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SOME PROPERTIES OF BINARY S_α OPEN AND CLOSED SETS IN BINARY TOPOLOGICAL SPACE

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Abstract: The primary aim of this paper is to discuss the interior and closure operators of binary S_α set in binary topological spaces. That is binary S_α-closure and binary S_α-interior are defined. Also, their basic properties are discussed with suitable examples. Furthermore, the basic relation with the other existing sets have been discussed.binary S_α-closure and binary S_α-interior are denoted as ${}_bS_\alpha cl$ and ${}_bS_\alpha int$ respectively.

Keywords and Phrases: Binary S_α set, Binary S_α-closure, Binary S_α-interior, Binary topological space, Binary interior, Binary closure.

2020 Mathematics Subject Classification: 54D70.

1. Introduction

A topological space is a space provided with a structure, called a topology, which involves study of properties of space that are fixed under continuous deformation. Topology plays a tremendous role in the field of mathematics research. In particular, binary topology is an recently developed part of topology. The concept of Binary topology was first introduced by S. Nithyanantha Jothi and P. Thangavelu [6] in 2011 where a single structure which carries the subsets of X as well as the subsets of Y in a ordered pair (A, B) of subsets of X and Y. This type of a structure is called Binary topology. S. N. Jothi and P. Thangavelu [7] introduced binary semiopen open sets in this Binary structure and discussed some of their properties in binary topological spaces. G. B. Navalagi [5] in 2000, defined semi-α open sets in topological spaces by combining the α set with semi set in a general topological space. This concept of semi-α open sets was further introduced in a binary topological space by J. Elekiah and G. Sindhu [2] in 2022 and studied its relationship with other existing sets. The closure and interior properties of a set are the basis properties of a set. In this paper, we studied about ${}_bS_\alpha$ closure and interior operators and how its properties behaves in binary topological space are been discussed.

2. Preliminaries

Definition 2.1. [6] *Let X and Y be any two nonempty sets. A binary topology is a binary structure $\mathcal{M} \subseteq P(X) \times P(Y)$ from X to Y which satisfies the following axioms:*

(i) $(\emptyset, \emptyset) \in \mathcal{M}$
$(X, Y) \in \mathcal{M}$.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ *where* $A_1, A_2, B_1, B_2 \in \mathcal{M}$

(iii) *If* $(A_\alpha, B_\alpha : \alpha \in A)$ *is a family of members of* $\mathcal{M}$*, then*
$(\cup_{\alpha \in A} A_\alpha, \cup_{\alpha \in A} B_\alpha) \in \mathcal{M}$

Definition 2.2. [6] *Let X and Y be any two nonempty sets and let (A, B) and $(C, D) \in P(X) \times P(Y)$. If $A \subseteq C$ and $B \subseteq D$, then $(A, B) \subseteq (C, D)$.*

Definition 2.3. [6] *Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. The ordered pair $((A, B)^{1^\circ}, (A, B)^{2^\circ})$ is called the binary interior of (A, B) where $(A, B)^{1^\circ} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)$ and $(A, B)^{2^\circ} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)$.*
The binary interior of (A, B) is denoted by b-int(A, B).

Definition 2.4. [6] *Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. The ordered pair $((A, B)^{1^\star}, (A, B)^{2^\star})$ is called the binary closure of (A, B) where $(A, B)^{1^\star} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B)_\alpha$ and $(A, B)^{2^\star} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A,B) \subseteq (A_\alpha, B_\alpha)$.*
The binary closure of (A, B) is denoted by b-cl(A, B).

Definition 2.5. *A subset (A, B) of a binary topological space $(X, Y, \mathcal{M})$ is called*

(i) *binary α open* [3] *if $(A, B) \subseteq$ b-int(b-cl(b-int(A, B))).*

(ii) *binary semi open set* [7] *if $(A, B) \subseteq$ b-int(b-cl(A, B)).*

Definition 2.6. [5] *In a topological space (X, τ), the subset A of X is said to be semi-α-open if there exists a α-open set U in X such that $U \subseteq A \subseteq cl(U)$. The family of all semi-α-open sets of X is denoted by $S_\alpha(X)$.*

Definition 2.7. [2] *Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. The subset (A,B) is said to be binary semi α -open (${}_bS_\alpha O$)if there exists an binary α-open set (U,V) in X such that $(U, V) \subseteq (A, B) \subseteq cl(U, V)$.*

Definition 2.8. [10] *A subset A of a space (X, τ_1, τ_2) is called (τ_1, τ_2)-b-locally open (in short $(\tau_1, \tau_2) - bLO$) if $A = G \cup F$, where G is $\tau_1 -$ b-closed and F is $\tau_2 -$ b-open in (X, τ_1, τ_2).*

Definition 2.9. [1] *A subset A of a topological space (X, τ) is called b-open if $A \subseteq cl(int(A)) \cup int(cl(A))$. The complement of b-open set is a b-closed set.*

Definition 2.10. [11] *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called pairwise b-locally open (in short, pairwise bLO) mapping if the image of each (τ_1, τ_2)-b-locally open set in X is σ_i-open set in Y , where i = 1,2.*

Definition 2.11. [12] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i, j)-weakly b-continuous if for each $x \in X$ and each σ_i -open set V of Y containing $f(x)$, there exists an (i, j)-b-open set U containing x such that $f(U) \subset jCl(V)$.*

Definition 2.12. [13] *Let (X, τ_1, τ_2, I) be an ideal bitopological space. A subset A of X is said to be (i, j) I-generalized b-closed (in short,(i, j) Igbclosed) set if $(j, i) - bcl(A) - B \in I$ whenever $A \subset B$ and B is τ_i-open in X, for $i, j = 1, 2$ and $i6 = j$*

3. ${}_bS_\alpha$ closure and interior operators

Definition 3.1. *Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y, \mathcal{M})$. The subset (A, B) is said to be binary semi α open set (${}_bS_\alpha$) if there exists an binary α open set (U, V) in $(X, Y, \mathcal{M})$ such that $(U, V) \subseteq (A, B) \subseteq_b cl(U, V)$.*

Definition 3.2. *The union of all binary semi α-open sets in a binary topological space $(X, Y, \mathcal{M})$ contained in (A, B) is called binary semi α interior of (A, B) and it is denoted by ${}_bS_\alpha int(A, B)$.*

(i) $(A, B)^{S_\alpha - 1^\circ} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ *is* ${}_bS_\alpha$*-open and* $(A_\alpha, B_\alpha) \subseteq (A, B)\}$

(ii) $(A, B)^{S_\alpha - 2^\circ} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ *is* ${}_bS_\alpha$*-open and* $(A_\alpha, B_\alpha) \subseteq (A, B)\}$

The ordered pair $((A, B)^{S_\alpha - 1^\circ}, (A, B)^{S_\alpha - 2^\circ})$ is called the binary S_α-interior of (A,B) where $(A, B) \subseteq (X, Y)$.

Definition 3.3. *The intersection of all binary semi α-open sets in a binary topological space $(X, Y, \mathcal{M})$ containing in (A, B) is called binary semi α closure of (A, B) and it is denoted by ${}_bS_\alpha cl(A, B)$.*

(i) $(A, B)^{S_\alpha - 1^\star} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ *is* ${}_bS_\alpha$*-closed and* $(A, B) \subseteq (A_\alpha, B_\alpha)\}$

(ii) $(A, B)^{S_\alpha - 2^\star} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ *is* ${}_bS_\alpha$*-closed and* $(A, B) \subseteq (A_\alpha, B_\alpha)\}$

The ordered pair $((A, B)^{S_\alpha - 1^\star}, (A, B)^{S_\alpha - 2^\star})$ *is called the binary* S_α*-closure of (A,B) where* $(A, B) \subseteq (X, Y)$.

Example 3.4. Let $X = \{x_1, x_2\}; Y = \{y_1, y_2, y_3\}; \mathcal{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2\}), (X, \{y_2, y_3\}), (\{x_1\}, \emptyset), (X, \{Y_3\}), (\{x_2\}, \{y_1\}), (X, \{y_1, y_2\}), (\{x_2\}, \emptyset), (X, \{y_2\}), (X, \{y_1\})\}, (X, \emptyset), (X, \{y_1, Y_3\})$ be binary topology from X to Y; binary S_α open set $= \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2\}), (X, \{y_2, y_3\}), (\{x_1\}, \emptyset), (X, \{Y_3\}), (\{x_2\}, \{y_1\}), (X, \{y_1, y_2\}), (\{x_2\}, \emptyset), (X, \{y_2\}), (X, \{y_1\})\}, (X, \emptyset), (X, \{y_1, Y_3\}), (\{x_1\}, \{y_2, y_3\}), (\{x_2\}, \{y_1, y_3\}), (\{x_1\}, Y)$; binary S_α closed set$=\{(\emptyset, \emptyset), (X, Y), (\{x_2\}, \{y_1, y_3\}), (\emptyset, \{y_1\}), (\{x_2\}, Y), (\emptyset, \{y_1, y_2\}), (\{x_1\}, \{y_2, y_3\}), (\emptyset, \{y_3\}), (\{x_1\}, Y), (\emptyset, \{y_1, y_3\}), (\emptyset, \{y_2, y_3\})\}, (\emptyset, Y), (\emptyset, \{y_2\}), (\{x_2\}, \{y_1\}), (\{x_1\}, \{y_2\}), (\{x_2\}, \emptyset)$. Let $(\{x_2\}, \{y_3\})$ be a set, then ${}_bS_\alpha int(\{x_2\}, \{y_3\}) = (\{x_2\}, \emptyset)$ and ${}_bS_\alpha\ cl(\{x_2\}, \{y_3\}) = (\{x_2\}, \{y_1, y_3\})$.

Theorem 3.5. *In a binary topological space $(X, Y, \mathcal{M})$, arbitrary union of ${}_bS_\alpha$ open set is ${}_bS_\alpha$ open set.*

Proof. Let $\{(A_i, B_i)\}_{i\in\Delta}$ be a family of ${}_bS_\alpha$ open set, To prove that $\cup_{i\in\Delta}(A_i, B_i)$ is a ${}_bS_\alpha$ open set. Since (A_i, B_i) is a ${}_bS_\alpha$ open set there exist a ${}_b\alpha$ open set (U_i, V_i) such that $(U_i, V_i) \subseteq (A_i, B_i) \subseteq cl(U_i, V_i)$ for all $i \in \Delta$ which implies that $\cup_{i\in\Delta}(U_i, V_i) \subseteq \cup_{i\in\Delta}(A_i, B_i) \subseteq \cup_{i\in\Delta}\ {}_bcl(U_i, V_i) \subseteq_b cl(\cup_{i\in\Delta}(U_i, V_i))$ by property arbitrary union of α Open set is α Open set. hence $\cup_{i\in\Delta}(U_i, V_i)$ is ${}_b\alpha$ open set. hence $\cup_{i\in\Delta}(A_i, B_i) \in_b S_\alpha$ open set. Hence arbitrary union of ${}_bS_\alpha$ open set is ${}_bS_\alpha$ open set.

Remark 3.6. *In a binary topological space $(X, Y, \mathcal{M})$, intersection of two ${}_bS_\alpha$ open set need not be ${}_bS_\alpha$ open set which is illustrated in the following example.*

Example 3.7. Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$. Let $\mathcal{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1, x_2\}, \{\ y_2\}), (\{x_3\}, \{y_1\}), (\{x_2\}, \{y_1\}), (\{X\}, \{y_2\}), (\{x_1, x_2\}, \{Y\}), (\{x_3\}, \emptyset), (\{x_2\}, \emptyset)\}$be binary topology from X to Y; ${}_bS_\alpha = \{(\emptyset, \emptyset), (X, Y), (\{x_1, x_2\}, \{\ y_2\}), (\{x_1, x_2\}, Y), (\{x_3\}, \{\ y_1\}), (\{x_2\}, \{\ y_1\}), (X, \{\ y_2\}), (\{x_3\}, \emptyset), (\{x_2\}, \emptyset), (\{x_2\}, \{\ y_2\}), (X, \{\ y_1\}), (\{x_2\}, Y)\}$. Let $(A, B) = (\{x_1, x_2\}, \{\ y_2\})$ and $(C, D) = (\{X\}, \{y_1\})$ be two ${}_bS_\alpha$ open set but their intersection $(A, B) \cap (C, D) = (\{x_1, x_2\}, \{\ y_2\}) \cap (\{X\}, \{y_1\}) = (\{x_1, x_2\}, \emptyset)$ which is not a ${}_bS_\alpha$ open set.

Proposition 3.8. *Let (A,B) be any binary set in a binary topological space $(X,Y,\mathcal{M})$, the following properties hold:*

(i) ${}_bS_\alpha int(A,B)=(A,B)$ *iff* (A,B) *is a* ${}_bS_\alpha OS$.

(ii) ${}_bS_\alpha cl(A,B)=(A,B)$ *iff* (A,B) *is a* ${}_bS_\alpha CS$.

(iii) ${}_bS_\alpha int(A,B)\subseteq(A,B)\subseteq_b S_\alpha cl(A,B)$

These properties is proved in the following example.

Example 3.9. Let $X=\{x_1,x_2\}$; $Y=\{y_1,y_2,y_3\}$; $\mathcal{M}=\{(\emptyset,\emptyset),(X,Y),(\{x_1\},\{y_2\}),(X,\{y_2,y_3\}),(\{x_1\},\emptyset),(X,\{Y_3\}),(\{x_2\},\{y_1\}),(X,\{y_1,y_2\}),(\{x_2\},\emptyset),(X,\{y_2\}),(X,\{y_1\})\},(X,\emptyset),(X,\{y_1,Y_3\})$ be binary topology from X to Y; binary S_α open set $=\{(\emptyset,\emptyset),(X,Y),(\{x_1\},\{y_2\}),(X,\{y_2,y_3\}),(\{x_1\},\emptyset),(X,\{Y_3\}),(\{x_2\},\{y_1\}),(X,\{y_1,y_2\}),(\{x_2\},\emptyset),(X,\{y_2\}),(X,\{y_1\})\},(X,\emptyset),(X,\{y_1,Y_3\}),(\{x_1\},\{y_2,y_3\}),(\{x_2\},\{y_1,y_3\}),(\{x_1\},Y)$; binary S_α closed set $=\{(\emptyset,\emptyset),(X,Y),(\{x_2\},\{y_1,y_3\}),(\emptyset,\{y_1\}),(\{x_2\},Y),(\emptyset,\{y_1,y_2\}),(\{x_1\},\{y_2,y_3\}),(\emptyset,\{y_3\}),(\{x_1\},Y),(\emptyset,\{y_1,y_3\}),(\emptyset,\{y_2,y_3\})\},(\emptyset,Y),(\emptyset,\{y_2\}),(\{x_2\},\{y_1\}),(\{x_1\},\{y_2\}),(\{x_2\},\emptyset)$. Which implies ${}_bS_\alpha int(\{x_1\},\{y_2,y_3\})=(\{x_1\},\{y_2,y_3\})$ where $(\{x_1\},\{y_2,y_3\})$ is a ${}_bS_\alpha OS$ and ${}_bS_\alpha cl(\emptyset,\{y_1\})=(\emptyset,\{y_1\})$ where $(\emptyset,\{y_1\})$ is a ${}_bS_\alpha CS$. Let $(\{x_2\},\{y_3\})$ be a set, then ${}_bS_\alpha int(\{x_2\},\{y_3\})=(\{x_2\},\emptyset)$ and ${}_bS_\alpha\ cl(\{x_2\},\{y_3\})=(\{x_2\},\{y_1,y_3\})$. Thus ${}_bS_\alpha\ int(\{x_2\},\{y_3\})\subseteq(\{x_2\},\{y_3\})\subseteq_b S_\alpha\ cl(\{x_2\},\{y_3\})$.

Proposition 3.10. *Let (A,B) be any binary set in a binary topological space $(X,Y,\mathcal{M})$, the following properties hold:*

(i) ${}_bS_\alpha int[(X,Y)-(A,B)]=(X,Y)-[{}_bS_\alpha cl(A,B)]$

(ii) ${}_bS_\alpha cl[(X,Y)-(A,B)]=(X,Y)-[{}_bS_\alpha int(A,B)]$

Proof. By definition, ${}_bS_\alpha cl(A,B)=\cap\{(A_\alpha,B_\alpha):(A_\alpha,B_\alpha)$ is ${}_bS_\alpha CS$ and $(A,B)\subseteq(A_\alpha,B)_\alpha$. Now, $(X,Y)-_b S_\alpha cl(A,B)=(X,Y)-[\cap\{(A_\alpha,B_\alpha):(A_\alpha,B_\alpha)$ is ${}_bS_\alpha CS$ and $(A,B)\subseteq(A_\alpha,B)_\alpha]=\cup[(X,Y)-\{(A_\alpha,B_\alpha):(A,B)\subseteq(A_\alpha,B_\alpha)$ and (A_α,B_α) is ${}_bS_\alpha CS=\cup[\{(A_\beta,B_\beta):(A,B)\subseteq[(X,Y)-(A_\beta,B_\beta)]$ and (A_β,B_β) is ${}_bS_\alpha OS=_b S_\alpha int[(X,Y)-(A,B)]$.
The proof for (ii) is similar.

Theorem 3.11. *Let (A,B) and (C,D) be two binary sets in a binary topological space $(X,Y,\mathcal{M})$, the following properties hold:*

(i) ${}_bS_\alpha\text{-}int(\emptyset,\emptyset)=(\emptyset,\emptyset)$
${}_bS_\alpha\text{-}int(X,Y)=(X,Y)$

(ii) ${}_bS_\alpha$-$int(A, B) \subseteq (A, B)$

(iii) $(A, B) \subseteq (C, D) \implies {}_bS_\alpha int(A, B) \subseteq_b S_\alpha int(C, D)$

(iv) ${}_bS_\alpha int[(A, B) \cap (C, D)] \subseteq_b S_\alpha int(A, B) \cap_b S_\alpha int(C, D)$

(v) ${}_bS_\alpha int(A, B) \cup_b S_\alpha int(C, D) \subseteq_b S_\alpha int[(A, B) \cup (C, D)]$

(vi) ${}_bS_\alpha$-$int({}_bS_\alpha$-$int(A, B)) =_b S_\alpha$-$int(A, B)$

Proof. (i), (ii), (iii), (iv), (v) and (vi) are obvious.

Theorem 3.12. *Let (A, B) and (C, D) be two binary sets in a binary topological space $(X, Y, \mathcal{M})$, the following properties hold:*

(i) ${}_bS_\alpha$-$cl(\emptyset, \emptyset) = (\emptyset, \emptyset)$
${}_bS_\alpha$-$cl(X, Y) = (X, Y)$

(ii) $(A, B) \subseteq_b S_\alpha$-$cl(A, B)$

(iii) $(A, B) \subseteq (C, D) \implies {}_bS_\alpha cl(A, B) \subseteq_b S_\alpha cl(C, D)$

(iv) ${}_bS_\alpha cl[(A, B) \cap (C, D)] \subseteq_b S_\alpha cl(A, B) \cap_b S_\alpha cl(C, D)$

(v) ${}_bS_\alpha cl(A, B) \cup_b S_\alpha cl(C, D) \subseteq_b S_\alpha cl[(A, B) \cup (C, D)]$

(vi) ${}_bS_\alpha$-$cl({}_bS_\alpha$-$cl(A, B)) =_b S_\alpha$-$cl(A, B)$

Proof. (i) and (ii) is easy, so omitted.
(iii) By part (ii) , $(C, D) \subseteq_b S_\alpha cl(C, D)$. Since $(A, B) \subseteq (C, D)$, we have $(A, B) \subseteq_b S_\alpha cl(C, D)$ but ${}_bS_\alpha cl(C, D)$ is a ${}_bS_\alpha CS$. Thus ${}_bS_\alpha cl(C, D)$ is a ${}_bS_\alpha CS$ containing (A, B). Since ${}_bS_\alpha cl(A, B)$ is the smallest ${}_bS_\alpha CS$ containing (A, B), we have ${}_bS_\alpha cl(A, B) \subseteq_b S_\alpha cl(C, D)$. Hence, $(A, B) \subseteq (C, D) \implies {}_bS_\alpha cl(A, B) \subseteq_b S_\alpha cl(C, D)$.
(iv) We know that, $(A, B) \cap (C, D) \subseteq (A, B)$ and $(A, B) \cap (C, D) \subseteq (C, D)$. By (iii), ${}_bS_\alpha cl[(A, B) \cap (C, D)] \subseteq_b S_\alpha cl(A, B)$ and ${}_bS_\alpha cl[(A, B) \cap (C, D)] \subseteq_b S_\alpha cl(C, D)$. Hence, ${}_bS_\alpha cl[(A, B) \cap (C, D)] \subseteq_b S_\alpha cl(A, B) \cap_b S_\alpha cl(C, D)$.
(v) Since $(A, B) \subseteq (A, B) \cup (C, D)$ and $(C, D) \subseteq (A, B) \cup (C, D)$.
By (iii), ${}_bS_\alpha cl(A, B) \subseteq_b S_\alpha cl[(A, B) \cup (C, D)]$ and ${}_bS_\alpha cl(C, D) \subseteq_b S_\alpha cl[(A, B) \cup (C, D)]$. Hence, ${}_bS_\alpha cl(A, B) \cup_b S_\alpha cl(C, D) \subseteq_b S_\alpha cl[(A, B) \cup (C, D)]$.
(vi) Since ${}_bS_\alpha cl(A, B)$ is a ${}_bS_\alpha CS$, by proposition 3.3, ${}_bS_\alpha cl(A, B) = (A, B)$.
Hence, ${}_bS_\alpha$-$cl({}_bS_\alpha$-$cl(A, B)) =_b S_\alpha$-$cl(A, B)$.

Proposition 3.13. *For any binary subset (A, B) of a binary topological space $(X, Y, \mathcal{M})$*
then

(i) ${}_bS_\alpha int(A,B) \subseteq_b \alpha int(A,B) \subseteq_b S_\alpha int(A,B) \subseteq_b S_\alpha cl(A,B) \subseteq_b \alpha cl(A,B) \subseteq_b cl(A,B)$

(ii) ${}_bint({}_bS_\alpha int(A,B)) = {}_bS_\alpha int({}_bint(A,B)) =_b int(A,B)$

(iii) ${}_b\alpha int({}_bS_\alpha int(A,B)) = {}_bS_\alpha int({}_b\alpha int(A,B)) =_b \alpha int(A,B)$

(iv) ${}_bcl({}_bS_\alpha cl(A,B)) = {}_bS_\alpha cl({}_bcl(A,B)) =_b cl(A,B)$

(v) ${}_b\alpha cl({}_bS_\alpha cl(A,B)) = {}_bS_\alpha cl({}_b\alpha cl(A,B)) = {}_b\alpha cl(A,B)$

(vi) ${}_bS_\alpha cl(A,B) = (A,B) \cup_b int({}_bcl({}_bint({}_bcl(A,B))))$

(vii) ${}_bS_\alpha int(A,B) = (A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B))))$

(viii) ${}_bint({}_bcl(A,B)) \subseteq_b S_\alpha int({}_bint(A,B))$

Proof. (i) is easy, so omitted. (ii) T.P ${}_bint({}_bS_\alpha int(A,B)) = {}_bS_\alpha int({}_bint(A,B)) =_b int(A,B)$
Since ${}_bint(A,B)$ is a binary open set, then ${}_bint(A,B)\ {}_bS_\alpha \cup S$.
Hence ${}_bint(A,B) = {}_bS_\alpha int({}_bint(A,B))$ by proposition 1.
Therefore

$$ {}_bint(A,B) =_b S_\alpha int({}_bint(A,B)). \tag{1} $$

Since ${}_bint(A,B) \subseteq_b S_\alpha int(A,B)$
$\Rightarrow {}_bint({}_bint(A,B)) \subseteq_b int({}_bS_\alpha int(A,B))$
$\Rightarrow {}_bint(A,B) \subseteq_b int({}_bS_\alpha int(A,B))$ also ${}_bS_\alpha int(A,B) \subseteq (A,B)$
$\Rightarrow {}_bint({}_bS_\alpha int(A,B)) \subseteq_b int(A,B)$
Hence,

$$ {}_bint(A,B) =_b int({}_bS_\alpha int(A,B)) \tag{2} $$

By 1 and 2, we get
${}_bint({}_bS_\alpha int(A,B)) = {}_bS_\alpha int({}_bint(A,B)) = {}_bint(A,B)$.
(iii) Since ${}_b\alpha int(A,B)$ is binary semi α open set by (1) ${}_b\alpha int(A,B)$ is binary semi α open set.
Therefore by proposition 1

$$ {}_b\alpha int(A,B) =_b S_\alpha int(\alpha)_bint(A,B) \tag{3} $$

Now to prove ${}_b\alpha int(A,B) = {}_b\alpha int({}_bS_\alpha int(A,B))$
Since ${}_b\alpha int(A,B) \subseteq {}_bS_\alpha int(A,B)$

$\implies {}_b\alpha int({}_b\alpha int(A,B)) \subseteq_b \alpha int({}_bS_\alpha int(A,B))$
$\implies {}_b\alpha int(A,B) \subseteq_b \alpha int({}_bS_\alpha int(A,B))$. Also, ${}_bS_\alpha int(A,B) \subseteq (A,B)$

$$ {}_b\alpha int({}_bS_\alpha int(A,B)) =_b \alpha int({}_bS_\alpha int(A,B)) \quad (4) $$

From 3 and 4, ${}_b\alpha int({}_bS_\alpha int(A,B)) =_b S_\alpha int({}_b\alpha int(A,B)) =_b \alpha int(A,B)$
(iv) We know that, ${}_bcl(A,B)$ is a binary closed set, So it is ${}_bS_\alpha CS$. Hence by proposition, we have

$$ {}_bcl(A,B) =_b S_\alpha cl({}_bcl(A,B)) \quad (5) $$

To prove: ${}_bcl(A,B) =_b cl({}_bS_\alpha cl(A,B))$. Since ${}_bS_\alpha cl(A,B) \subseteq_b cl(A,B)$ by (i). Then ${}_bcl({}_bS_\alpha cl(A,B)) \subseteq_b cl({}_bcl(A,B)) =_b cl(A,B) \implies {}_bcl({}_bS_\alpha cl(A,B)) \subseteq_b cl(A,B)$. Since, $(A,B) \subseteq_b S_\alpha cl(A,B) \subseteq_b cl({}_bS_\alpha cl(A,B))$. Then $(A,B) \subseteq_b cl({}_bS_\alpha cl(A,B))$. Hence ${}_bcl(A,B) \subseteq_b cl({}_bcl({}_bS_\alpha cl(A,B)))$ and therefore

$$ {}_bcl(A,B) =_b cl({}_bS_\alpha cl(A,B)). \quad (6) $$

Now from 5 and 6 , we get, ${}_bcl({}_bS_\alpha cl(A,B)) =_b S_\alpha cl({}_bcl(A,B))$.
Hence, ${}_bcl({}_bS_\alpha cl(A,B)) =_b S_\alpha cl({}_bcl(A,B)) =_b cl(A,B)$.
(vii) Since ${}_bS_\alpha int(A,B) \in_b S_\alpha OS$
$\implies {}_bS_\alpha int(A,B) \subseteq_b cl({}_bint({}_bcl({}_bint({}_bS_\alpha int(A,B))))) =_b cl({}_bint({}_bcl({}_bint(A,B))))$.
Hence ${}_bS_\alpha int(A,B) \subseteq_b cl({}_bint({}_bcl({}_bint(A,B))))$. Also ${}_bS_\alpha int(A,B) \subseteq (A,B)$. Then

$$ {}_bS_\alpha int(A,B) \subseteq (A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B)))). \quad (7) $$

To prove: $(A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B))))$ is a ${}_bS_\alpha OS$ contained in (A,B). It is clear that $(A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B)))) \subseteq_b cl({}_bint({}_bcl({}_bint(A,B))))$ and also ${}_bint(A,B) \subseteq_b cl({}_bint(A,B)) \implies {}_bint({}_bint(A,B)) \subseteq_b int({}_bcl({}_bint(A,B))) \implies {}_bint(A,B) \subseteq_b int({}_bcl({}_bint(A,B))) \implies {}_bcl({}_bint(A,B)) \subseteq_b cl({}_bint({}_bcl({}_bint(A,B))))$ and ${}_bint(A,B) \subseteq_b cl({}_bint(A,B)) \implies {}_bint(A,B) \subseteq_b cl({}_bint({}_bcl({}_bint(A,B))))$ and ${}_bint(A,B) \subseteq (A,B) \implies {}_bint(A,B) \subseteq (A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B))))$.
Therefore, we get
${}_bint(A,B) \subseteq (A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B)))) \subseteq_b cl({}_bint({}_bcl({}_bint(A,B))))$.
Hence $(A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B))))$ is a ${}_bS_\alpha OS$ by 1.
Also, $(A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B))))$ is contained in (A,B).
Then

$$ (A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B)))) \subseteq_b S_\alpha int(A,B) \quad (8) $$

By 7 and 8, ${}_bS_\alpha int(A,B) = (A,B) \cap_b cl({}_bint({}_bcl({}_bint(A,B))))$
(viii) To prove ${}_bint({}_bcl(A,B)) \subseteq_b S_\alpha int({}_bS_\alpha cl(A,B))$. Since ${}_bS_\alpha cl(A,B)$ is a ${}_bS_\alpha CS$. Therefore $({}_bint({}_bcl({}_bint({}_bcl({}_bS_\alpha cl(A,B)))) \subseteq_b S_\alpha cl(A,B)$.

Hence, ${}_bint({}_bcl(A,B)) \subseteq_b int({}_bcl({}_bint({}_bcl(A,B)))) \subseteq_b S_\alpha cl(A,B)$ by (iv).
Therefore, ${}_bS_\alpha int({}_bint({}_bcl(A,B))) \subseteq_b S_\alpha int({}_bS_\alpha cl(A,B))$.
Hence, ${}_bint({}_bcl(A,B)) \subseteq_b S_\alpha int({}_bS_\alpha cl(A,B))$ by (i).

4. Novelty of this paper

The Binary topological space is a recent new emerging concept in the field of topology. A new set is been introduced in this new topological space called the Binary semi α set (${}_bS_\alpha$). The interior and closure operators of the Binary semi α set (${}_bS_\alpha$) are been defined which forms the base for the set in binary topological space. This operators further helps to study and have a detailed examination of the Binary semi α set (${}_bS_\alpha$) in binary topological space.

5. Conclusion

The interior and closure operators of the Binary semi α set (${}_bS_\alpha$) is formed and its properties have been discussed in the form of propositions and remarks with suitable examples. Further this can be used in detailed study of the Binary semi α set (${}_bS_\alpha$) in Binary topological space.

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