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ON ALMOST CONTRA-SOMEWHAT FUZZY CONTINUOUS FUNCTIONS

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Abstract: In this paper, a new notion of fuzzy contra-continuity, called almost contra-somewhat fuzzy continuity between fuzzy topological spaces,is introduced. Several characterizations of these functions are obtained and it is shown that fuzzy almost contra-semi continuous functions are almost contra-somewhat fuzzy continuous functions. The conditions under which fuzzy hyper connected spaces become the fuzzy Baire spaces, fuzzy second category spaces and fuzzy almost irresolvable spaces, are obtained by means of almost contra-somewhat fuzzy continuous functions.

Keywords and Phrases: Fuzzy dense set, fuzzy regular closed set, fuzzy β-open set, Fuzzy resolvable sets, fuzzy almost contra-semi continuous function, Fuzzy hyper connected space, fuzzy Baire space.

2020 Mathematics Subject Classification: 54A40, 03E72.

1. Introduction

The concept of fuzzy sets as a new approach for modelling uncertainties was introduced by Zadeh [27] in 1965. This concept provides a natural foundation for treating mathematically the fuzzy phenomena, which exist pervasively in our real world, and for building new branches of fuzzy mathematics. Topology provided the most natural framework for the concepts of fuzzy sets to flourish. In 1968, Chang [7] introduced the notion of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.In the recent years, a considerable amount of research has been done on many types of continuity in general topology. In 1971, Gentry and Hoyle [11] introduced the class of somewhat continuous functions. The concept of contra-continuous functions which is a modification of continuity requiring inverse images of open sets to be closed rather than open, was introduced by Dontchev [8] in 1996. In 2015, Baker [5] introduced the notion of almost contra somewhat continuous functions in general topology. In 1981, Azad [2] introduced the concept of fuzzy regular open sets in fuzzy topological spaces. The notions of somewhat fuzzy continuous functions and somewhat fuzzy open functions between fuzzy topological spaces were introduced and studied by Thangaraj and Balasubramanian in [13]. The notions of fuzzy contra-continuous functions were introduced and investigated by Ekici and Kerre [9].

The aim of this paper is to introduce, a new notion of contra-fuzzy continuity called almost contra-somewhat fuzzy continuity between fuzzy topological spaces, several characterizations of these functions are obtained and it is shown that fuzzy almost contra-semi continuous functions are almost contra-somewhat fuzzy continuous functions, the conditions under which fuzzy hyper connected spaces become fuzzy Baire spaces, fuzzy second category spaces and fuzzy almost irresolvable spaces, are also obtained by means of almost contra-somewhat fuzzy continuous functions.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1. [7] *A fuzzy topology is a family τ of fuzzy sets in X, which satisfies the following conditions :*

(1) $0_X,\ 1_X \in \tau$;

(2) If $\lambda, \mu \in \tau$, *then* $\lambda \wedge \mu \in \tau$;

(3) *If* $\lambda_i \in \tau$ *for each* $i \in I$, *then* $\vee_{i=1}^{\infty}(\lambda_i) \in \tau$.

τ *is called the fuzzy topology for* X, *and the pair* (X, τ) *is an fuzzy topological space.*

Definition 2.2. [7] *Let* λ *be any fuzzy set in the fuzzy topological space* (X, T). *The fuzzy interior, the fuzzy closure and the fuzzy complement of* λ *are defined respectively as follows :*

(i) $int(\lambda) = \vee\{\mu | \mu \leq \lambda, \mu \in T\}$;

(ii) $cl(\lambda) = \wedge\{\mu | \lambda \leq \mu, 1 - \mu \in T\}$.

(iii) $\lambda'(x) = 1 - \lambda(x)$, *for all* $x \in X$.

The notions union $\psi = \vee_i(\lambda_i)$ *and intersection* $\delta = \wedge_i(\lambda_i)$, *are defined respectively, for the family* $\{(\lambda_i) | i \in I\}$ *of fuzzy sets in* (X, T) *as follows :*

(iv) $\psi(x) = \sup_i\{\lambda_i(x) | x \in X\}$

(v) $\delta(x) = \inf_i\{\lambda_i(x) | x \in X\}$.

Lemma 2.1. [2] *For a fuzzy set* λ *of a fuzzy topological space* X, $(i) 1 - int(\lambda) = cl(1 - \lambda)$ *and* $(ii) 1 - cl(\lambda) = int(1 - \lambda)$

Definition 2.3. *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called an*

(1) *fuzzy dense set if there exists no fuzzy closed set* μ *in* (X, T) *such that* $\lambda < \mu < 1$. *That is,* $cl\ (\lambda) = 1$, *in* (X, T) [13].

(2) *fuzzy nowhere dense set if there exists no non-zero fuzzy open set* μ *in* (X, T) *such that* $\mu < cl(\lambda)$. *That is,* $intcl(\lambda) = 0$, *in* (X, T) [13].

(3) *fuzzy resolvable set if for each fuzzy closed set* μ *in* (X, T), $\{cl\ (\mu \wedge \lambda) \wedge cl(\mu \wedge [1 - \lambda])\}$ *is a fuzzy nowhere dense in* (X, T) [18].

(4) *fuzzy somewhere dense set if* $intcl(\lambda) \neq 0$ *for a fuzzy set* λ *defined on* X *in* (X, T) *and the fuzzy complement of fuzzy somewhere dense set in* (X, T) *is called a fuzzy cs dense set in* (X, T) [23].

(5) *fuzzy* σ*-boundary set if* $\lambda = \wedge_{i=1}^{\infty}(\mu_i)$, *where* $\mu_i = cl\ (\lambda_i) \wedge (1 - \lambda_i)$ *and* $(\lambda_i)'s$ *are fuzzy regular open sets in* (X, T) [19] .

(6) *fuzzy regular-open if* $\lambda = int\ cl(\lambda)$ *and fuzzy regular-closed if* $\lambda = cl\ int(\lambda)$ [2] .

(7) *fuzzy semi-open if* $\lambda \leq cl\ int(\lambda)$ *and fuzzy semi-closed if* $int\ cl(\lambda) \leq \lambda$ [2].

(8) *fuzzy* β*-open if* $\lambda \leq cl\ int\ cl(\lambda)$ *and fuzzy* β*-closed if* $int\ cl\ int(\lambda) \leq \lambda$ [4].

(9) *fuzzy pre-open if* $\lambda \leq int\ cl(\lambda)$ *and fuzzy pre-closed if* $cl\ int(\lambda) \leq \lambda$ [6].

(10) *fuzzy* G_δ *- set if* $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$*, where* $\lambda_i \in T$ *for* $i \in I$ [3].

(11) *fuzzy* F_σ *- set if* $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$*, where* $1 - \lambda_i \in T$ *for* $i \in I$ [3].

Definition 2.4. *Let* (X, T) *be a fuzzy topological space and* (X, T) *is called a*

(1) *fuzzy Baire space if* $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$*, where* $(\lambda_i)'s$ *are fuzzy nowhere dense sets in* (X, T) [15].

(2) *fuzzy hyper connected space if every non - null fuzzy open subset of* (X, T) *is fuzzy dense in* (X, T) [12].

(3) *fuzzy almost resolvable space if* $\vee_{i=1}^{\infty}(\lambda_i) = 1_X$,*, where the fuzzy sets* $(\lambda_i)'s$ *in* (X, T) *are such that int* $(\lambda_i) = 0$*. Otherwise* (X, T) *is called a fuzzy almost irresolvable space* [25] .

(4) *fuzzy open hereditarily irresolvable space if* $int\ cl(\lambda) \neq 0$*, for any non- zero fuzzy set* λ *defined on* X*, then* $int(\lambda) \neq 0$*, in* (X, T) [14].

(5) *fuzzy nodec space if each non-zero fuzzy nowhere dense set is fuzzy closed in* (X, T) [16] .

(6) *fuzzy resolvable space if there exists fuzzy dense set* λ *in* (X, T) *such that* $cl(1 - \lambda) = 1$*. Otherwise* (X, T) *is called the fuzzy irresolvable space* [14].

(7) *fuzzy perfectly disconnected space if for any two non - zero fuzzy sets* λ *and* μ *defined on* X *with* $\lambda \leq 1 - \mu$*,* $cl(\lambda) \leq 1 - cl(\mu)$*, in* (X, T) [21].

(8) *fuzzy nodef space if each fuzzy nowhere dense set is a fuzzy* F_σ *- set in* (X, T) [20].

(9) *weak fuzzy P-space if* $\wedge_{i=1}^{\infty}(\lambda_i)$ *is a fuzzy regular open set in* (X, T)*, where* $(\lambda_i)'s$ *are fuzzy regular open sets in* (X, T) [26].

Definition 2.5. [13] *Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called a somewhat fuzzy continuous function if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$, there exists a non-zero fuzzy open set μ in (X, T) such that $\mu \leq f^{-1}(\lambda)$. That is, $int[f^{-1}(\lambda)] \neq 0$, in (X, T).*

Definition 2.6. [9] *Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called a fuzzy contra- somewhat continuous function if for each open set λ in (Y, S) such that $f^{-1}(\lambda) \neq 0$, there exists a non-zero fuzzy closed set μ in (X, T) such that $\mu \leq f^{-1}(\lambda)$.*

Definition 2.7. [10] *Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function $f : (X, T) \to (Y, S)$ is called a fuzzy almost contra-semi continuous function if the inverse image of each fuzzy regular open set in Y is fuzzy semi-closed in X.*

Theorem 2.1. [2] *In a fuzzy space X,*

(a) The closure of a fuzzy open set is a fuzzy regular closed set.

(b) The interior of a fuzzy closed set is a fuzzy regular open set.

Theorem 2.2. [24] *If λ is a fuzzy somewhere dense set in the fuzzy hyperconnected space (X, T), then λ is a fuzzy dense set in (X, T).*

Theorem 2.3. [24] *If the fuzzy set λ is a fuzzy cs dense set in the fuzzyhyper connected space (X, T), then $int(\lambda) = 0$, in (X, T).*

Theorem 2.4. [24] *If λ is a fuzzy somewhere dense set in the fuzzy hyperconnected space (X, T), then λ is a fuzzy β-open set in (X, T).*

Theorem 2.5. [17] *If λ is a non-zero fuzzy β -open set in the fuzzy hyperconnected space (X, T), then there exists an fuzzy resolvable set μ in (X, T) such that $\mu \leq cl(\lambda)$.*

Theorem 2.6. [1] *For any fuzzy set λ in a fuzzy topological space X, if λ is fuzzy semi- pre open, then $cl(\lambda)$ is fuzzy regularly closed.*

Theorem 2.7. [24] *If λ is a fuzzy somewhere dense set in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space, then the fuzzy set $1 - \lambda$ is a fuzzy nowhere dense set in (X, T).*

Theorem 2.8. [23] *If λ is a fuzzy somewhere dense set in the fuzzy topological space (X, T), then there exist a fuzzy regular closed set η in (X, T) such that $\eta \leq cl(\lambda)$.*

Theorem 2.9. [24] *If λ is a fuzzy somewhere dense set in the fuzzy hyperconnected* *space* (X, T), *then* $int(1 - \lambda) = 0$ *in* (X, T).

Theorem 2.10. [24] *If* $cl[\wedge_{i=1}^{\infty}(\lambda_i)] = 1$, *where* $(\lambda_i)'s$ *are fuzzy somewhere dense sets in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space* (X, T), *then* (X, T) *is an fuzzy Baire space.*

Theorem 2.11. [16] *If* (X, T) *is the fuzzy Baire space, then* (X, T) *is the fuzzy second category space.*

Theorem 2.12. [24] *If* $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, *where* $(\lambda_i)'s$ *are fuzzy cs dense sets in the fuzzy hyperconnected space* (X, T), *then* (X, T) *is the fuzzy almost irresolvable space.*

Theorem 2.13. [17] *If* $int[\vee_{i=1}^{\infty}(\lambda_i)] = 0$, *where* $(\lambda_i)'s$ *are fuzzy resolvable sets in a fuzzy topological space* (X, T), *then* (X, T) *is a fuzzy Baire space.*

Theorem 2.14. [22] *If* λ *is a fuzzy pre-closed set in a fuzzy perfectly disconnected space* (X, T), *then* $int(\lambda)$ *is a fuzzy regular closed set in* (X, T).

Theorem 2.15. [22] *If* λ *is a fuzzy semi-closed set in a fuzzy perfectly disconnected space* (X, T), *then* λ *is a fuzzy pre-closed set in* (X, T).

Theorem 2.16. [22] *If* $(\lambda_i)'s$ $(i = 1$ *to* $\infty)$ *are fuzzy somewhere dense sets in a fuzzy perfectly disconnected space* (X, T), *then there exists a fuzzy* G_δ *-set* η *in* (X, T) *such that* $\eta \leq \wedge_{i=1}^{\infty}[cl(\lambda_i)]$.

Theorem 2.17. [19] *If* λ *is a fuzzy* σ*-boundary set in the fuzzy topological space* (X, T), *then* $\lambda = \vee_{i=1}^{\infty}(\mu_i)$, *where* $(\mu_i)'s$ *are fuzzy pre-closed sets in* (X, T).

Theorem 2.18. [24] *A fuzzy topological space* (X, T) *is a weak fuzzy P-space if and only if each fuzzy regular* F_σ *-set is a fuzzy regular closed set in* (X, T).

3. Almost Contra-somewhat Fuzzy Continuous Functions

Definition 3.1. *Let* (X, T) *and* (Y, S) *be any two fuzzy topological spaces. A function* $f : (X, T) \to (Y, S)$ *is called a almost contra - somewhat fuzzy continuous function if for each fuzzy regular closed set* μ *in* (Y, S) *such that* $f^{-1}(\mu) \neq 0$, *there exists a non-zero fuzzy open set* λ *in* (X, T) *such that* $\lambda \leq f^{-1}(\mu)$.

That is, $f : (X, T) \to (Y, S)$ *is a almost contra - somewhat fuzzy continuous function if* $int[f^{-1}(\mu)] \neq 0$ *in* (X, T), *for a fuzzy regular closed set* μ *such that* $f^{-1}(\mu) \neq 0$ *in* (Y, S).

Example 3.1. Let $X = \{a, b, c\}$. Consider the fuzzy sets P, Q, R, E, F and G defined on X as follows :

$P : X \to [0, 1]$ is defined as $P(a) = 0.6; P(b) = 0.4; P(c) = 0.5$,

$Q : X \to [0, 1]$ is defined as $Q(a) = 0.4; Q(b) = 0.7; Q(c) = 0.6,$

$R : X \to [0, 1]$ is defined as $R(a) = 0.5; R(b) = 0.3; R(c) = 0.7,$

$E : X \to [0, 1]$ is defined as $E(a) = 0.5; E(b) = 0.6; E(c) = 0.7,$

$F : X \to [0, 1]$ is defined as $F(a) = 0.8; F(b) = 0.4; F(c) = 0.2,$

$G : X \to [0, 1]$ is defined as $G(a) = 0.7; G(b) = 0.5; G(c) = 0.8.$

Then $T = \{0, P, Q, R, P\vee Q, P\vee R, Q\vee R, P\wedge Q, P\wedge R, Q\wedge R, [P\wedge(Q\vee R)], [Q\wedge(P\vee R)], [R\wedge(P\vee Q)], [P\vee(Q\wedge R)], [Q\vee(P\wedge R)], [R\vee(P\wedge Q)], [P\wedge Q\wedge R], [P\vee Q\vee R], 1\}$ and $S = \{0, E, F, G, E\vee F, E\vee G, F\vee G, E\wedge F, E\wedge G, F\wedge G, [E\vee(F\wedge G)], [F\vee(E\wedge G)], [G\wedge(E\vee F)], [E\vee(F\vee G)], 1\}$ are fuzzy topologies on X. On computation $(1 - E)$ and $[1 - (E \wedge F)]$ are fuzzy regular closed sets in (X, S). Now define a function $f : (X, T) \to (X, S)$ by $f(a) = b; f(b) = c; f(c) = a$. On computation, for the fuzzy regular closed set $1-E$ in (X, S), there exists a fuzzy open set $P\wedge Q\wedge R$ in (X, T) such that $P \wedge Q \wedge R \leq f^{-1}(1 - E)$ and for the fuzzy regular closed set $[1 - (E \wedge F)]$ in (X, S), there exists a fuzzy open set E in (X, T) such that $E \leq f^{-1}[1 - (E \wedge F)]$. Hence $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from (X, T) into (X, S).

Remark 3.1. *It is to be noted that almost contra- somewhat fuzzy continuous function between topological spaces need not be a somewhat fuzzy continuous function. For, in example 3.1, for the fuzzy open set μ in (X, S), there is no fuzzy open set η in (X, T) such that $\eta \leq f^{-1}(\mu)$.*

Proposition 3.1. *For the function $f : (X, T) \to (Y, S)$ from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S), the following are equivalent :*

(1) $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function.

(2) for each fuzzy regular open set λ such that $f^{-1}(\lambda) \leq 1$, in (Y, S), there exits a fuzzy closed set $\mu(\neq 1)$ in (X, T) such that $f^{-1}(\lambda) < \mu$.

Proof. (1) $\Rightarrow$ (2). Let λ be the fuzzy regular open set such that $f^{-1}(\lambda) \neq 1$, in (Y, S). Then, $1-\lambda$ is a fuzzy regular closed set in (Y, S) such that $f^{-1}(1-\lambda) \neq 0$, in (X, T). Since $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function, $int[f^{-1}(1 - \lambda)] \neq 0$. Then, $1 - int[f^{-1}(1 - \lambda)] \neq 1$ and by the lemma 2.1, $cl[1 - f^{-1}(1 - \lambda)] \neq 1$. This implies that $cl[f^{-1}(\lambda)] \neq 1$, in (X, T). Then, there exists a fuzzy closed set $\mu(\neq 1)$ in (X, T) such that $f^{-1}(\lambda) < \mu$.

(2) $\Rightarrow$ (1). Let η be a non-zero fuzzy regular closed set in (Y, S) such that $f^{-1}(\eta) \neq 0$. Then, $1-\eta$ is a fuzzy regular open set in (Y, S) such that $f^{-1}(1-\eta) \neq 1$.

Then, by (2), there exists a fuzzy closed set $\mu(\neq 1)$ in (X, T) such that $f^{-1}(1-\eta) < \mu$. This implies that $1 - f^{-1}(\eta) \leq \mu$ and then $1 - \mu \leq f^{-1}(\eta)$. Since μ is a fuzzy closed set in (X, T), $1-\mu$ is a fuzzy open set in (X, T). Let $\delta = 1-\mu$. Then, $\delta \neq 0$ in (X, T). Thus, for the non- zero fuzzy regular closed set η in (Y, S), there exists a fuzzy open set $\delta \neq 0$ in (X, T) such that $\delta \leq f^{-1}(\eta)$. Hence $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from (X, T) into (Y, S).

Proposition 3.2. *If $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and μ is a fuzzy regular closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$, then there exists a fuzzy regular closed set $\delta \neq 1$, in (X, T) such that $\delta < cl[f^{-1}(\mu)]$.*

Proposition 3.3. *If $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and μ is a fuzzy regular closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$, then $f^{-1}(1 - \mu)$ is not a fuzzy dense set in (X, T).*

Proposition 3.4. *If $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and μ is a fuzzy regular closed in (Y, S) such that $f^{-1}(\mu) \neq 0$, then $f^{-1}(\mu)$ is a fuzzy somewhere dense set in (X, T).*

Proposition 3.5. *If $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and is a fuzzy regular open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$, then $f^{-1}(\lambda)$ is a fuzzy cs dense set in (X, T).*

Proposition 3.6. *If $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy hyper connected space (X, T) into the fuzzy topological space (Y, S) and μ is a fuzzy regular closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$, then $f^{-1}(\mu)$ is a fuzzy dense set in (X, T).*

Proposition 3.7. *If $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy hyper connected space (X, T) into the fuzzy topological space (Y, S) and λ is a fuzzy regular open set in (Y, S) such that $f^{-1}(\lambda) \neq 1$, then $int[f^{-1}(\lambda)] = 0$, in (X, T).*

Proposition 3.8. *If $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy hyper connected space (X, T) into the fuzzy topological space (Y, S) and μ is a fuzzy regular closed in (Y, S) such that $f^{-1}(\mu) \neq 0$, then $f^{-1}(\mu)$ is a fuzzy β - open set in (X, T).*

Proof. Let μ be a fuzzy regular closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$. Since

$f : (X,T) \to (Y,S)$ is the almost contra -somewhat fuzzy continuous function, by the proposition 3.4, $f^{-1}(\mu)$ is a fuzzy somewhere dense set in (X,T). Since (X,T) is a fuzzy hyperconnected space, by the theorem 2.4, $f^{-1}(\mu)$ is a fuzzy β - open set in (X,T).

Proposition 3.9. *If $f : (X,T) \to (Y,S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy hyper connected space (X,T) into the fuzzy topological space (Y,S) and μ is a fuzzy regular closed in (Y,S) such that $f^{-1}(\mu) \neq 0$, then there exists a fuzzy resolvable set δ in (X,T) such that $\delta \leq cl[f^{-1}(\lambda)]$.*

The following proposition shows that fuzzy almost contra-semi continuous functions are almost contra- somewhat fuzzy continuous functions.

Proposition 3.10. *If $f : (X,T) \to (Y,S)$ is the fuzzy almost contra-semi continuous function from the fuzzy topological space (X,T) into the fuzzy topological space (Y,S), then $(X,T) \to (Y,S)$ is the almost contra- somewhat fuzzy continuous function from (X,T) into (Y,S).*
Proof. Let μ be a fuzzy regular closed in (Y,S) such that $f^{-1}(\mu) \neq 0$. Then, $1-\mu$ is a fuzzy regular open set in (Y,S). Since $f : (X,T) \to (Y,S)$ is the fuzzy almost contra-semi continuous function from (X,T) into (Y,S), $f^{-1}(1-\mu)$ is a fuzzy semi-closed set in (X,T). Now $f^{-1}(1-\mu) = 1 - f^{-1}(\mu)$, implies that $1 - f^{-1}(\mu)$, is a fuzzy semi-closed set in (X,T) and thus $f^{-1}(\mu)$ is a fuzzy semi-open set in (X,T). Then, $f^{-1}(\mu) \leq cl\ int[f^{-1}(\mu)]$ in (X,T). This implies that $int[f^{-1}(\mu)] \neq 0$, in (X,T). Hence $(X,T) \to (Y,S)$ is the almost contra- somewhat fuzzy continuous function from (X,T) into (Y,S).

Proposition 3.11. *If $f : (X,T) \to (Y,S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy topological space (X,T) into the fuzzy topological space (Y,S) and λ is fuzzy β -open set (fuzzy semi-preopen set) in (Y,S) then $int[f^{-1}(cl(\lambda))] \neq 0$, in (X,T).*

The following proposition gives the condition for fuzzy almost semi- continuous functions to become the almost contra- somewhat fuzzy continuous functions.

Proposition 3.12. *If $f : (X,T) \to (Y,S)$ is the fuzzy almost semi- continuous function from the fuzzy topological space (X,T) in which there are no fuzzy open and fuzzy dense sets into the fuzzy topological space (Y,S), then $f : (X,T) \to (Y,S)$ is the almost contra- somewhat fuzzy continuous function from (X,T) into (Y,S).*
Proof. Let μ be a fuzzy regular open set in (Y,S) such that $f^{-1}(\mu) \neq 1$. Since $f : (X,T) \to (Y,S)$ is the fuzzy almost semi- continuous function from (X,T) into (Y,S), $f^{-1}(\mu)$ is a fuzzy semi-open set in (X,T). Then $f^{-1}(\mu) \leq cl\ int[f^{-1}(\mu)]$ in (X,T). Since $int[f^{-1}(\mu)]$ is a fuzzy open set in (X,T), by the hypothesis,

$int[f^{-1}(\mu)]$ is not a fuzzy dense set in (X, T). Then, $f^{-1}(\mu) \leq cl\ int[f^{-1}(\mu)] \neq 1$. Let $\delta = cl\ int[f^{-1}(\mu)]$. Thus, for the fuzzy regular open set μ such that $f^{-1}(\mu) \neq 1$, in (Y, S), there exits a fuzzy closed set $\delta(\neq 1)$ in (X, T) such that $f^{-1}(\mu) < \delta$ and thus by the proposition 3.1 , $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from (X, T) Into (Y, S).

The following proposition shows that fuzzy contra - continuous functions are almost contra- somewhat fuzzy continuous functions.

Proposition 3.13. *If $f : (X, T) \to (Y, S)$ is the fuzzy contra - continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S), then $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from (X, T) into (Y, S).*

Proof. Let μ be a fuzzy regular closed in (Y, S) such that $f^{-1}(\mu) \neq 0$. Then, μ is a fuzzy closed set in (Y, S). Since $f : (X, T) \to (Y, S)$ is the fuzzy contra-continuous function from (X, T) into (Y, S), $f^{-1}(\mu)$ is a fuzzy open set in (X, T) and thus $int[f^{-1}(\mu)] = f^{-1}(\mu) \neq 0$. Hence $f : (X, T) \to (Y, S)$ is the almost contra-somewhat fuzzy continuous function from (X, T) into (Y, S).

The following proposition gives the condition for fuzzy functions between fuzzy topological spaces to become almost contra- somewhat fuzzy continuous functions.

Proposition 3.14. *If the function $f : (X, T) \to (X, T)$ has the property that $f(\mu) \leq \mu$, for each fuzzy closed set μ in (X, T), then $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from (X, T) into (Y, S).*

Proof. Let μ be a fuzzy regular closed in (Y, S) such that $f^{-1}(\mu) \neq 0$. Now, $f(\mu) \leq \mu$ implies that $f^{-1}f(\mu) \leq f^{-1}(\mu)$ and $\mu \leq f^{-1}f(\mu) \leq f^{-1}(\mu)$. Then $int(\mu) \leq int[f^{-1}(\mu)]$, in (X, T). Since μ is a fuzzy regular closed in (Y, S), $cl\ int(\mu) = \mu$, implies that $int(\mu) \neq 0$. Thus, $int[f^{-1}(\mu)] \neq 0$, in (X, T). Hence $f : (X, T) \to (Y, S)$ is the almost contra-somewhat fuzzy continuous function from (X, T) into (Y, S).

Proposition 3.15. *If $f : (X, T) \to (Y, S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S), then $f : (X, T) \to (Y, S)$ is not the somewhat fuzzy continuous function from (X, T) into (Y, S).*

Proposition 3.16. *If $f : (X, T) \to (Y, S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S) and μ is a fuzzy regular closed in (Y, S), then $f^{-1}(\mu)$ is a fuzzy dense set in (X, T).*

Proposition 3.17. *If $f : (X, T) \to (Y, S)$ is the almost contra-somewhat fuzzy* *continuous function from the fuzzy hyperconnected space (X,T) into the fuzzy topological space (Y,S) and μ is a fuzzy regular closed in (Y,S), $f^{-1}(\lambda)$ is a fuzzy β -open set in (X,T).*

Proposition 3.18. *If $f:(X,T)\to(Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space (X,T) into the fuzzy topological space (Y,S) and μ is a fuzzy regular closed in (Y,S), then $f^{-1}(1-\mu)$ is a fuzzy nowhere dense set in (X,T).*

Proof. Let μ be a fuzzy regular closed in (Y,S) such that $f^{-1}(\mu)\neq 0$. Since $f:(X,T)\to(Y,S)$ is the almost contra- somewhat fuzzy continuous function from (X,T) into (Y,S), by the proposition 3.4, $f^{-1}(\lambda)$ is a fuzzy somewhere dense set in (X,T). Since (X,T) is the fuzzy hyperconnected space and fuzzy open hereditarily irresolvable space , by the theorem 2.7, for the fuzzy somewhere dense set $f^{-1}(\mu)$, $[1-f^{-1}(\mu)]$ is a fuzzy nowhere dense set in (X,T). Because $f^{-1}(1-\mu)=1-f^{-1}(\mu)$, $f^{-1}(1-\mu)$ is a fuzzy nowhere dense set in (X,T).

Proposition 3.19. *If $f:(X,T)\to(Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected , fuzzy open hereditarily irresolvable and fuzzy nodec space (X,T) into the fuzzy topological space (Y,S) and μ is a fuzzy regular closed set in (Y,S), then $f^{-1}(\mu)$ is a fuzzy open set in (X,T).*

Proposition 3.20. *If $f:(X,T)\to(Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X,T) into the fuzzy topological space (Y,S) and μ is a fuzzy regular closed set in (Y,S), then $int[f^{-1}(1-\mu)]=0$, in (X,T).*

Proposition 3.21. *If $f:(X,T)\to(Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy topological space (X,T) into the fuzzy perfectly disconnected space (Y,S) and μ is a fuzzy pre- closed in (Y,S) such that $f^{-1}(\mu)\neq 0$, then $int[f^{-1}(\mu)]\neq 0$, in (X,T).*

Proposition 3.22. *If $f:(X,T)\to(Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyper connected space (X,T) into the fuzzy perfectly disconnected space (Y,S) and μ is a fuzzy pre- closed set in (Y,S) such that $f^{-1}(\mu)\neq 0$, then $f^{-1}(\mu)$ is a fuzzy dense set in (X,T).*

Proposition 3.23. *If $f:(X,T)\to(Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyper connected space (X,T) into the fuzzy perfectly disconnected space (Y,S) and μ is a fuzzy semi- closed set in (Y,S) such that $f^{-1}(\mu)\neq 0$, then $f^{-1}(\mu)$ is a fuzzy dense set in (X,T).*

Proposition 3.24. *If $f:(X,T)\to(Y,S)$ is the almost contra- somewhat fuzzy* *continuous function from the fuzzy perfectly disconnected space (X,T) into the fuzzy topological space (Y,S) and $(\mu_i)'s(i = 1$ to $\infty)$ are fuzzy regular closed sets in (Y,S) such that $f^{-1}(\mu_i) \neq 0$, then there exists a fuzzy G_δ - set η in (X,T) such that $\eta \leq \wedge_{i=1}^{\infty}[cl\{f^{-1}(\mu_i)\}]$.*

Proposition 3.25. *If $f : (X,T) \to (Y,S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy perfectly disconnected space (X,T) into the fuzzy topological space (Y,S) and $(\mu_i)'s(i = 1$ to $\infty)$ are fuzzy regular closed sets in (Y,S) such that $f^{-1}(\mu_i) \neq 0$, then there exists a fuzzy F_σ - set δ in (X,T) such that $\vee_{i=1}^{\infty}[int(f^{-1}(1-\mu_i))] \leq \delta$.*

Proposition 3.26. *If $f : (X,T) \to (Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyper connected space (X,T) into the fuzzy perfectly disconnected space (Y,S) and is a fuzzy σ - boundary set in (Y,S), then $f^{-1}(\lambda) = \vee_{i=1}^{\infty} f^{-1}(\mu_i)$, where $[f^{-1}(\mu_i)]'s$ are fuzzy dense sets in (X,T).*

Proof. Let λ be a fuzzy σ - boundary set in (Y,S). Then, by the theorem 2.17, $\lambda = \vee_{i=1}^{\infty}(\mu_i)$, where (μ_i)'s are fuzzy pre-closed sets in (Y,S). Since $f : (X,T) \to (Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyper connected space (X,T) into the fuzzy perfectly disconnected space (Y,S), by the proposition 3.22, $f^{-1}(\mu_i)$ are fuzzy dense sets in (X,T). Now $\lambda = \vee_{i=1}^{\infty}(\mu_i)$, implies that $f^{-1}(\lambda) = f^{-1}[\vee_{i=1}^{\infty}(\mu_i)] = [\vee_{i=1}^{\infty} f^{-1}(\mu_i)]$ and hence $f^{-1}(\lambda) = \vee_{i=1}^{\infty}[f^{-1}(\mu_i)]$, where $[f^{-1}(\mu_i)]'$s are fuzzy dense sets in (X,T).

Proposition 3.27. *If $f : (X,T) \to (Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected fuzzy open hereditarily irresolvable and fuzzy nodef space (X,T) into the fuzzy topological space (Y,S) and μ is a fuzzy regular closed in (Y,S), then*

(1) $f^{-1}(1-\mu)$ is a fuzzy F_σ - set in (X,T).

(2) $f^{-1}(\mu)$ is a fuzzy G_δ - set in (X,T).

Proposition 3.28. *If $f : (X,T) \to (Y,S)$ is the almost contra - somewhat fuzzy continuous function from the fuzzy topological space (X,T) into the weak fuzzy P-space (Y,S) and μ is a fuzzy regular F_σ - set in (Y,S), then $int[f^{-1}(\mu)] \neq 0$, in (X,T).*

Proof. Let μ be a fuzzy regular F_σ - set in (Y,S). Since (Y,S) is the weak fuzzy P- space , by the theorem 2.18, μ is a fuzzy regular closed set in (Y,S). Since $f : (X,T) \to (Y,S)$ is the almost contra - somewhat fuzzy continuous function from (X,T) into (Y,S), $int[f^{-1}(\mu)] \neq 0$, in (X,T).

4. Applications of Almost Contra-somewhat Fuzzy Continuous Functions

The following proposition gives the condition for the fuzzy hyperconnected and fuzzy open hereditarily irresolvable spaces to become fuzzy Baire spaces by means of fuzzy regular open sets.

Proposition 4.1. *If $f : (X,T) \to (Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected and fuzzy open hereditarily irresolvable space (X,T) into the fuzzy topological space (Y,S) and $int((\vee_{i=1}^{\infty} f^{-1}(\delta_i)) = 0$, where $(\delta_i)'s$ are fuzzy regular open sets in (Y,S) such that $f^{-1}(\delta_i) \neq 1$, then (X,T) is a fuzzy Baire space.*

Proof. Let $(\delta_i)'$s ($i = 1$ *to* ∞) be fuzzy regular open sets in (Y,S) such that $f^{-1}(\delta_i) \neq 0$. Then, $(1 - \delta_i)'$s are fuzzy regular closed sets in (Y,S) such that $f^{-1}(1-\delta_i) \neq 0$. Since $f : (X,T) \to (Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X,T) into the fuzzy topological space (Y,S), by the proposition 3.4, $[f^{-1}(1-\delta_i)]'$s are fuzzy somewhere dense sets in (X,T). From the hypothesis, $int((\vee_{i=1}^{\infty} f^{-1}(\delta_i)) = 0, int(f^{-1}(\vee_{i=1}^{\infty}(\delta_i)) = 0$, and this implies that $1 - int(f^{-1}(\vee_{i=1}^{\infty}(\delta_i)) = 1 - 0 = 1$ and $cl(1 - f^{-1}(\vee_{i=1}^{\infty}(\delta_i) = 1$ and $cl[f^{-1}(\wedge_{i=1}^{\infty}(1-\delta_i)] = 1$. Hence $cl[\wedge_{i=1}^{\infty} f^{-1}(1-\delta_i)] = 1$, where $[f^{-1}(1-\delta_i)]'$s are fuzzy somewhere dense sets in the fuzzy hyper connected and fuzzy open hereditarily irresolvable space (X,T). Then, by the theorem 2.10 , (X,T) is a fuzzy Baire space .

The following proposition gives the condition for the fuzzy hyperconnected spaces to become fuzzy Baire spaces by means of fuzzy resolvable sets.

Proposition 4.2. *If $f : (X,T) \to (Y,S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy hyper connected space (X,T) into the fuzzy topological space (Y,S) and $int[\vee_{i=1}^{\infty} cl\{f^{-1}(\mu_i)\}] = 0$, where $(\mu_i)'s$ are fuzzy regular closed sets in (Y,S) such that $f^{-1}(\mu_i) \neq 0$, then (X,T) is the fuzzy Baire space.*

Proof. Let $(\mu_i)'$s ($i = 1$ *to* ∞) be fuzzy regular closed sets in (Y,S) such that $f^{-1}((\mu_i) \neq 0$. Since $f : (X,T) \to (Y,S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy hyper connected space (X,T) into the fuzzy topological space (Y,S), by the proposition 3.9, for the fuzzy regular closed sets in (Y,S), there exist fuzzy resolvable sets $(\delta_i)'$s in (X,T) such that $\delta_i \leq cl[f^{-1}(\mu_i)]$. Then, $int[\vee_{i=1}^{\infty}(\delta_i)] \leq int[\vee_{i=1}^{\infty} cl\{f^{-1}(\mu_i)\}]$. By the hypothesis, $int[\vee_{i=1}^{\infty} cl\{f^{-1}(\mu_i)\}] = 0$ and thus, $int[\vee_{i=1}^{\infty}(\delta_i)] = 0$, where $(\delta_i)'$s are fuzzy resolvable sets in (X,T). Hence, by the theorem 2.13 , (X,T) is the fuzzy Baire space.

Proposition 4.3. *If $f : (X,T) \to (Y,S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected and fuzzy open hereditarily irresolv-*

able space (X, T) into the fuzzy topological space (Y, S) and $int((\vee_{i=1}^{\infty} f^{-1}(\delta_i)) = 0$, where $(\delta_i)'s$ are fuzzy regular open sets in (Y, S) such that $f^{-1}(\delta_i) \neq 1$, then (X, T) is a fuzzy second category space.

Proof. The proof follows from the proposition 4.1 and the theorem 2.11.

Proposition 4.4. *If $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy hyper connected space (X, T) into the fuzzy topological space (Y, S) and $int[\vee_{i=1}^{\infty} cl\{f^{-1}(\mu_i)\}] = 0$, where $(\mu_i)'s$ are fuzzy regular closed sets in (Y, S) such that $f^{-1}(\mu_i) \neq 0$, then (X, T) is the fuzzy second category space.*

Proof. The proof follows from the proposition 4.2 and the theorem 2.11.

The following proposition gives the condition for the fuzzy hyperconnected spaces to become fuzzy almost irresolvable spaces.

Proposition 4.5. *If $f : (X, T) \to (Y, S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S) and $\wedge_{i=1}^{\infty}[f^{-1}(\mu_i)] \neq 0$, where $(\mu_i)'s$ are fuzzy regular closed sets in (X, T) such that $f^{-1}(\mu_i) \neq 0$, then (X, T) is the fuzzy almost irresolvable space.*

Proof. Let $(\mu_i)'$s ($i = 1$ *to* ∞) be fuzzy regular closed sets in (Y, S) such that $f^{-1}((\mu_i) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S), by the proposition 3.4, $[f^{-1}(\mu_i)]'$s are fuzzy somewhere dense sets in (X, T). Then, $[1 - f^{-1}(\mu_i)]'$s are fuzzy cs dense sets in (X, T). Now the hypothesis, $\wedge_{i=1}^{\infty}[f^{-1}(\mu_i)] \neq 0$, implies that $1 - \wedge_{i=1}^{\infty}[f^{-1}(\mu_i)] \neq 1$ and then $\vee_{i=1}^{\infty}[1 - f^{-1}(\mu_i)] \neq 1$ and $\vee_{i=1}^{\infty}[f^{-1}(1 - \mu_i)] \neq 1$, where $[f^{-1}(1 - \mu_i)]'$s are fuzzy cs dense sets in (X, T). Then, by the theorem 2.12, (X, T) is the fuzzy almost irresolvable space.

Proposition 4.6. *If $f : (X, T) \to (Y, S)$ is the almost contra-somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S), then (X, T) is a fuzzy irresolvable space.*

Proof. Let μ be a fuzzy regular closed set in (Y, S) such that $f^{-1}(\mu) \neq 0$. Since $f : (X, T) \to (Y, S)$ is the almost contra- somewhat fuzzy continuous function from the fuzzy hyperconnected space (X, T) into the fuzzy topological space (Y, S), by the proposition 3.6, $f^{-1}(\mu)$ is a fuzzy dense set in (X, T). Suppose that $1 - f^{-1}(\mu)$ is a fuzzy dense set in (X, T) and then $f^{-1}(1 - \mu)$ is a fuzzy dense set in (X, T), a contradiction, by the proposition 3.3. Thus, $1 - f^{-1}(\mu)$ is not a fuzzy dense set in (X, T) and hence the fuzzy hyperconnected space (X, T) is a fuzzy irresolvable space.

5. Conclusion

In this paper, a new notion of contra- fuzzy continuity, called almost contra -somewhat fuzzy continuity between fuzzy topological spaces, is introduced. Several characterizations of these functions are obtained and it is shown that fuzzy almost contra-semi continuous functions are almost contra- somewhat fuzzy continuous functions. The condition for fuzzy almost semi- continuous functions to become the almost contra- somewhat fuzzy continuous functions is also obtained. It is shown that fuzzy contra - continuous functions are almost contra- somewhat fuzzy continuous functions. The condition for the fuzzy hyperconnected and fuzzy open hereditarily irresolvable spaces to become fuzzy Baire spaces and fuzzy hyper connected spaces to become fuzzy second category spaces and fuzzy almost irresolvable spaces, are obtained by means of almost contra- somewhat fuzzy continuous functions.

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