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FUZZY PRE β-COMPACT SPACE

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Abstract: This paper deals with a new type of compactness, viz., fuzzy pre β-compactness by using fuzzy pre β-open set [1] as a basic tool. We characterize this newly defined compactness by fuzzy net and prefilterbase. It is shown that this compactness implies fuzzy almost compactness [3] and the converse is true only on fuzzy pre β-regular space [1]. Afterwards, it is shown that this compactness remains invariant under fuzzy pre β-irresolute function [1].

Keywords and Phrases: Fuzzy pre β-open set, fuzzy pre β-regular space, fuzzy regularly pre β-closed set, fuzzy pre β-compact set (space), pre β-adherent point of a prefilterbase, pre β-cluster point of a fuzzy net.

2020 Mathematics Subject Classification: 54A40, 03E72.

1. Introduction

After introducing fuzzy compactness by Chang [2], many mathematicians have engaged themselves to introduce different types of fuzzy compactness. In [3], fuzzy almost compactness is introduced. In this paper we introduce fuzzy pre β-compactness which is weaker than fuzzy almost compactness. Here we use fuzzy net [8] and prefilterbase [6] to characterize fuzzy pre β-compactness.

2. Preliminaries

Throughout this paper, (X,τ) or simply by X we shall mean an fts. In 1965, L.A. Zadeh introduced fuzzy set [9] A which is a function from a non-empty set X

into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support [9] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [9] of a fuzzy set A in an fts X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [9] while AqB means A is quasi-coincident (q-coincident, for short) [8] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy set A, clA and $intA$ will stand for fuzzy closure [2] and fuzzy interior [2] of A respectively. A fuzzy set A in X is called a fuzzy neighbourhood (fuzzy nbd, for short) [8] of a fuzzy point x_t if there exists a fuzzy open set G in X such that $x_t \in G \leq A$. If, in addition, A is fuzzy open, then A is called fuzzy open nbd of x_t. A fuzzy set A is said to be a fuzzy q-nbd of a fuzzy point x_t in an fts X if there is a fuzzy open set U in X such that $x_t qU \leq A$. If, in addition, A is fuzzy open, then A is called a fuzzy open q-nbd [8] of x_t.

A fuzzy set A in an fts (X, τ) is called fuzzy β-open [4] if $A \leq cl(int(clA))$. The complement of a fuzzy β-open set is called fuzzy β-closed [4]. The union (intersection) of all fuzzy β-open (resp., fuzzy β-closed) sets contained in (resp., containing) a fuzzy set A is called fuzzy β-interior [4] (resp., fuzzy β-closure [4]) of A, denoted by $\beta intA$ (resp., βclA).

Let $(D, \geq)$ be a directed set and X be an ordinary set. Let J denote the collection of all fuzzy points in X. A function $S : D \to J$ is called a fuzzy net in X [8]. It is denoted by $\{S_n : n \in (D, \geq)\}$. A non empty family $\mathcal{F}$ of fuzzy sets in X is called a prefilterbase on X if (i) $0_X \notin \mathcal{F}$ and (ii) for any $U, V \in \mathcal{F}$, there exists $W \in \mathcal{F}$ such that $W \leq U \bigcap V$ [6].

3. Fuzzy Pre β-Open Sets : Some Results

In this section we recall some definitions and results from [1, 2, 3, 5, 7] for ready references.

Definition 3.1. [1] *A fuzzy set A in an fts (X, τ) is called fuzzy pre β-open if $A \leq \beta int(clA)$. The complement of this set is called fuzzy pre β-closed set.*
The union (resp., intersection) of all fuzzy pre β-open (resp., fuzzy pre β-closed) sets contained in (containing) a fuzzy set A is called fuzzy pre β-interior (resp., fuzzy pre β-closure) of A, denoted by $p\beta intA$ (resp., $p\beta clA$).

Definition 3.2. [1] *A fuzzy set A in an fts (X, τ) is called fuzzy pre β-nbd of a fuzzy point x_α in X if there exists a fuzzy pre β-open set U in X such that $x_\alpha \in U \leq A$. If, in addition, A is fuzzy pre β-open, then A is called fuzzy pre β-open nbd of x_α.*

Definition 3.3. [1] *A fuzzy set A in an fts (X, τ) is called fuzzy pre β-q-nbd of a fuzzy point x_α in X if there exists a fuzzy pre β-open set U in X such that $x_\alpha q U \leq A$. If, in addition, A is fuzzy pre β-open, then A is called fuzzy pre β-open q-nbd of x_α.*

Result 3.4. [1] *Union (resp., intersection) of any two fuzzy pre β-open (resp., fuzzy pre β-closed) sets is also so.*

Result 3.5. [1] *$x_\alpha \in p\beta cl A$ if and only if every fuzzy pre β-open q-nbd U of x_α, UqA.*

Result 3.6. [1] *$p\beta cl(p\beta cl A) = p\beta cl A$ for any fuzzy set A in an fts (X, τ).*

Result 3.7. *$p\beta cl(A \bigvee B) = p\beta cl A \bigvee p\beta cl B$, for any two fuzzy sets A, B in X.*
Proof. It is clear that

$$p\beta cl A \bigvee p\beta cl B \subseteq p\beta cl(A \bigvee B)...(1)$$

Conversely, let $x_\alpha \in p\beta cl(A \bigvee B)$. Then for any fuzzy pre β-open q-nbd U of x_α, $Uq(A \bigvee B) \Rightarrow$ there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1 \Rightarrow$ either $U(y) + A(y) > 1 \Rightarrow UqA$ or $U(y) + B(y) > 1 \Rightarrow UqB \Rightarrow$ either $x_\alpha \in p\beta cl A$ or $x_\alpha \in p\beta cl B \Rightarrow x_\alpha \in p\beta cl A \bigvee p\beta cl B$.

Result 3.8. *For any fuzzy set A in an fts (X, τ),*
(i) $p\beta cl(1_X \setminus A) = 1_X \setminus p\beta int A$,
(ii) $p\beta int(1_X \setminus A) = 1_X \setminus p\beta cl A$.
Proof. (i). Let $x_t \in p\beta cl(1_X \setminus A)$ for any $A \in I^X$. If possible, let $x_t \notin 1_X \setminus p\beta int A$. Then $x_t q p\beta int A$. Then there exists a fuzzy pre β-open set B in X with $B \leq A$ such that $x_t q B$. Then B is a fuzzy pre β-open q-nbd of x_t. By assumption, $Bq(1_X \setminus A) \Rightarrow Aq(1_X \setminus A)$, which is absurd.
Conversely, let $x_t \in 1_X \setminus p\beta int A$ for any $A \in I^X$. Then $x_t \not{q} p\beta int A$ and so $x_t \not{q} U$ for any fuzzy pre β-open set U in X with $U \leq A \Rightarrow x_t \in 1_X \setminus U$ which is fuzzy pre β-closed set in X with $1_X \setminus A \leq 1_X \setminus U$. So $x_t \in p\beta cl(1_X \setminus A)$.
(ii) Writing $1_X \setminus A$ for A in (i), we get the result.

Definition 3.9. *Let A be a fuzzy set in an fts (X, τ). A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $sup\{U(x) : U \in \mathcal{U}\} = 1$, for each $x \in supp A$* [5]*. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy pre β-open), we call $\mathcal{U}$ is fuzzy open* [5] *(resp., fuzzy pre β-open) cover of A. In particular, if $A = 1_X$, we get the definition of fuzzy cover of X* [2].

Definition 3.10. *A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts (X, τ) is said to have a finite (resp., finite proximate) subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such* *that* $\bigvee \mathcal{U}_0 \geq A$ [5] *(resp.,* $\bigvee\{clU : U \in \mathcal{U}_0\} \geq A$ [7]*). In particular, if* $A = 1_X$*, we get* $\bigvee \mathcal{U}_0 = 1_X$ [2] *(resp.,* $\bigvee\{clU : U \in \mathcal{U}_0\} = 1_X$ [3]*).*

Definition 3.11. [3] *An fts* (X, τ) *is called fuzzy almost compact space if every fuzzy open cover has a finite proximate subcover.*

4. Fuzzy Pre β-compact Space : Some Characterizations

In this section fuzzy pre β-compactness is introduced and studied by fuzzy pre β-open and fuzzy regularly pre β-open sets and characterize this space via fuzzy net and prefilterbase.

Definition 4.1. *A fuzzy set* A *in an fts* (X, τ) *is said to be a fuzzy pre* β*-compact set if every fuzzy pre* β*-open cover* $\mathcal{U}$ *of* A *has a finite* $p\beta$*-proximate subcover, i.e., there exists a finite subcollection* $\mathcal{U}_0$ *of* $\mathcal{U}$ *such that* $\bigvee\{p\beta clU : U \in \mathcal{U}_0\} \geq A$*. If, in addition,* $A = 1_X$*, we say that the fts* X *is fuzzy pre* β*-compact space.*

Definition 4.2. *Let* x_α *be a fuzzy point in an fts* (X, τ)*. A prefilterbase* $\mathcal{F}$ *on* X *is called*
(a) $p\beta$*-adhere at* x_α*, written as* $x_\alpha \in p\beta\text{-}ad\mathcal{F}$*, if for each fuzzy pre* β*-open q-nbd* U *of* x_α *and each* $F \in \mathcal{F}$*,* $Fqp\beta clU$*, i.e.,* $x_\alpha \in p\beta clF$*, for each* $F \in \mathcal{F}$*;*
(b) $p\beta$*-converge to* x_α*, written as* $\mathcal{F}\overrightarrow{p\beta}x_\alpha$*, if to each fuzzy pre* β*-open q-nbd* U *of* x_α*, there corresponds some* $F \in \mathcal{F}$ *such that* $F \leq p\beta clU$*.*

Definition 4.3. *Let* x_α *be a fuzzy point in an fts* (X, τ)*. A fuzzy net* $\{S_n : n \in (D, \geq)\}$ *is said to*
(a) $p\beta$*-adhere at* x_α*, denoted by* $x_\alpha \in p\beta\text{-}ad(S_n)$*, if for each fuzzy pre* β*-open q-nbd* U *of* x_α *and each* $n \in D$*, there exists* $m \in D$ *with* $m \geq n$ *such that* $S_m qp\beta clU$*;*
(b) $p\beta$*-converge to* x_α*, denoted by* $S_n\overrightarrow{p\beta}x_\alpha$*, if for each fuzzy pre* β*-open q-nbd* U *of* x_α*, there exists* $m \in D$ *such that* $S_n qp\beta clU$*, for all* $n \geq m (n \in D)$*.*

Theorem 4.4. *For a fuzzy set* A *in an fts* X*, the following statements are equivalent:*
(a) A *is a fuzzy pre* β*-compact set,*
(b) for every prefilterbase $\mathcal{B}$ *in* X*,* $[\bigwedge\{p\beta clB : B \in \mathcal{B}\}] \bigwedge A = 0_X \Rightarrow$ *there exists a finite subcollection* $\mathcal{B}_0$ *of* $\mathcal{B}$ *such that* $\bigwedge\{p\beta intB : B \in \mathcal{B}_0\} \not{q} A$*,*
(c) for any family $\mathcal{F}$ *of fuzzy pre* β*-closed sets in* X *with* $\bigwedge\{F : F \in \mathcal{F}\} \bigwedge A = 0_X$*, there exists a finite subcollection* $\mathcal{F}_0$ *of* $\mathcal{F}$ *such that* $\bigwedge\{p\beta intF : F \in \mathcal{F}_0\} \not{q} A$*,*
(d) every prefilterbase on X*, each member of which is q-coincident with* A*,* $p\beta$*-adheres at some fuzzy point in* A*.*

Proof. (a) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{p\beta clB : B \in \mathcal{B}\}] \bigwedge A = 0_X$. Then for any $x \in suppA$, $[\bigwedge\{p\beta clB : B \in \mathcal{B}\}](x) = 0 \Rightarrow 1 -$

$[\bigwedge\{p\beta clB(x) : B \in \mathcal{B}\}] = 1 \Rightarrow \bigvee[(1_X \setminus p\beta clB)(x) : B \in \mathcal{B}] = 1 \Rightarrow sup\{p\beta int(1_X \setminus B)(x) : B \in \mathcal{B}\} = 1 \Rightarrow \{p\beta int(1_X \setminus B) : B \in \mathcal{B}\}$ is a fuzzy pre β-open cover of A. By (a), there exists a finite $p\beta$-proximate subcover $\{p\beta int(1_X \setminus B_1), p\beta int(1_X \setminus B_2), ..., p\beta int(1_X \setminus B_n)\}$ (say) of it for A. Thus $A \leq \bigvee_{i=1}^{n} p\beta cl(p\beta int(1_X \setminus B_i)) = \bigvee_{i=1}^{n}[1_X \setminus p\beta int(p\beta clB_i)] = 1_X \setminus \bigwedge_{i=1}^{n} p\beta int(p\beta clB_i) \Rightarrow \bigwedge_{i=1}^{n} p\beta int(p\beta clB_i) \leq 1_X \setminus A \Rightarrow A \not{q} \bigwedge_{i=1}^{n} p\beta int(p\beta clB_i) \Rightarrow A \not{q} \bigwedge_{i=1}^{n} p\beta int B_i$.

(b) $\Rightarrow$ (a). Let the condition (b) hold, and suppose that there exists a fuzzy pre β-open cover $\mathcal{U}$ of A having no finite $p\beta$-proximate subcover for A. Then for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in suppA$ such that $sup\{p\beta clU(x) : U \in \mathcal{U}_0\} < A(x)$, i.e., $1 - sup\{(p\beta clU)(x) : U \in \mathcal{U}_0\} > 1 - A(x) \geq 0 \Rightarrow inf\{(1_X \setminus p\beta clU)(x) : U \in \mathcal{U}_0\} > 0$. Thus $\{\bigwedge_{U \in \mathcal{U}_0}(1_X \setminus p\beta clU) : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ ($=\mathcal{B}$, say) is a prefilterbase in X. If there exists a finite subcollection $\{U_1, U_2, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigwedge_{i=1}^{n} p\beta int(1_X \setminus p\beta clU_i) \not{q} A$, then $A \leq 1_X \setminus \bigwedge_{i=1}^{n} p\beta int(1_X \setminus p\beta clU_i) = \bigvee_{i=1}^{n}[1_X \setminus p\beta int(1_X \setminus p\beta clU_i)] = \bigvee_{i=1}^{n} p\beta cl(p\beta clU_i) = \bigvee_{i=1}^{n} p\beta clU_i$ (by Result 3.6). Thus $\mathcal{U}$ has a finite $p\beta$-proximate subcover for A, contradicts our hypothesis. Hence for every finite subcollection $\{\bigwedge_{U \in \mathcal{U}_1}(1_X \setminus p\beta clU), ..., \bigwedge_{U \in \mathcal{U}_k}(1_X \setminus p\beta clU)\}$ of $\mathcal{B}$, where $\mathcal{U}_1, ..., \mathcal{U}_k$ are finite subset of $\mathcal{U}$, we have $[\bigwedge_{U \in \mathcal{U}_1 \bigvee ... \bigvee \mathcal{U}_k} p\beta int(1_X \setminus p\beta clU)] q A$. By(b), $[\bigwedge_{U \in \mathcal{U}} p\beta cl(1_X \setminus p\beta clU)] \bigwedge A \neq 0_X$. Then there exists $x \in suppA$, such that $\inf_{U \in \mathcal{U}}[p\beta cl(1_X \setminus p\beta clU)](x) > 0 \Rightarrow 1 - \inf_{U \in \mathcal{U}}[p\beta cl(1_X \setminus p\beta clU)](x) < 1 \Rightarrow \sup_{U \in \mathcal{U}}[1_X \setminus p\beta cl(1_X \setminus p\beta clU)](x) < 1 \Rightarrow \sup_{U \in \mathcal{U}} U(x) \leq \sup_{U \in \mathcal{U}} p\beta int(p\beta clU)(x) < 1$ which contradicts that $\mathcal{U}$ is a fuzzy pre β-open cover of A.

(a) $\Rightarrow$ (c). Let $\mathcal{F}$ be a family of fuzzy pre β-closed sets in X such that $\bigwedge\{F : F \in \mathcal{F}\} \bigwedge A = 0_X$. Then for each $x \in suppA$ and for each positive integer n, there exists some $F_n \in \mathcal{F}$ such that $F_n(x) < 1/n \Rightarrow 1 - F_n(x) > 1 - 1/n \Rightarrow \sup_{F \in \mathcal{F}}[(1_X \setminus F)(x)] = 1$ and so $\{1_X \setminus F : F \in \mathcal{F}\}$ is a fuzzy pre β-open cover of A. By (a), there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $A \leq \bigvee_{F \in \mathcal{F}_0} p\beta cl(1_X \setminus F) \Rightarrow 1_X \setminus A \geq 1_X \setminus \bigvee_{F \in \mathcal{F}_0} p\beta cl(1_X \setminus F) = \bigwedge_{F \in \mathcal{F}_0} (1_X \setminus p\beta cl(1_X \setminus F)) = \bigwedge_{F \in \mathcal{F}_0} p\beta int F$. Hence $A \not q (\bigwedge_{F \in \mathcal{F}_0} p\beta int F)$, where $\mathcal{F}_0$ is a finite subcollection of $\mathcal{F}$.

(c) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{p\beta cl B : B \in \mathcal{B}\}] \bigwedge A = 0_X$. Then the family $\mathcal{F} = \{p\beta cl B : B \in \mathcal{B}\}$ is a family of fuzzy pre β-closed sets in X with $(\bigwedge F) \bigwedge A = 0_X$. By (c), there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $[\bigwedge\{p\beta int(p\beta cl B) : B \in \mathcal{B}_0\}] \not q A \Rightarrow (\bigwedge_{B \in \mathcal{B}_0} p\beta int B) \not q A$.

(a) $\Rightarrow$ (d). Let $\mathcal{F}$ be a prefilterbase in X, each member of which is q-coincident with A. If possible, let $\mathcal{F}$ do not $p\beta$-adhere at any fuzzy point in A. Then for each $x \in suppA$, there exists $n_x \in \mathcal{N}$ (the set of all natural numbers) such that $x_{1/n_x} \in A$. Then there are a fuzzy pre β-open set $U^x_{n_x}$ and a member $F^x_{n_x}$ of $\mathcal{F}$ such that $x_{1/n_x} q U^x_{n_x}$ and $p\beta cl U^x_{n_x} \ /q F^x_{n_x}$. Thus $U^x_{n_x}(x) > 1 - 1/n_x$ so that $sup\{U^x_n(x) : n \in \mathcal{N}, n \geq n_x\} = 1$. Thus $\{U^x_n : n \in \mathcal{N}, n \geq n_x, x \in suppA\}$ forms a fuzzy pre β-open cover of A. By (a), there exist finitely many points $x_1, x_2, ..., x_k \in suppA$ and $n_1, n_2, ..., n_k \in \mathcal{N}$ such that $A \leq \bigvee_{i=1}^{k} p\beta cl U^{x_i}_{n_{x_i}}$. Choose $F \in \mathcal{F}$ such that $F \leq \bigwedge_{i=1}^{k} F^{x_i}_{n_i}$. Then $F \not q [\bigvee_{i=1}^{k} p\beta cl U^{x_i}_{n_{x_i}}]$, i.e., $F \not q A$, a contradiction.

(d) $\Rightarrow$ (a). If possible, let there exist a fuzzy pre β-open cover $\mathcal{U}$ of A such that for every finite subset $\mathcal{U}_0$ of $\mathcal{U}$, $\bigvee\{p\beta cl U : U \in \mathcal{U}_0\} \not\geq A$. Then $\mathcal{F} = \{1_X \setminus \bigvee_{U \in \mathcal{U}_0} p\beta cl U : \mathcal{U}_0$ is a finite subset of $\mathcal{U}\}$ is a prefilterbase on X such that $F q A$, for each $F \in \mathcal{F}$. By (d), $\mathcal{F}$ $p\beta$-adheres at some fuzzy point $x_\alpha \in A$. As $\mathcal{U}$ is a fuzzy cover of A, $\sup_{U \in \mathcal{U}} U(x) = 1 \Rightarrow$ there exists $U_0 \in \mathcal{U}$ such that $U_0(x) > 1 - \alpha \Rightarrow x_\alpha q U_0$. As $x_\alpha \in p\beta$-ad$\mathcal{F}$ and $1_X \setminus p\beta cl U_0 \in \mathcal{F}$, we have $p\beta cl U_0 q (1_X \setminus p\beta cl U_0)$, a contradiction.

Theorem 4.5. *For a fuzzy set A in an fts X, the following implications hold :*
(a) every fuzzy net in A $p\beta$-adheres at some fuzzy point in A,
$\Leftrightarrow$(b) every fuzzy net in A has a $p\beta$-convergent fuzzy subnet,
$\Leftrightarrow$ (c) every prefilterbase in A $p\beta$-adheres at some fuzzy point in A,
$\Rightarrow$ (d) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\alpha \in \Lambda} p\beta cl B_\alpha] \bigwedge A =$

0_X, *there is a finite subset* Λ_0 *of* Λ *such that* $(\bigwedge_{\alpha\in\Lambda_0} B_\alpha)\bigwedge A = 0_X$,
$\Rightarrow$ *(e) A is fuzzy pre β-compact set.*

Proof. (a) $\Rightarrow$ (b). Let a fuzzy net $\{S_n : n \in (D,\geq)\}$ in A where $(D,\geq)$ is a directed set, $p\beta$-adhere at a fuzzy point $x_\alpha \in A$. Let Q_{x_α} denote the set of the fuzzy $p\beta$-closures of all fuzzy pre β-open q-nbds of x_α. For any $B \in Q_{x_\alpha}$, we can choose some $n \in D$ such that $S_n qB$. Let E denote the set of all ordered pairs (n, B) with the property that $n \in D$, $B \in Q_{x_\alpha}$ and $S_n qB$. Then $(E, \gg)$ is a directed set where $(m, C) \gg (n, B)$ if and only if $m \geq n$ in D and $C \leq B$. Then $T : (E, \gg) \to (X, \tau)$ given by $T(n, B) = S_n$, is a fuzzy subnet of $\{S_n : n \in (D,\geq)\}$. Let V be any fuzzy pre β-open q-nbd of x_α. Then there is $n \in D$ such that that $(n, p\beta clV) \in E$ and hence $S_n qp\beta clV$. Now, for any $(m, U) \gg (n, p\beta clV)$, $T(m, U) = S_m qU \leq p\beta clV \Rightarrow T(m, U)qp\beta clV$. Hence $T\overrightarrow{p\beta}x_\alpha$.
(b) $\Rightarrow$ (a). If a fuzzy net $\{S_n : n \in (D,\geq)\}$ does not $p\beta$-adhere at a fuzzy point x_α, then there is a fuzzy pre β-open q-nbd U of x_α and an $n \in D$ such that $S_m \not{q} p\beta clU$, for all $m \geq n$. Then obviously no fuzzy subnet of the fuzzy net can $p\beta$-converge to x_α.
(a) $\Rightarrow$ (c). Let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a prefilterbase in A. For each $\alpha \in \Lambda$, choose a fuzzy point $x_{F_\alpha} \in F_\alpha$ and construct the fuzzy net $S = \{x_{F_\alpha} : F_\alpha \in \mathcal{F}\}$ in A with $(\mathcal{F}, \gg)$ as domain, where for two members $F_\alpha, F_\beta \in \mathcal{F}$, $F_\alpha \gg F_\beta$ if and only if $F_\alpha \leq F_\beta$. By (a), the fuzzy net S $p\beta$-adheres at some fuzzy point x_t $(0 < t \leq 1) \in A$. Then for any fuzzy pre β-open q-nbd U of x_t and any $F_\alpha \in \mathcal{F}$, there exists $F_\beta \in \mathcal{F}$ such that $F_\beta \gg F_\alpha$ and $x_{F_\beta} qp\beta clU$. Then $F_\beta qp\beta clU$ and hence $F_\alpha qp\beta clU$. Thus $\mathcal{F}$ $p\beta$-adheres at x_t.
(c) $\Rightarrow$ (a). Let $\{S_n : n \in (D,\geq)\}$ be a fuzzy net in A. Consider the prefilterbase $\mathcal{F} = \{T_n : n \in D\}$ generated by the net, where $T_n = \{S_m : m \in D, m \geq n\}$. By (c), there exists a fuzzy point $a_\alpha \in A$ such that $\mathcal{F}$ $p\beta$-adheres at a_α. Then for each fuzzy pre β-open q-nbd U of a_α and each $F \in \mathcal{F}$, $Fqp\beta clU$, i.e., $p\beta clUqT_n$, for all $n \in D$. Hence the given fuzzy net $p\beta$-adheres at a_α.
(c) $\Rightarrow$ (d). Let $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy sets in X such that for every finite subset Λ_0 of Λ, $(\bigwedge_{\alpha\in\Lambda_0} B_\alpha)\bigwedge A \neq 0_X$. Then $\mathcal{F} = \{(\bigwedge_{\alpha\in\Lambda_0} B_\alpha)\bigwedge A : \Lambda_0$ is a finite subset of $\Lambda\}$ is a prefilterbase in A. By (c), $\mathcal{F}$ $p\beta$-adheres at some fuzzy point $a_t \in A$ $(0 < t \leq 1)$. Then for each $\alpha \in \Lambda$ and each fuzzy pre β-open q-nbd U of a_t, $B_\alpha qp\beta clU$, i.e., $a_t \in p\beta clB_\alpha$, for each $\alpha \in \Lambda$. Consequently, $(\bigwedge_{\alpha\in\Lambda} p\beta clB_\alpha)\bigwedge A \neq 0_X$.
(d) $\Rightarrow$ (e). Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy pre β-open cover of a fuzzy set A.

Then by (d), $A\bigwedge[\bigwedge_{\alpha\in\Lambda}(1_X\setminus U_\alpha)] = A\bigwedge[1_X\setminus\bigvee_{\alpha\in\Lambda}U_\alpha] = 0_X$. If for some $\alpha\in\Lambda$, $1_X\setminus p\beta clU_\alpha = 0_X$, then we are done. If $1_X\setminus p\beta clU_\alpha$ $(=B_\alpha$, say$)\neq 0_X$, then for each $\alpha\in\Lambda$, $\mathcal{B}=\{B_\alpha:\alpha\in\Lambda\}$ is a family of non-null fuzzy sets. We show that $\bigwedge_{\alpha\in\Lambda}p\beta clB_\alpha\leq\bigwedge_{\alpha\in\Lambda}(1_X\setminus U_\alpha)$. In fact, let x_t $(0<t\leq 1)$ be a fuzzy point such that $x_t\in p\beta clB_\alpha = p\beta cl(1_X\setminus p\beta clU_\alpha)$. If x_tqU_α, then $p\beta clU_\alpha q(1_X\setminus p\beta clU_\alpha)$, which is absurd. Hence $x_t\not{q}U_\alpha\Rightarrow x_t\in 1_X\setminus U_\alpha$. Then $[\bigwedge_{\alpha\in\Lambda}p\beta clB_\alpha]\bigwedge A\leq A\bigwedge[\bigwedge_{\alpha\in\Lambda}(1_X\setminus U_\alpha)] = 0_X$. By (d), there exists a finite subset Λ_0 of Λ such that $[\bigwedge_{\alpha\in\Lambda_0}B_\alpha]\bigwedge A = 0_X$, i.e., $A\leq 1_X\setminus\bigwedge_{\alpha\in\Lambda_0}B_\alpha = \bigvee_{\alpha\in\Lambda_0}(1_X\setminus B_\alpha) = \bigvee_{\alpha\in\Lambda_0}p\beta clU_\alpha$ and (e) follows.

Definition 4.6. *A fuzzy set A in an fts (X,τ) is said to be fuzzy regularly pre β-open if $A = p\beta int(p\beta clA)$. The complement of such a set is called fuzzy regularly pre β-closed.*

Definition 4.7. *A fuzzy point x_α in X is said to be a fuzzy $p\beta$-cluster point of a prefilterbase $\mathcal{B}$ if $x_\alpha\in p\beta clB$, for all $B\in\mathcal{B}$. If, in addition, $x_\alpha\in A$, for a fuzzy set A, then $\mathcal{B}$ is said to have a fuzzy $p\beta$-cluster point in A.*

Theorem 4.8. *A fuzzy set A in an fts (X,τ) is fuzzy pre β-compact if and only if for each prefilterbase $\mathcal{F}$ in X which is such that for each set of finitely many members $F_1, F_2, ..., F_n$ from $\mathcal{F}$ and for any fuzzy regularly pre β-closed set C containing A, one has $(F_1\bigwedge ...\bigwedge F_n)qC$, $\mathcal{F}$ has a fuzzy $p\beta$-cluster point in A.*

Proof. Let A be fuzzy pre β-compact set and suppose $\mathcal{F}$ be a prefilterbase in X such that $[\bigwedge\{p\beta clF : F\in\mathcal{F}\}]\bigwedge A = 0_X$...(1). Let $x\in suppA$. Consider any $n\in\mathcal{N}$ (the set of all natural numbers) such that $1/n < A(x)$, i.e., $x_{1/n}\in A$. By (1), $x_{1/n}\notin p\beta clF^n_x$, for some $F^n_x\in\mathcal{F}$. Then there exists a fuzzy pre β-open q-nbd U^n_x of $x_{1/n}$ such that $p\beta clU^n_x\not{q}F^n_x$. Now $U^n_x(x) > 1-1/n\Rightarrow sup\{U^n_x(x) : 1/n < A(x), n\in\mathcal{N}\} = 1\Rightarrow\mathcal{U} = \{U^n_x : x\in suppA, n\in\mathcal{N}\}$ forms a fuzzy pre β-open cover of A such that for U^n_x, there exists $F^n_x\in\mathcal{F}$ with $U^n_x\not{q}F^n_x$. Since A is fuzzy pre β-compact, there exist finitely many members $U^{n_1}_{x_1},,,,,U^{n_k}_{x_k}$ of $\mathcal{U}$ such that $A\leq\bigvee_{i=1}^{k}p\beta clU^{n_i}_{x_i} = p\beta cl(\bigvee_{i=1}^{k}U^{n_i}_{x_i})$ (by Result 3.7) $(=U$, say$)$. Now $F^{n_1}_{x_1}, ..., F^{n_k}_{x_k}\in\mathcal{F}$ such that $U^{n_i}_{x_i}\not{q}F^{n_i}_{x_i}$ for $i=1,2,...,k$. Now U is a fuzzy regularly pre β-closed set containing A such that $p\beta clU\not{q}(F^{n_1}_{x_1}\bigwedge ..\bigwedge F^{n_k}_{x_k})\Rightarrow U\not{q}(F^{n_1}_{x_1}\bigwedge ..\bigwedge F^{n_k}_{x_k})$.

Conversely, let $\mathcal{B}$ be a prefilterbase in X having no fuzzy $p\beta$-cluster point in A.

Then by hypothesis, there is a fuzzy regularly pre β-closed set C containing A such that for some finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$, $(\bigwedge \mathcal{B}_0) \not{q} C$. Then $(\bigwedge \mathcal{B}_0) \not{q} A$. By Theorem 4.4 (b) $\Rightarrow$ (a), A is fuzzy pre β-compact set.

From Theorem 4.4, Theorem 4.5 and Theorem 4.8, we have the characterizations of fuzzy pre β-compact space as follows.

Theorem 4.9. *For an fts X, the following statements are equivalent :*
(a) X is fuzzy pre β-compact,
(b) every fuzzy net in X $p\beta$-adheres at some fuzzy point in X,
(c) every fuzzy net in X has a $p\beta$-convergent fuzzy subnet,
(d) every prefilterbase in X $p\beta$-adheres at some fuzzy point in X,
(e) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\alpha\in\Lambda} p\beta cl B_\alpha] = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\alpha\in\Lambda_0} B_\alpha) = 0_X$,
(f) for every prefilterbase $\mathcal{B}$ in X with $\bigwedge\{p\beta cl B : B \in \mathcal{B}\} = 0_X$, there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge\{p\beta int B : B \in \mathcal{B}_0\} = 0_X$,
(g) for any family $\mathcal{F}$ of fuzzy pre β-closed sets in X with $\bigwedge \mathcal{F} = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge\{p\beta int F : F \in \mathcal{F}_0\} = 0_X$.

Theorem 4.10. *An fts X is fuzzy pre β-compact if and only if for any collection $\{F_\alpha : \alpha \in \Lambda\}$ of fuzzy pre β-open sets in X having finite intersection property $\bigwedge\{p\beta cl F_\alpha : \alpha \in \Lambda\} \neq 0_X$.*

Proof. Let X be fuzzy pre β-compact space and $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a collection of fuzzy pre β-open sets in X with finite intersection property. Suppose $\bigwedge\{p\beta cl F_\alpha : \alpha \in \Lambda\} = 0_X$. Then $\{1_X \setminus p\beta cl F_\alpha : \alpha \in \Lambda\}$ is a fuzzy pre β-open cover of X. By hypothesis, there exists a finite subset Λ_0 of Λ such that $1_X = \bigvee\{p\beta cl(1_X \setminus p\beta cl F_\alpha) : \alpha \in \Lambda_0\} = \bigvee\{1_X \setminus p\beta int(p\beta cl F_\alpha) : \alpha \in \Lambda_0\} \leq \bigvee\{1_X \setminus F_\alpha : \alpha \in \Lambda_0\} = 1_X \setminus \bigwedge_{\alpha\in\Lambda_0} F_\alpha \Rightarrow \bigwedge_{\alpha\in\Lambda_0} F_\alpha = 0_X$ which contradicts the fact that $\mathcal{F}$ has finite intersection property.

Conversely, suppose that X is not fuzzy pre β-compact space. Then there is a fuzzy pre β-open cover $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ of X such that for every finite subset Λ_0 of Λ, $\bigvee\{p\beta cl F_\alpha : \alpha \in \Lambda_0\} \neq 1_X$. Then $1_X \setminus \bigvee\{p\beta cl F_\alpha : \alpha \in \Lambda_0\} \neq 0_X \Rightarrow \bigwedge_{\alpha\in\Lambda_0} (1_X \setminus p\beta cl F_\alpha) \neq 0_X$, for every finite subset Λ_0 of Λ. Thus $\{1_X \setminus p\beta cl F_\alpha : \alpha \in \Lambda\}$ is a collection of fuzzy pre β-open sets with finite intersection property. By hypothesis, $\bigwedge_{\alpha\in\Lambda} p\beta cl(1_X \setminus p\beta cl F_\alpha) \neq 0_X$, i.e., $1_X \setminus \bigvee_{\alpha\in\Lambda} p\beta int(p\beta cl F_\alpha) \neq 0_X \Rightarrow$

$\bigvee_{\alpha\in\Lambda} p\beta int(p\beta cl F_\alpha) \neq 1_X$. Hence $\bigvee_{\alpha\in\Lambda} F_\alpha \neq 1_X$, a contradiction as $\mathcal{F}$ is a fuzzy pre β-open cover of X.

Definition 4.11. *Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy pre β-open sets in X, i.e., for each member n of a directed set $(D, \geq)$, S_n is a fuzzy pre β-open set in X. A fuzzy point x_α in X is said to be a fuzzy $p\beta$-cluster point of the fuzzy net if for every $n \in D$ and every fuzzy pre β-open q-nbd V of x_α, there exists $m \in D$ with $m \geq n$ such that $S_m q V$.*

Theorem 4.12. *An fts X is fuzzy pre β-compact if and only if every fuzzy net of fuzzy pre β-open sets in X has a fuzzy $p\beta$-cluster point in X.*

Proof. Let $\mathcal{U} = \{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy pre β-open sets in a fuzzy pre β-compact space X. For each $n \in D$, let us put $F_n = p\beta cl[\bigvee\{S_m : m \in D$ and $m \geq n\}]$. Then $\mathcal{F} = \{F_n : n \in D\}$ is a family of fuzzy pre β-closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{p\beta int F : F \in \mathcal{F}_0\} \neq 0_X$. By Theorem 4.9 (a)$\Rightarrow$ (g), $\bigwedge_{n\in D} F_n \neq 0_X$. Let $x_\alpha \in \bigwedge_{n\in D} F_n$. Then $x_\alpha \in F_n$, for all $n \in D$. Thus for any fuzzy pre β-open q-nbd A of x_α and any $n \in D$, $Aq[\bigvee\{S_m : m \geq n\}]$ and so there exists some $m \in D$ with $m \geq n$ and $AqS_m \Rightarrow x_\alpha$ is a fuzzy $p\beta$-cluster point of $\mathcal{U}$.

Conversely, let $\mathcal{F}$ be a collection of fuzzy pre β-closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{p\beta int F : F \in \mathcal{F}_0\} \neq 0_X$. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of $\mathcal{F}$ directed by the relation '$\gg$' such that for $F_1, F_2 \in \mathcal{F}^*$, $F_1 \gg F_2$ if and only if $F_1 \leq F_2$. Let $F^* = p\beta int F$, for each $F \in \mathcal{F}^*$. Then $F^* \neq 0_X$. Consider the fuzzy net $\mathcal{U} = \{F^* : F \in (\mathcal{F}^*, \gg)\}$ of non-null fuzzy pre β-open sets of X. By hypothesis, $\mathcal{U}$ has a fuzzy $p\beta$-cluster point, say x_α. We claim that $x_\alpha \in \bigwedge \mathcal{F}$. In fact, let $F \in \mathcal{F}$ be arbitrary and A be any fuzzy pre β-open q-nbd of x_α. Since $F \in \mathcal{F}^*$ and x_α is a fuzzy $p\beta$-cluster point of $\mathcal{U}$, there exists $G \in \mathcal{F}^*$ such that $G \gg F$ (i.e., $G \leq F$) and $G^* qA \Rightarrow GqA \Rightarrow FqA \Rightarrow x_\alpha \in p\beta cl F = F$, for each $F \in \mathcal{F} \Rightarrow x_\alpha \in \bigwedge \mathcal{F} \Rightarrow \bigwedge \mathcal{F} \neq 0_X$. By Theorem 4.9 (g) $\Rightarrow$ (a), X is fuzzy pre β-compact space.

Definition 4.13. *A fuzzy cover $\mathcal{U}$ by fuzzy pre β-closed sets of an fts (X, τ) will be called a fuzzy $p\beta$-cover of X if for each fuzzy point x_α $(0 < \alpha < 1)$ in X, there exits $U \in \mathcal{U}$ such that U is a fuzzy pre β-open nbd of x_α.*

Theorem 4.14. *An fts (X, τ) is fuzzy pre β-compact if and only if every fuzzy $p\beta$-cover of X has a finite subcover.*

Proof. Let X be fuzzy pre β-compact space and $\mathcal{U}$ be any fuzzy $p\beta$-cover of X.

Then for each $n \in \mathcal{N}$ (the set of all natural numbers) with $n > 1$, there exist $U_x^n \in \mathcal{U}$ and a fuzzy pre β-open set V_x^n in X such that $x_{1-1/n} \leq V_x^n \leq U_x^n$. Then $V_x^n(x) \geq 1 - 1/n \Rightarrow sup\{V_x^n(x) : n \in \mathcal{N}\} = 1 \Rightarrow \mathcal{V} = \{V_x^n : x \in X, n \in \mathcal{N}, n > 1\}$ is a fuzzy pre β-open cover of X. As X is fuzzy pre β-compact, there exist finitely many points $x_1, x_2, ..., x_m \in X$ and $n_1, n_2, ..., n_m \in N \setminus \{1\}$ such that $1_X = \bigvee_{k=1}^{m} p\beta cl V_{x_k}^{n_k} \leq \bigvee_{k=1}^{m} p\beta cl U_{x_k}^{n_k} = \bigvee_{k=1}^{m} U_{x_k}^{n_k}$.
Conversely, let $\mathcal{U}$ be fuzzy pre β-open cover of X. For any fuzzy point x_α $(0 < \alpha < 1)$ in X, as $\sup_{U \in \mathcal{U}} U(x) = 1$, there exists $U_{x_\alpha} \in \mathcal{U}$ such that $U_{x_\alpha}(x) \geq \alpha$ $(0 < \alpha < 1)$. Then $\mathcal{V} = \{p\beta cl U : U \in \mathcal{U}\}$ is a fuzzy $p\beta$-cover of X and the rest is clear.

The following theorem gives a necessary condition for an fts to be fuzzy pre β-compact.

Theorem 4.15. *If an fts X is fuzzy pre β-compact, then every prefilterbase on X with at most one $p\beta$-adherent point is $p\beta$-convergent.*

Proof. Let $\mathcal{F}$ be a prefilterbase with at most one $p\beta$-adherent point in a fuzzy pre β-compact fts X. Then by Theorem 4.9, $\mathcal{F}$ has at least one $p\beta$-adherent point in X. Let x_α be the unique $p\beta$-adherent point of $\mathcal{F}$ and if possible, let $\mathcal{F}$ do not $p\beta$-converge to x_α. Then for some fuzzy pre β-open q-nbd U of x_α and for each $F \in \mathcal{F}$, $F \not\leq p\beta cl U$, so that $F \bigwedge \{1_X \setminus p\beta cl U\} \neq 0_X$. Then $\mathcal{G} = \{F \bigwedge (1_X \setminus p\beta cl U) : F \in \mathcal{F}\}$ is a prefilterbase in X and hence has a $p\beta$-adherent point y_t (say) in X. Now $p\beta cl U \not{q} G$, for all $G \in \mathcal{G}$ so that $x_\alpha \neq y_t$. Again, for each fuzzy pre β-open q-nbd V of y_t and each $F \in \mathcal{F}$, $p\beta cl V q (F \bigwedge (1_X \setminus p\beta cl U)) \Rightarrow p\beta cl V q F \Rightarrow y_t$ is a fuzzy $p\beta$-adherent point of $\mathcal{F}$, where $x_\alpha \neq y_t$. This contradicts the fact that x_α is the only fuzzy $p\beta$-adherent point of $\mathcal{F}$.

Some results on fuzzy pre β-compactness of an fts are given by the following theorem.

Theorem 4.16. *Let (X, τ) be an fts and $A \in I^X$. Then the following statements are true :*
(a) If A is fuzzy pre β-compact, then so is $p\beta cl A$,
(b) Union of two fuzzy pre β-compact sets is also so,
(c) If X is fuzzy pre β-compact, then every fuzzy regularly pre β-closed set A in X is fuzzy pre β-compact.

Proof. (a). Let $\mathcal{U}$ be a fuzzy pre β-open cover of $p\beta cl A$. Then $\mathcal{U}$ is also a fuzzy pre β-open cover of A. As A is fuzzy pre β-compact, there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $A \leq \bigvee\{p\beta cl U : U \in \mathcal{U}_0\} = p\beta cl\{\bigvee U : U \in \mathcal{U}_0\} \Rightarrow p\beta cl A \leq p\beta cl\{p\beta cl[\bigvee\{U : U \in \mathcal{U}_0\}]\} = p\beta cl\{\bigvee U : U \in \mathcal{U}_0\} = \bigvee\{p\beta cl U : U \in \mathcal{U}_0\}$. Hence

the proof.
(b). Obvious.
(c). Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy pre β-open cover of a fuzzy regularly pre β-closed set A in X. Then for each $x \notin suppA$, $A(x) = 0 \Rightarrow (1_X \setminus A)(x) = 1 \Rightarrow \mathcal{U} \bigvee\{(1_X \setminus A)\}$ is a fuzzy pre β-open cover of X. Since X is fuzzy pre β-compact, there are finitely many members $U_1, U_2, ..., U_n$ in $\mathcal{U}$ such that $1_X = (p\beta clU_1 \bigvee ... \bigvee p\beta clU_n) \bigvee p\beta cl(1_X \setminus A)$. We claim that $p\beta intA \leq p\beta clU_1 \bigvee ... \bigvee p\beta clU_n$. If not, there exists a fuzzy point $x_t \in p\beta intA$, but $x_t \notin (p\beta clU_1 \bigvee ... \bigvee p\beta clU_n)$, i.e., $t > max\{(p\beta clU_1)(x), ..., (p\beta clU_n)(x)\}$. As $1_X = (p\beta clU_1 \bigvee ... \bigvee p\beta clU_n) \bigvee p\beta cl(1_X \setminus A)$, $[p\beta cl(1_X \setminus A)](x) = 1 \Rightarrow 1 - p\beta intA(x) = 1 \Rightarrow p\beta intA(x) = 0 \Rightarrow x_t \notin p\beta intA$, a contradiction. Hence $A = p\beta cl(p\beta intA) \leq p\beta cl(p\beta clU_1 \bigvee .... \bigvee p\beta clU_n) = p\beta clU_1 \bigvee ... \bigvee p\beta clU_n$ (by Result 3.6 and Result 3.7) $\Rightarrow A$ is fuzzy pre β-compact set.

5. Mutual Relationship

Here we establish the mutual relationship between fuzzy almost compactness [3] and fuzzy pre β-compactness. Then it is shown that fuzzy pre β-compactness implies fuzzy almost compactness, but converse is true in fuzzy pre β-regular space [1]. It is also established that fuzzy pre β-compactness remains invariant under fuzzy pre β-irresolute function [1].

Since for any fuzzy set A in an fts X, $p\beta clA \leq clA$ (as every fuzzy closed set is fuzzy pre β-closed [1]), we can state the following theorem easily.

Theorem 5.1. *Every fuzzy pre β-compact space is fuzzy almost compact.*

To get the converse we have to recall the following definition and theorem for ready references.

Definition 5.2. [1] *An fts (X, τ) is said to be fuzzy pre β-regular if for each fuzzy pre β-closed set F in X and each fuzzy point x_α in X with $x_\alpha q(1_X \setminus F)$, there exists a fuzzy open set U in X and a fuzzy pre β-open set V in X such that $x_\alpha qU$, $F \leq V$ and $U \not{q} V$.*

Theorem 5.3. [1] *An fts (X, τ) is fuzzy pre β-regular iff every fuzzy pre β-closed set is fuzzy closed.*

Theorem 5.4. *A fuzzy pre β-regular, fuzzy almost compact space X is fuzzy pre β-compact.*

Proof. Let $\mathcal{U}$ be a fuzzy pre β-open cover of a fuzzy pre β-regular, fuzzy almost compact space X. Then by Theorem 5.3, $\mathcal{U}$ is a fuzzy open cover of X. As X is fuzzy almost compact, there is a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee\{clU : U \in \mathcal{U}_0\} = \bigvee\{p\beta clU : U \in \mathcal{U}_0\}$ (by Theorem 5.3) $= 1_X \Rightarrow X$ is fuzzy pre β-compact.

Next we recall the following definition and theorem for ready references.

Definition 5.5. [1] *A function $f : X \to Y$ is said to be fuzzy pre β-irresolute if the inverse image of every fuzzy pre β-open set in Y is fuzzy pre β-open in X.*

Theorem 5.6. [1] *For a function $f : X \to Y$, the following statements are equivalent :*
(i) f is fuzzy pre β-irresolute,
(ii) $f(p\beta clA) \leq p\beta cl(f(A))$, for all $A \in I^X$,
(iii) for each fuzzy point x_α in X and each fuzzy pre β-open q-nbd V of $f(x_\alpha)$ in Y, there exists a fuzzy pre β-open q-nbd U of x_α in X such that $f(U) \leq V$.

Theorem 5.7. *Fuzzy pre β-irresolute image of a fuzzy pre β-compact space is fuzzy pre β-compact.*

Proof. Let $f : X \to Y$ be fuzzy pre β-irresolute surjection from a fuzzy pre β-compact space X to an fts Y, and let $\mathcal{V}$ be a fuzzy pre β-open cover of Y. Let $x \in X$ and $f(x) = y$. Since $sup\{V(y) : V \in \mathcal{V}\} = 1$, for each $n \in \mathcal{N}$ (the set of all natural numbers), there exists some $V_x^n \in \mathcal{V}$ with $V_x^n(y) > 1-1/n$ and so $y_{1/n} q V_x^n$. By fuzzy pre β-irresoluteness of f, by Theorem 5.6 (i)$\Rightarrow$ (iii), $f(U_x^n) \leq V_x^n$, for some fuzzy pre β-open set U_x^n in X q-coincident with $x_{1/n}$. Since $U_x^n(x) > 1-1/n$, $sup\{U_x^n(x) : n \in \mathcal{N}\} = 1$. Then $\mathcal{U} = \{U_x^n : n \in \mathcal{N}, x \in X\}$ is a fuzzy pre β-open cover of X. By fuzzy pre β-compactness of X, $\bigvee_{i=1}^{k} p\beta clU_{x_i}^{n_i} = 1_X$, for some finite subcollection $\{U_{x_1}^{n_1}, ..., U_{x_k}^{n_k}\}$ of $\mathcal{U}$. Then $1_Y = f(\bigvee_{i=1}^{k} p\beta clU_{x_i}^{n_i}) = \bigvee_{i=1}^{k} f(p\beta clU_{x_i}^{n_i}) \leq \bigvee_{i=1}^{k} p\beta cl(f(U_{x_i}^{n_i}))$ (by Theorem 5.6 (i) $\Rightarrow$ (ii)) $\leq \bigvee_{i=1}^{k} p\beta clV_{x_i}^{n_i} \Rightarrow Y$ is fuzzy pre β-compact space.

References

[1] Bhattacharyya, Anjana, Fuzzy pre β-open set and its applications, J. of Ramanujan Society of Mathematics and Mathematical Sciences, 10 (2) (2023), 187-198.

[2] Chang, C. L., Fuzzy topological spaces, J. Math. Anal. Appl., 24 (1968), 182-190.

[3] DiConcillio, A. and Gerla, G., Almost compactness in fuzzy topological spaces, Fuzzy Sets and Systems, 13 (1984), 187-192.

[4] Fath Alla, M. A., On fuzzy topological spaces, Ph.D. Thesis, Assiut Univ., Sohag, Egypt, 1984.

[5] Ganguly, S. and Saha, S., A note on compactness in fuzzy setting, Fuzzy Sets and Systems, 34 (1990), 117-124.

[6] Lowen, R., Convergence in fuzzy topological spaces, General Topology and Its Appl., 10 (1979), 147-160.

[7] Mukherjee, M. N. and Sinha, S. P., Almost compact fuzzy sets in fuzzy topological spaces, Fuzzy Sets and Systems, 38 (1990), 389-396.

[8] Pu, Pao Ming and Liu, Ying Ming, Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore-Smith Convergence, J. Math. Anal. Appl., 76 (1980), 571-599.

[9] Zadeh, L. A.; Fuzzy Sets, Inform. Control, 8 (1965), 338-353.