{R}$-perfect if and only if all its biconnected components are $\mathcal{R}$-perfect.*
Proof. The forward implication is a direct consequence of the definition of $\mathcal{R}$-perfect graphs, since every biconnected component is also an induced subgraph of G.
Conversely, let every biconnected component of G be $\mathcal{R}$-perfect. It is to be proved

that G is $\mathcal{R}$-perfect. This is done by method of induction on the number of biconnected components in G, say $b(G)$.
Initially, let $b(G) = 1$, that is G is a graph with exactly one biconnected component. This implies that either G is a biconnected graph, in which case G is trivially $\mathcal{R}$-perfect or G is formed by adjoining trees to the vertices of a biconnected graph. If G is a such a graph then by Corollary 3.10 and the fact that the biconnected component in G is $\mathcal{R}$-perfect, it is obtained that G is $\mathcal{R}$-perfect.
Now, let G be a graph with k biconnected components, all of which are $\mathcal{R}$-perfect, then assume that G is $\mathcal{R}$-perfect. It is to be proved that, if G is a graph with $k+1$ biconnected components, all of which are $\mathcal{R}$-perfect, then G is $\mathcal{R}$-perfect. Let H be a biconnected component of G. Clearly, H is connected to G by cut vertices as it is a maximal biconnected subgraph of G. Now, removing H from G results in a graph G' that has k biconnected components, all of which are $\mathcal{R}$-perfect. Therefore by induction hypothesis it is obtained that G' is $\mathcal{R}$-perfect. Also H is $\mathcal{R}$-perfect as it is also a biconnected component of G.
Now we connect H to G' to get G and prove that G is $\mathcal{R}$-perfect. Since H and G are $\mathcal{R}$-perfect it is obtained that $\alpha_{\mathcal{R}}(H) = p = \theta_{\mathcal{R}}(H)$ and $\alpha_{\mathcal{R}}(G') = q = \theta_{\mathcal{R}}(G')$, respectively.
Case 1: Let H be connected to G' by trees. We prove $\mathcal{R}$-perfection when H is connected to G' by one such tree and $\mathcal{R}$-perfection would follow for H being connected to G' by multiple trees. Let v be the common vertex in H and G'.
Subcase 1: Let $v \in \mathrm{I}_{\mathcal{R}}(H)$ and $v \in \mathrm{I}_{\mathcal{R}}(G')$. Since v belongs to a tree in G' and it also belongs to the induced regular independent set of G' we can observe that v belongs to the induced regular independent set of the tree, containing v, as well. Therefore, $\mathrm{I}_{\mathcal{R}}(G)$ can be formed by using all vertices in $\mathrm{I}_{\mathcal{R}}(H)$ and every vertex in $\mathrm{I}_{\mathcal{R}}(G')$ so as to form a maximum induced regular independent set for G.
That is, $\alpha_{\mathcal{R}}(G) = |\mathrm{I}_{\mathcal{R}}(G') \bigcup \mathrm{I}_{\mathcal{R}}(H)|$. This implies that, $\alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(G') + \alpha_{\mathcal{R}}(H) - |\mathrm{I}_{\mathcal{R}}(G') \cap \mathrm{I}_{\mathcal{R}}(H)| = \alpha_{\mathcal{R}}(G') + \alpha_{\mathcal{R}}(H) - 1$.
Now in order to form $\mathrm{T}_{\mathcal{R}}(G)$, every regular subgraph from $\mathrm{T}_{\mathcal{R}}(H)$ must be considered into the set so as to cover the vertex set of H and also we must include every regular subgraph from $\mathrm{T}_{\mathcal{R}}(G')$, other than the K_2 containing v into $\mathrm{T}_{\mathcal{R}}(G)$, as v is already covered by $\mathrm{T}_{\mathcal{R}}(H)$. Therefore, $\mathrm{T}_{\mathcal{R}}(G) \subseteq \mathrm{T}_{\mathcal{R}}(H) \bigcup \mathrm{T}_{\mathcal{R}}(G') \setminus \{K_2\}$.
$\Rightarrow \theta_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(H) + \theta_{\mathcal{R}}(G') - 1$. Now, by induction hypothesis; $\theta_{\mathcal{R}}(G) \leq \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G') - 1 = \alpha_{\mathcal{R}}(G)$. But from Equation 1, $\alpha_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(G)$ for all G. Therefore it is obtained that $\alpha_{\mathcal{R}}(G) = \theta_{\mathcal{R}}(G)$.
Subcase 2: Let $v \in \mathrm{I}_{\mathcal{R}}(H)$ and $v \notin \mathrm{I}_{\mathcal{R}}(G')$ or vice versa. Then $\mathrm{I}_{\mathcal{R}}(G)$ can be obtained simply by taking the union of $\mathrm{I}_{\mathcal{R}}(H)$ and $\mathrm{I}_{\mathcal{R}}(G')$ since both are disjoint sets, hence obtaining the maximum induced regular independence set for G. That

is, $\alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(G') + \alpha_{\mathcal{R}}(H)$. Now it can be easily observed that an induced regular cover of $V(G)$ can be formed by taking the disjoint union of the sets $\mathrm{T}_{\mathcal{R}}(G')$ and $\mathrm{T}_{\mathcal{R}}(H)$ as they form the minimum induced regular covers of $V(G')$ and $V(H)$ respectively. That is, $\mathrm{T}_{\mathcal{R}}(G) \subseteq \mathrm{T}_{\mathcal{R}}(H) \bigcup \mathrm{T}_{\mathcal{R}}(G')$.
$\Rightarrow \theta_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(H) + \theta_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(G)$ (by induction hypothesis). That is, $\theta_{\mathcal{R}}(G) \leq \alpha_{\mathcal{R}}(G)$. But it is obtained from Equation 1 that, $\alpha_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(G)$ for all G. Therefore, $\alpha_{\mathcal{R}}(G) = \theta_{\mathcal{R}}(G)$.

Case 2: Let H be connected to G' by another biconnected component in G'. If G' is connected to H by a bridge, then $\mathrm{I}_{\mathcal{R}}(G)$ can be formed by taking the union of $\mathrm{I}_{\mathcal{R}}(H)$ and $\mathrm{I}_{\mathcal{R}}(G')$ as they are disjoint. Therefore, $\alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G')$. Now, since the union of $\mathrm{T}_{\mathcal{R}}(H)$ and $\mathrm{T}_{\mathcal{R}}(G')$ forms a cover for $V(G)$ and also since these sets are disjoint it is obtained that $\theta_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(H) + \theta_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(G)$ (by induction hypothesis). That is $\theta_{\mathcal{R}}(G) \leq \alpha_{\mathcal{R}}(G)$. But it is obtained from Equation 1 that, $\alpha_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(G)$ for all G. Therefore, $\theta_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(G)$. Now we consider the condition where H is connected to G' by a common vertex, say v.
Let ${R_{d_i}}^i$ denote induced regular subgraphs in G' containing v such that it also contains a vertex u_i, where i denotes the label given to the regular graph, d_i is the degree of the vertices in the regular graph with label i and $deg_G(u_i) = d_i$. Similarly, let ${R_{d_j}}^j$ represent such regular graphs in H that contains v.

Subcase 1: Let v belong to at least one induced regular subgraph in both G' and H such that, there exist a vertex u_i(or w_j) in each ${R_{d_i}}^i$(or ${R_{d_j}}^j$) where u_i(or w_j) is a vertex of degree d_i(or d_j) in G. Computing for $\mathrm{I}_{\mathcal{R}}(G)$, let us initially consider all vertices of $\mathrm{I}_{\mathcal{R}}(G')$ into $\mathrm{I}_{\mathcal{R}}(G)$. Clearly the vertices u_i in G belongs exclusively to the induced regular subgraphs ${R_{d_i}}^i$ in G. Hence all such vertices u_i must be included in $\mathrm{I}_{\mathcal{R}}(G')$ so as to maximise the number of elements in it. This implies that $v \notin \mathrm{I}_{\mathcal{R}}(G')$ as $v \in {R_{d_i}}^i$ for all i. Similarly, since w_js are vertices that exclusively belong to the induced regular subgraphs ${R_{d_j}}^j$, all these vertices must be considered in $\mathrm{I}_{\mathcal{R}}(H)$. This implies that $v \notin \mathrm{I}_{\mathcal{R}}(H)$ as $v \in {R_{d_j}}^j$ for all j. Therefore, $v \notin \mathrm{I}_{\mathcal{R}}(G')$ and $v \notin \mathrm{I}_{\mathcal{R}}(H)$, implying that $\mathrm{I}_{\mathcal{R}}(G')$ and $\mathrm{I}_{\mathcal{R}}(H)$ are disjoint. With this information in hand, to maximise the number of vertices in $\mathrm{I}_{\mathcal{R}}(G)$ we take the union of $\mathrm{I}_{\mathcal{R}}(G')$ and $\mathrm{I}_{\mathcal{R}}(H)$. Therefore, $\alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G')$. Now, since $\mathrm{T}_{\mathcal{R}}(G')$ and $\mathrm{T}_{\mathcal{R}}(H)$ covers the vertex set of G' and H respectively, the union of these two disjoint sets forms a vertex cover of G. That is, $\mathrm{T}_{\mathcal{R}}(G) \subseteq \mathrm{T}_{\mathcal{R}}(G') \bigcup \mathrm{T}_{\mathcal{R}}(H)$. This implies that, $\theta_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(H) + \theta_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(G)$ (by induction hypothesis). That is, $\theta_{\mathcal{R}}(G) \leq \alpha_{\mathcal{R}}(G)$. But by Equation 1, $\alpha_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(G)$ for all G. Therefore, $\alpha_{\mathcal{R}}(G) = \theta_{\mathcal{R}}(G)$.

Subcase 2: Let v belong to no such induced regular subgraph ${R_{d_i}}^i$ which contains a vertex u_i that has degree d_i in G. To be more clear, every vertex that share an induced regular subgraph with v has a degree greater than that of the induced regular subgraph it is contained in.

Since $V(G') \cap V(H) = \{v\}$, we need to analyse only the selection of vertices from the induced regular subgraphs containing v, to $\mathrm{I}_{\mathcal{R}}(G)$. All other vertices in $\mathrm{I}_{\mathcal{R}}(G)$ are taken directly from the union of $\mathrm{I}_{\mathcal{R}}(G')$ and $\mathrm{I}_{\mathcal{R}}(H)$ so as to maximise the set.

Subcase 2.1: Let u' be a vertex in G' that shares an induced regular subgraph with v, such that no other vertex that shares an induced regular subgraph with u' is contained in $\mathrm{I}_{\mathcal{R}}(G')$, hence implying that either $u' \in \mathrm{I}_{\mathcal{R}}(G')$ or $v \in \mathrm{I}_{\mathcal{R}}(G')$. Therefore while forming the maximum induced regular independence set of G, either one of u' or v can be considered along with all other vertices in $\mathrm{I}_{\mathcal{R}}(G')$ and $\mathrm{I}_{\mathcal{R}}(H)$. This selection of u' or v, into $\mathrm{I}_{\mathcal{R}}(G)$, depends on availability of v in $\mathrm{I}_{\mathcal{R}}(H)$. If $v \in \mathrm{I}_{\mathcal{R}}(H) \subset \mathrm{I}_{\mathcal{R}}(G)$, then u' cannot be taken into $I_{\mathcal{R}}(G)$, since u' and v share the same induced regular subgraph. Therefore, $v \in \mathrm{I}_{\mathcal{R}}(G') \subset \mathrm{I}_{\mathcal{R}}(G)$. Hence, maximising the induced regular independence set of G, it is obtained that $\mathrm{I}_{\mathcal{R}}(G) = \mathrm{I}_{\mathcal{R}}(H) \bigcup \mathrm{I}_{\mathcal{R}}(G')$.

$\Rightarrow \alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G') - |\mathrm{I}_{\mathcal{R}}(H) \cap \mathrm{I}_{\mathcal{R}}(G')|$.

$\Rightarrow \alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G') - |\{v\}|$

$\Rightarrow \alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G') - 1$.

Now, $\theta_{\mathcal{R}}(G)$ is to be analysed for the above condition. In order to form $\mathrm{T}_{\mathcal{R}}(G)$, all elements from $\mathrm{T}_{\mathcal{R}}(G')$ are included so as to cover $V(G')$ including v. Now, since $v \in \mathrm{I}_{\mathcal{R}}(H)$ and also every vertex that share an induced regular subgraph with v also belongs to other induced regular subgraphs, it can be inferred that all these vertices can be covered by these induced regular graphs in H that does not contain v. That is, we need not consider the induced regular graph from $\mathrm{T}_{\mathcal{R}}(H)$ which contains v, say R_v, while forming an induced regular cover for $V(G)$. Therefore, it can be observed that $\mathrm{T}_{\mathcal{R}}(G) \subseteq \mathrm{T}_{\mathcal{R}}(G') \bigcup \mathrm{T}_{\mathcal{R}}(H) \setminus \{R_v\}$.

$\Rightarrow \theta_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(G') + \theta_{\mathcal{R}}(H) - 1 = \alpha_{\mathcal{R}}(G') + \alpha_{\mathcal{R}}(H) - 1 = \alpha_{\mathcal{R}}(G)$ (by induction hypothesis). That is, $\theta_{\mathcal{R}}(G) \leq \alpha_{\mathcal{R}}(G)$. But it is observed from Equation 1 that, $\alpha_{\mathcal{R}}(G) \leq \theta_{\mathcal{R}}(G)$ for all G, implying that $\alpha_{\mathcal{R}}(G) = \theta_{\mathcal{R}}(G)$.

If $v \notin \mathrm{I}_{\mathcal{R}}(H) \subseteq \mathrm{I}_{\mathcal{R}}(G)$. Then there exists a vertex w that shares an induced regular subgraph of H with v such that $w \in \mathrm{I}_{\mathcal{R}}(H) \subseteq \mathrm{I}_{\mathcal{R}}(G)$. Therefore, v cannot be included in $\mathrm{I}_{\mathcal{R}}(G)$, hence we can consider u' into $\mathrm{I}_{\mathcal{R}}(G')$ and also it forces u' into $\mathrm{I}_{\mathcal{R}}(G)$. This implies that $\mathrm{I}_{\mathcal{R}}(G')$ and $\mathrm{I}_{\mathcal{R}}(H)$ are disjoint, as neither of these sets contain v. Also since these sets form the maximum induced regular independent sets of G' and H respectively, we can form the $\mathrm{I}_{\mathcal{R}}(G)$ set by taking the disjoint union of these two sets. That is, $\mathrm{I}_{\mathcal{R}}(G) = \mathrm{I}_{\mathcal{R}}(H) \bigcup \mathrm{I}_{\mathcal{R}}(G') \Rightarrow \alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G')$. Now, since $\mathrm{T}_{\mathcal{R}}(G')$ and $\mathrm{T}_{\mathcal{R}}(H)$ covers the vertex set of G' and H respectively, the union of these two disjoint sets forms a vertex cover of G. That is, $\mathrm{T}_{\mathcal{R}}(G) \subseteq$

$T_{\mathcal{R}}(G') \bigcup T_{\mathcal{R}}(H)$. This implies that, $\theta_{\mathcal{R}}(G) \le \theta_{\mathcal{R}}(H) + \theta_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(G)$ (by induction hypothesis). That is, $\theta_{\mathcal{R}}(G) \le \alpha_{\mathcal{R}}(G)$. But it is observed from Equation 1 that $\alpha_{\mathcal{R}}(G) \le \theta_{\mathcal{R}}(G)$ for all G. Therefore, $\alpha_{\mathcal{R}}(G) = \theta_{\mathcal{R}}(G)$.
Subcase 2.2: Let there exist no such vertex u'. That is, for every vertex u' that shares an induced regular subgraph with v, there exist a vertices w' that shares an induced regular subgraph with u', not containing v, such that $w' \in I_{\mathcal{R}}(G')$. This implies that $v \in I_{\mathcal{R}}(G')$ and also v is forced into $I_{\mathcal{R}}(G)$. Now if $v \in I_{\mathcal{R}}(H)$ then a similar approach used in Subcase 1.1 can be used to obtain the result that $\alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(G') + \alpha_{\mathcal{R}}(H) - 1 = \theta_{\mathcal{R}}(G') + \theta_{\mathcal{R}}(H) - 1 = \theta_{\mathcal{R}}(G)$. Also if $v \notin I_{\mathcal{R}}(H)$ then the disjoint union of $I_{\mathcal{R}}(G')$ and $I_{\mathcal{R}}(H)$ form the maximum induced regular independent set of G. That is, $\alpha_{\mathcal{R}}(G) = \alpha_{\mathcal{R}}(G') + \alpha_{\mathcal{R}}(H)$. Also since $T_{\mathcal{R}}(G')$ and $T_{\mathcal{R}}(H)$ covers the vertex set of G' and H respectively, the union of these two disjoint sets forms a vertex cover of G. That is, $T_{\mathcal{R}}(G) \subseteq T_{\mathcal{R}}(G') \bigcup T_{\mathcal{R}}(H)$. This implies that, $\theta_{\mathcal{R}}(G) \le \theta_{\mathcal{R}}(H) + \theta_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(H) + \alpha_{\mathcal{R}}(G') = \alpha_{\mathcal{R}}(G)$ (by induction hypothesis). That is, $\theta_{\mathcal{R}}(G) \le \alpha_{\mathcal{R}}(G)$. But it is observed from Equation 1 that $\alpha_{\mathcal{R}}(G) \le \theta_{\mathcal{R}}(G)$ for all G. Therefore, $\alpha_{\mathcal{R}}(G) = \theta_{\mathcal{R}}(G)$.
From all the above Cases it is obtained that a graph G with $k+1$ biconnected components, all of which are $\mathcal{R}$-perfect, is $\mathcal{R}$-perfect. Hence by induction method, the theorem is proved.

The following Theorem characterises 2-connected, $\mathcal{R}$-perfect graphs. This is obtained by identifying the forbidden class: $\{\overline{ISK4}, ISK_4^+, ISK_{2,3}\}$. For convenience, let us denote the above forbidden class by $\mathcal{A}$. This theorem is further used to characterise $\mathcal{R}$-perfect graphs.

Corollary 3.16. *A 2-connected graph is $\mathcal{R}$-perfect if and only if it is $\mathcal{A}$-free.*
Proof. Given that G is a 2-connected, $\mathcal{R}$-perfect graph it can be obtained as a direct consequence of Lemmas 3.7, 3.8, and 3.13, that G is $\overline{ISK4}$-free, $ISK_{2,3}$-free and $ISK4^+$-free. This proves the forward implication.
Conversely, assume that G is $\mathcal{A}$-free that is, G is $\{\overline{ISK4}, ISK_4^+, ISK_{2,3}\}$-free. It is to be proved that the biconnected graph G is $\mathcal{R}$-perfect. Since G is $\overline{ISK4}$-free, Corollary 3.14 implies that G is either series parallel or contains $K_{3,3}$, prism or wheel as an induced subgraph. Since $K_{2,3} \in K_{3,3}$ and G is $K_{2,3}$-free, G is $K_{3,3}$-free. Also G is $W_{1,n}$-free for all $n \ge 4$. Therefore the only wheel graph in G is K_4. Let G contain a prism graph, $C_n \square P_2$ it can be easily observed that the prism graphs contains an $ISK4$ for all $n \ge 4$, and contains $ISK4^+$ for all $n \ge 5$ as an induced subgraph. Therefore G is prism free for all $n \ge 4$. Hence the only prism in G is $C_3 \square P_2$, which is $\mathcal{R}$-perfect.
Now, let G be a series parallel graph. Since G is a biconnected, $ISK_{2,3}$-free graph, Theorem 2.10 implies that G is $\mathcal{C}$-perfect and hence by Theorem 3.9, G is $\mathcal{R}$-

perfect.
From all the above inferences, we obtain that G is a series parallel, $ISK_{2,3}$-free graph or may contain K_4s or $C_3\Box P_2$ as induced subgraphs, all of which are obtained to be $\mathcal{R}$-perfect. Hence the theorem.

With the help of Theorems 3.16 and 3.15 we prove the characterization of $\mathcal{R}$-perfect graphs.

Corollary 3.17. *A graph G is $\mathcal{R}$-perfect if and only if G is $\mathcal{A}$-free.*
Proof. Given G is an $\mathcal{R}$-perfect graph it is obtained from Theorem 3.15 that every biconnected component of G is $\mathcal{R}$-perfect. Further, Theorem 3.16 implies that all these biconnected components are $\mathcal{A}$-free. This proves the forward implication. Conversely, let G be $\mathcal{A}$-free, that is, G is a graph such that every biconnected component in G is $\{\overline{ISK4}, ISK_4^+, ISK_{2,3}\}$-free. Theorem 3.16 implies that every biconnected component in G is $\mathcal{R}$-perfect. Further, since every biconnected component in G is $\mathcal{R}$-perfect, Theorem 3.15 implies that G is $\mathcal{R}$-perfect.

4. Summary and Conclusion

In this paper we have extended the notion of $\mathcal{C}$-perfect graphs and introduced a super class for it in $\mathcal{R}$-perfect graphs. The main objective and inspiration that lead to the birth of this topic was the interest in extending the concept of $\mathcal{C}$-perfect graphs to all possible classes of graphs. Through this paper we have studied the structural properties of $\mathcal{R}$-perfect graphs, which lead to identifying a forbidden class for the same. We have characterised biconnected $\mathcal{R}$-perfect graphs as either series parallel or containing K_4 or prism as induced subgraphs, which further helped in deriving a general characterization for induced regular perfect graphs. Series parallel graphs have wide range of applications in electronics. The scope of applications for $\mathcal{R}$-perfect graphs can be explored. The future scope of this topic lay vast and works on studying the structural properties of $\mathcal{R}$-perfect graphs for various derived graphs are in progress.

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