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Basic Hypergeometric Series and Fourier Transform

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Abstract: In this paper, we shall make use of Bailey's transform and fourier transform, in order to establish summations formulae for basic hypergeometric series.

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1. Introduction, Notations and Definitions

We employ the usual notations

$$[a;q]_n = (1-a)(1-aq)...(1-aq^{n-1}), \quad n = 1, 2, 3, ...,$$

$$[a;q]_0 = 1,$$

$$[a_1, a_2, ..., a_r; q]_n = [a_1;q]_n[a_2;q]_n...[a_r;q]_n.$$

and for $|q| < 1$,

$$[a;q]_\infty = \prod_{r=0}^{\infty}(1-aq^r)$$

$$[a_1, a_2, ..., a_r; q]_\infty = [a_1;q]_\infty[a_2;q]_\infty...[a_r;q]_\infty.$$

The basic hypergeometric series is defined as,

$${}_r\Phi_s\left[\begin{matrix} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s \end{matrix}\right] = \sum_{n=0}^{\infty}\frac{(a_1, a_2, ..., a_r; q)_n z^n}{(q, b_1, b_2, ..., b_s; q)_n}, \tag{1.1}$$

where for convergence $(|z|, |q|) < 1$, if $1+s = r$ and for $1+s > r$, $|z| < \infty$.

W.N. Bailey [1] in 1944 established a simple but a very useful transform called as Bailey transform: if

$$\beta_n = \sum_{r=0}^{n}\alpha_r u_{n-r} v_{n+r}$$

and

$$\gamma_n = \sum_{r=0}^{\infty} \delta_r u_{r-n} v_{r+n},$$

where, α_r, δ_r, u_r and v_r be any functions of r only. Such that the series γ_n exists, then

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n. \tag{1.2}$$

If we take, $u_r = v_r = 1$, in the above lemma, then it take the following form

$$\beta_n = \sum_{r=0}^{n} \alpha_r, \qquad \text{and} \qquad \gamma_n = \sum_{r=0}^{\infty} \delta_{r+n}$$

then under suitable convergence conditions

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n \tag{1.3}$$

Let,

$$\alpha_r = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \sin rx f(x) dx = \bar{F}_r(r)$$

and

$$\delta_r = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \cos rx f(x) dx = \bar{F}_c(r)$$

then

$$\beta_n = \sum_{r=0}^{n} \sqrt{\frac{2}{\pi}} \int_0^{\infty} \sin rx f(x) dx = \sum_{r=0}^{n} \bar{F}_s(r).$$

$$\gamma_n = \sum_{r=0}^{\infty} \sqrt{\frac{2}{\pi}} \int_0^{\infty} \cos(r+n) x f(x) dx = \sum_{r=0}^{\infty} \bar{F}_c(r+n).$$

Putting the value of $\alpha_n, \gamma_n, \beta_n$ and δ_n in (1.3), we get

$$\sum_{n,r=0}^{\infty} \bar{F}_s(n) \bar{F}_c(r+n) = \sum_{n=0}^{\infty} \left(\sum_{r=0}^{n} \bar{F}_s(r) \right) \bar{F}_c(n) \tag{1.4}$$

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