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IN M_v^b- COMPLETE METRIC SPACE, COMMON FIXED POINT THEOREMS FOR TWO AND FOUR SELF-MAPS UNDER DIFFERENT CONTRACTION PRINCIPLES

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Abstract: Two distinct theorems are presented in this manuscript. The first one establishes the existence of coincidence points and the g-weakness of M_v^b metric space. The Reich contraction principle produces a unique common fixed point for two maps, as illustrated in various examples. Second, same concept is used to identify common fixed point for four self maps. The Kannan and Banach contraction principles were applied in conjunction with extra requirements to get the fixed points as corollaries. This theorem's approach was used to solve several examples.

Keywords and Phrases: Fixed point, self maps, Complete M_v^b metric space, m_v^b-convergent, coincidence point.

2020 Mathematics Subject Classification: 47H10, 54H25.

1. Introduction and Preliminaries

Many academics have tried to generalize and enlarge metric spaces. Take a non-empty set M and distance function $d : M \times M \to \mathbb{R}^+$. Pick any x, y, z in M. The pair (M, d) is a metric space if it satisfies $d(x, y) = 0 \Leftrightarrow x = y; d(x, y) = d(y, x); d(x, y) \leq d(x, z) + d(z, y)$. For example Mitrovic' and Radenovic' [7], Karahan and Isik [4], and Asim et al. [1] suggested the $b_v(s)$- metric, p_v^b- metric, and M_v - metric spaces. The generalization of $b_v(s)$—- metric, p_v^b- metric, and M_v - metric spaces is M_v^b- metric space, introduced by Joshi et al. [3] in 2021. M_v^b- metric space's definition and theorems that aid in understanding the proof are provided below. One of the most common real-world applications of metric spaces is distributed similarity queries [10]. However, as the internet grows, data is getting more complex. Thus it is challenging to use traditional metric similarity searches due to the data's richness, and diversity in both space and time. One of the most crucial aspects of deep metric learning is to find the correct distance metric [5] based on the data. Distributed data processing is now seeing tremendous growth in both the data management sector and academia due to the widespread use of mobile devices. Scalable metric space is necessary to provide successful query services or accurate distance metric. The structure of this work is as follows. The idea is to explain the presence of coincidence points for self-maps in this M_v^b- metric space. If the self-maps are weakly compatible pairs, then these maps have a unique common fixed point. This claim is ascertained through resolving issues. The examples are written in such a way that distance function between the identical locations have a non-zero value. Inspired by Rangamma's et al. [8] theorem, the common fixed point for four self-maps is determined in a cone rectangular metric space, by substituting the cone rectangular metric space with the excellent metric space introduced by Joshi et al. The literature has comprehensive information[1-10]. This theorem is shown using instances.

The following information would help to understand the theorems.

Definition 1.1. [6] *Let (M, d) be a metric space and $A, B : M \to M$ be two single valued functions. The functions A and B have coincidence point x if $Ax = Bx = w$ for $w \in M$, and w is called a point of coincidence of A and B. The point x is called a common fixed point of A and B if $w = x$.*

In 2017, Mitrovic' and Radenovic' initiated the $b_v(s)$ metric space.

Definition 1.2. [7] *For a non-empty set M with a real number $s \geq 1$, $v \in \mathbb{N}$ and a distance functionf $: M \times M \to \mathbb{R}^+$ satisfying*
1) $f(x, y) = 0 \Leftrightarrow x = y$
2) $f(x, y) \geq 0$
3) $f(x, y) = f(y, x)$
4) $f(x, y) \leq s[f(x, z_1) + f(z_1, z_2) + ... + f(z_v, y)]$
where $x, z_1, z_2, ...z_v, y$ are distinct and belong to M, the pair (M, f) is the $b_v(s)$ -metric space.

In 2018, Karahan and Isik created the p_v^b partial metric space.

Definition 1.3. [4] *For a non-empty set M with a real number $s \geq 1$, $v \in \mathbb{N}$ and a function $h : M \times M \to \mathbb{R}^+$ satisfying*
1) $h(x, y) = h(x, x) = h(y, y) \Leftrightarrow x = y$

2) $h(x,x) \leq h(x,y)$
3) $h(x,y) = h(y,x)$
4) $h(x,y) \leq s[h(x,z_1) + ... + h(z_v,y)] - \sum_{j=1}^{v} h(z_j,z_j)$
where $\{x, z_1, z_2, ...z_v, y\} \in M$*, the pair* (M,h) *is a* p_v^b *- partial metric space.*

In 2019, Asim et al. introduced the M_v- metric space.

Definition 1.4. [1] *For a non-empty set* M*,* $v \in \mathbb{N}$ *and a map* $g : M \times M \to \mathbb{R}^+$ *satisfying*
1) $g(x,y) = g(x,x) = g(y,y) \Leftrightarrow x = y$
2) $g_{x,y} = \min\{g(x,x), g(y,y)\}$ *and* $g_{x,y} \leq g(x,y)$
3) $g(x,y) = g(y,x)$
4) $[g(x,y) - g_{x,y}] \leq \{[g(x,z_1) - g_{x,z_1}] + ... + [g(z_v,y) - g_{z_v,y}]\}$.
where $x, z_1, z_2, ...z_v, y$ *are distinct and belong to* M*, the pair* (M,g) *is a* M_v *-metric space.*

In 2021, Joshi et al. proposed the M_v^b - metric space.

Definition 1.5. [3] *For a non-empty set* M *with a real number* $s \geq 1, v \in \mathbb{N}$ *and a distance function* $f_v^b : M \times M \to \mathbb{R}^+$ *satisfying.*
1) $f_v^b(x,y) = f_v^b(x,x) = f_v^b(y,y) \Leftrightarrow x = y,$
2) $f_{vx,y}^b \leq f_v^b(x,y)$
where $f_{vx,y}^b = \min\{f_v^b(x,x), f_v^b(y,y)\}$*, and* $F_{vx,y}^b = \max\{f_v^b(x,x), f_v^b(y,y)\}$.
3) $f_v^b(x,y) = f_v^b(y,x)$
4) $[f_v^b(x,y) - f_{vx,y}^b] \leq s\{[f_v^b(x,z_1) - f_{vx,z_1}^b] + ... + [f_v^b(z_v,y) - f_{vz_v,y}^b]\} - \sum_{j=1}^{v} f_v^b(z_j,z_j)$.
where $x, z_1, z_2, ...z_v, y$ *are terms of* M*, the pair* (M, f_v^b) *is an* M_v^b *- metric space.*

Definition 1.6. [3] *i) A sequence* $\{y_p\}$ *in* (M, f_v^b) *is* f_v^b*-convergent to* $y \in M$ *if and only if* lim *as* p *tends to +infinity* $(f_v^b(y_p,y) - f_{vy_p,y}^b)$ *tends to zero.*
ii) A sequence $\{y_p\}$ *in* (M, f_v^b) *is* f_v^b*- Cauchy Sequence if and only if* lim *as* p*,* q *tends to +infinity* $(f_v^b(y_p,y_q) - f_{vy_p,y_q}^b)$ *and* $(F_{vy_p,y_q}^b - f_{vy_p,y_q}^b)$ *exists and are finite.*
iii) Every f_v^b *- Cauchy sequence* $\{y_p\}$ *in* (M, f_v^b) *converges to* $y \in M$ *such that* lim *as* p, q *tends to +infinity* $(f_v^b(y_p,y_q) - f_{vy_p,y_q}^b)$ *and* $(F_{vy_p,y_q}^b - f_{vp,y_q}^b)$ *tends to zero.*

Definition 1.7. [3] *Let* (M, f_v^b) *be an* M_v^b *- metric space with coefficient* $s \geq 1$*,* $v \in \mathbb{N}$ *and self map* $\mathcal{B} : M \to M$ *satisfies* $f_v^b(\mathcal{B}x, \mathcal{B}y) \leq \mu f_v^b(x,y)$ *with* $0 < \mu < \frac{1}{2s}$ *and* $x, y \in M$*. Construct a sequence* $\{y_q\}$ *such that* $y_{q+1} = \mathcal{B}y_q, q \in \mathbb{N} y \in M$*. If* $\{y_q\}$ *converges to* y *then* $\{\mathcal{B}y_q\}$ *converges to* $\mathcal{B}y$*, when limit* $q \to \infty$ *is applied.*

Definition 1.8. [3] *Let* (M, f_v^b) *be an* M_v^b *- metric space with coefficient* $s \geq 1$*,* $v \in \mathbb{N}$ *and self map* $\mathcal{B} : M \to M$ *satisfies* $f_v^b(\mathcal{B}x, \mathcal{B}y) \leq \mu[f_v^b(x, \mathcal{B}x) + f_v^b(y, \mathcal{B}y)]$ *with* $\mu < \frac{1}{2s}$ *and* $x, y \in M$*. Then,* $\mathcal{B}$ *has a unique fixed point* y^* *such that* $f_v^b(y^*, y^*) = 0$*. Select* $y_0 \in M$*, the sequence of iterates* $\{\mathcal{B}^n y_0\} \subseteq M$ *converges to* $y^* \in M$*.*

Definition 1.9. [6] *Let (M, d) be a metric space and $A, B : M \to M$ be two single valued maps. If the maps A and B are weakly compatible then they commute at their coincidence point x, i.e. $Ax = Bx \Rightarrow ABx = BAx$.*

Definition 1.10. [6] *Let $f, g : X \to X$ be two self-mappings and (X, d) be a metric space. The mapping f is called g-weak contraction for any x, y in X and $\alpha, \beta, \gamma \in [0, 1); \alpha + \beta + \gamma < 1$ satisfies $d(fx, fy) \leq \alpha d(gx, gy) + \beta d(gx, fx) + \gamma d(gy, fy)$.*

Definition 1.11. [9] *Let (X, d) be a metric space and $\alpha, \beta, \gamma \in [0, 1)$. For any $x, y \in X, kx, ky \in X$ Since $k : X \to X$ be a self-map. For $\alpha < 1$, if k satisfies $d(kx, ky) \leq \alpha d(x, y)$ then k is known as Banach-type contraction. For $\beta + \gamma < 1$, if k satisfies $d(kx, ky) \leq \beta d(x, kx) + \gamma d(y, ky)$ then k is known as Kannan-type contraction. For $\alpha + \beta + \gamma < 1$, if k satisfies $d(kx, ky) \leq \alpha d(x, y) + \beta d(x, kx) + \gamma d(y, ky)$ then k is known as Reich-type contraction.*

Example 1.12. i) If $W = [0, 1]$, and distance function $d(x, y) = |x - y|^2$, then we can see that (W, d) is a metric space.
ii) Let $W = [0, 1]$, and a map is defined as $n_v^b(x, y) = |x-y|^2$. By simple calculation, it can be shown that (W, n_v^b) is an M_v^b-metric space for any v and s. And it is also a metric space.
iii) If $W = [0, 1]$ and $n_v^b(x, y) = \min\{|x|, |y|\}$, then it is clear that (W, n_v^b) is an M_v^b-metric space for any v and s. But (W, n_v^b) fails to be a metric space because $n_v^b(x, x) = |x|$ and $n_v^b(x, x) \neq 0$.
iv) Every metric space is an M_v^b-metric space, but not every M_v^b-metric space is a metric space.
v) If $W = [0, 1]$ and a map is defined as $n_v^b(x, y) = |x - y| + |x|$, then (W, n_v^b) is neither an M_v^b-metric space nor a metric space. Symmetry condition fails $n_v^b(x, y) = |x - y| + |x| \neq n_v^b(y, x) = |y - x| + |y|$.

2. Main First Result

In this section we have formulated the theorem given by Malhotra et al. [6] within an M_v^b -complete metric space, a generalized metric space.

Theorem 2.1. *Let $(\mathcal{D}, m_v^b)$ be an M_v^b - complete metric space with coefficients $s \geq 1$ and $v \in \mathbb{N}$. Let two self maps $\mathcal{A}, \mathcal{B} : \mathcal{D} \to \mathcal{D}$ satisfy*

$$m_v^b(\mathcal{B}x, \mathcal{B}y) \leq \alpha[m_v^b(\mathcal{A}x, \mathcal{A}y)] + \beta[m_v^b(\mathcal{B}x, \mathcal{A}x)] + \gamma[m_v^b(\mathcal{B}y, \mathcal{A}y)].$$

Assume that $\mathcal{B}$ satisfies $\mathcal{A}$ -weak contraction with $\alpha+\beta+\gamma < \frac{1}{s}$; $\alpha \geq 0, \beta \geq 0, \gamma \geq 0$ and for all $x, y \in \mathcal{D}$.
If $\mathcal{B}(\mathcal{D}) \subseteq \mathcal{A}(\mathcal{D})$ and $\mathcal{B}(\mathcal{D})$ or $\mathcal{A}(\mathcal{D})$ is a complete subspace of $\mathcal{D}$ then the mapping $\mathcal{B}$, $\mathcal{A}$ have a unique coincidence point in $\mathcal{D}$. Moreover, if $\mathcal{B}$ and $\mathcal{A}$ are weakly

compatible pairs, then $\mathcal{B}$ and $\mathcal{A}$ have a unique common fixed point in $\mathcal{D}$.

Proof. Let x_0 represent any random point of $\mathcal{D}$. Because $\mathcal{B}(\mathcal{D}) \subseteq \mathcal{A}(\mathcal{D})$, we may select a point $x_1 \in \mathcal{D}$ at which $\mathcal{B}x_0 = \mathcal{A}x_1$. In this manner, a sequence may be gathered. For $x_n \in \mathcal{D}$ we can locate $x_{n+1} \in \mathcal{D}$ such that $\mathcal{B}x_n = \mathcal{A}x_{n+1}$. Let's say that the sequence be $\{y_n\} \subset \mathcal{D}$ and that $y_n = \mathcal{B}x_n = \mathcal{A}x_{n+1}$ where $n = 0, 1, 2, 3....$

$$\begin{aligned} m_v^b(y_n, y_{n+1}) &= m_v^b(\mathcal{B}x_n, \mathcal{B}x_{n+1}) \leq \alpha[m_v^b(\mathcal{A}x_n, \mathcal{A}x_{n+1})] \\ &\quad + \beta[m_v^b(\mathcal{B}x_n, \mathcal{A}x_n) + \gamma[m_v^b(\mathcal{B}x_{n+1}, \mathcal{A}x_{n+1})] \\ &\leq \alpha[m_v^b(y_{n-1}, y_n)] + \beta[m_v^b(y_n, y_{n-1})] + \gamma[m_v^b(y_{n+1}, y_n)] \\ (1-\gamma)m_v^b(y_n, y_{n+1}) &\leq \alpha[m_v^b(y_{n-1}, y_n)] + \beta[m_v^b(y_n, y_{n-1})] \\ m_v^b(y_n, y_{n+1}) &\leq \frac{\alpha+\beta}{(1-\gamma)}[m_v^b(y_n, y_{n-1})] = \mu[m_v^b(y_{n-1}, y_n)] \\ &\leq (\mu)^n[m_v^b(y_0, y_1)];\ \frac{\alpha+\beta}{1-\gamma} < 1;\ \mu = \frac{\alpha+\beta}{1-\gamma}; \end{aligned}$$

For any m, n as $\lim n, m \to \infty$ $\lim_{n,m\to\infty} m_v^b(y_n y_m) \to 0$; $\lim_{n,m\to\infty} m_{vy_n,y_m}^b \to 0$; $\lim_{n,m\to\infty} m_v^b(y_n y_m) - m_{vy_n,y_m}^b \to 0$ and $\lim_{n,m\to\infty} M_{vy_n,y_m}^b - m_{vy_n,y_m}^b = 0$.

From the definition 1.6, we see that $\{y_n\}$ as the Cauchy sequence in the set $\mathcal{D}$. Because M_v^b is a complete metric space, all Cauchy sequences are convergent. As a result $\{y_n\}$ is m_v^b-convergent sequence in the set $\mathcal{D}$ and there exists $z \in \mathcal{D}$ such that $y_n \to z$.

If $\mathcal{B}(\mathcal{D})$ is a complete subspace of $\mathcal{D}$, then z exists in $\mathcal{B}(\mathcal{D}) \subseteq \mathcal{A}(\mathcal{D})$ such that $y_n \to z$ and $\mathcal{B}x_n \to z$. In order to prove that $z = \mathcal{B}x$, we can identify $x \in \mathcal{D}$ such that $z = \mathcal{A}x$. As inferred from the definition $m_v^b(z, \mathcal{B}x) = m_v^b(z, z)$ or $m_v^b(z, \mathcal{B}x) = m_v^b(\mathcal{B}x, \mathcal{B}x)$ is implied when $m_v^b(z, \mathcal{B}x) - m_{vz,\mathcal{B}x}^b = 0$. Hence, we arrive at $z = \mathcal{B}x$ and $z = \mathcal{B}x = \mathcal{A}x$. x is the coincidence point in $\mathcal{D}$ as a result. Similarly, if $\mathcal{A}(\mathcal{D})$ is a complete subspace of $\mathcal{D}$, then there exists $z' \in \mathcal{A}(\mathcal{D})$ such that $y_n \to z'$ and $\mathcal{A}x_{n+1} \to z'$. we may locate $x' \in \mathcal{D}$ such that $z' = \mathcal{A}x'$ to get $z' = \mathcal{B}x'$. So, $z' = \mathcal{B}x' = \mathcal{A}x'$. As a result x' is the coincidence point in $\mathcal{D}$. In both situations, coincidence points x, x' in $\mathcal{D}$ exist. The points of coincidence are z, z'.

To demonstrate that $z = z'$, and the point of coincidence z, z' are distinct. $z = \mathcal{B}x = \mathcal{A}x$ and $z' = \mathcal{B}x' = \mathcal{A}x'$. $m_v^b(z, z) \leq [m_v^b(z, z')]$ and $m_v^b(z', z') \leq [m_v^b(z, z')]$ taken from the definition 1.5.

$m_v^b(z, z') \leq \alpha[m_v^b(z', z)] + \beta[m_v^b(z, z)] + \gamma[m_v^b(z', z')] \leq \{\alpha + \beta + \gamma\}[m_v^b(z, z')] < [m_v^b(z, z')]$. Due to $\alpha + \beta + \gamma < 1/s < 1$, an inconsistency exists here. So $z = z'$ and $z' = \mathcal{B}x' = \mathcal{A}x' = z = \mathcal{B}x = \mathcal{A}x$ are true. Therefore $z = z'$ is the point of coincidence in $\mathcal{D}$ and the point of coincidence in $\mathcal{D}$ is unique.

If $\mathcal{B}$ and $\mathcal{A}$ are weakly compatible, then $\mathcal{B}z = \mathcal{B}\mathcal{A}x = \mathcal{A}\mathcal{B}x = \mathcal{A}z \Rightarrow \mathcal{B}z = \mathcal{A}z = w$. w is another point of coincidence, however because of the uniqueness point of coincidence $w = z$. There is only one point $z \in \mathcal{D}$ so that $\mathcal{B}z = \mathcal{A}z = z.z$ is the singular common fixed point of the self-mappings $\mathcal{B}$ and $\mathcal{A}$. Hence the theory is established.

Example 2.2. To show the distance function between same points is not zero. The definition of $d : W \times W \to W$ is $d(x, y) = \max\{x, y\}$ for every x, y in the nonempty set $W = \mathbb{R}^+$. d satisfies all the characteristics of definition 1.5. (W, d) is an M_v^b- metric space for any $s \geq 1$ and v. Metric space means $d(x, y) = 0 \Leftrightarrow x = y$. Due to the fact that $d(x, y) \neq 0$ when $x = y\{d(2, 2) = 2 \neq 0\}$, this example is not a metric space.

Example 2.3. Let $Q = \{a, b, c, d\}$ and the map $m_v^b : Q \times Q \to \mathbb{R}^+$ be defined as
1) $m_v^b(x, y) = (0, 0)$ for $x = y$ and $m_v^b(x, y) = m_v^b(y, x)$.
2) $m_v^b(a, b) = (3, 9)$.
3) $m_v^b(a, c) = m_v^b(b, c) = (1, 3)$.
4) $m_v^b(a, d) = m_v^b(b, d) = m_v^b(c, d) = (4, 12)$.
If $B, A : Q \to Q$ are self maps given by $B(a) = B(b) = B(c) = c, B(d) = a$ and $A(a) = b, A(b) = a, A(c) = c, A(d) = d$, then find the common fixed point for A and B. Apply Theorem 2.1.
Solution. The map m_v^b is expressed in Table 1.

m_v^b	a	b	c	d
a	(0,0)	(3,9)	(1,3)	(4,12)
b	(3,9)	(0,0)	(1,3)	(4,12)
c	(1,3)	(1,3)	(0,0)	(4,12)
d	(4,12)	(4,12)	(4,12)	(0,0)

Table 1: The Map m_v^b

With values of $s = 1$ and $v = 2$, the pair (Q, m_v^b) is an M_v^b-metric space. There are several potential values for α, β, γ with $\alpha + \beta + \gamma < \frac{1}{s}$; $\alpha \geq 0, \beta \geq 0, \gamma \geq 0$. Specifically, pick $\alpha > 1/8$ and $\alpha + \beta + \gamma = 3\alpha < \frac{1}{s}$, if $\alpha = \beta = \gamma$. Select $\alpha = \frac{1}{4}$; $s = 1$. $m_v^b(Bx, By) \leq \alpha[m_v^b(Ax, Ay)] + \beta[m_v^b(Bx, Ax)] + \gamma[m_v^b(By, Ay)]$. $(1, 3) \leq 1/4[(4, 12) + (0, 0) + (4, 12)]$. For various combinations listed in Table 2, the conditions of theorem may be confirmed. This pair (Q, m_v^b) fulfils all of Theorem 2.1's inequalities and is an M_v^b -metric space for $v \in \mathbb{N}$; $\alpha = \beta = \gamma = \frac{1}{4}$ and $s = 1$. So the unique common fixed point is "c".

x, y	Bx, By	Ax, Ay	Ax, Bx	Ay, By	$m_v^b(Bx, By)$	$m_v^b(Ax, Ay)$	$m_v^b(Ax, Bx)$	$m_v^b(Ay, By)$
a, a	c, c	b, b	b, c	b, c	(0, 0)	(0, 0)	(1, 3)	(1, 3)
a, b	c, c	b, a	b, c	a, c	(0, 0)	(3, 9)	(1, 3)	(1, 3)
a, c	c, c	b, c	b, c	c, c	(0, 0)	(1, 3)	(1, 3)	(0, 0)
a, d	c, a	b, d	b, c	d, a	(1, 3)	(4, 12)	(1, 3)	(4, 12)
b, b	c, c	a, a	a, c	a, c	(0, 0)	(0, 0)	(1, 3)	(1, 3)
b, c	c, c	a, c	a, c	c, c	(0, 0)	(1, 3)	(1, 3)	(0, 0)
b, d	c, a	a, d	a, c	d, a	(1, 3)	(4, 12)	(1, 3)	(4, 12)
c, c	c, c	c, c	c, c	c, c	(0, 0)	(0, 0)	(0, 0)	(0, 0)
c, d	c, a	c, d	c, c	d, a	(1, 3)	(4, 12)	(0, 0)	(4, 12)
d, d	a, a	d, d	d, a	d, a	(0, 0)	(4, 12)	(4, 12)	(4, 12)

Table 2: Different values of x, y

Example 2.4. Validate Theorem 2.1. The map $m_v^b : M \times M \to \mathbb{R}^+$, where $M = [0, 1]$ be defined by $m_v^b(x, y) = |x - y|$. Let the self maps be $B, A : [0, 1] \to [0, 1]$, where

$$\begin{aligned} Bx &= 7/8 && x \in [0, 1/2] \\ &= 5/6 && x \in (1/2, 1] \\ Ax &= 1 && x \in [0, 1/2] \\ &= 5/6 && x \in (1/2, 1) \\ &= 7/8 && x = 1 \end{aligned}$$

Solution. The map $m_v^b : M \times M \to \mathbb{R}^+$ is an M_v^b- metric space. There are a wide range of options for α, β, γ with $\alpha + \beta + \gamma < \frac{1}{s}$; $\alpha \geq 0, \beta \geq 0, \gamma \geq 0$. If $\alpha = \beta = \gamma$ then $\frac{1}{7} < \alpha$ and $\alpha + \beta + \gamma = 3\alpha < \frac{1}{s}$. Take $\alpha = \frac{1}{5}$; $s = 1$. When $x = 1, y \in [0, 1/2]$

$$\begin{aligned} m_v^b(Bx, By) &\leq \alpha[m_v^b(Ax, Ay)] + \beta[m_v^b(Bx, Ax)] + \gamma[m_v^b(By, Ay)]. \\ |5/6 - 7/8| &\leq (|7/8 - 1| + |5/6 - 7/8| + |7/8 - 1|)\alpha \\ 1/24 &\leq (1/8 + 1/24 + 1/8)1/5 \end{aligned}$$

Note: $m_v^b(x, x) = 0$.
All of Theorem 2.1's inequalities are satisfied by this pair (M, m_v^b), which is an M_v^b-metric space for $v \in \mathbb{N}$; $\alpha = \beta = \gamma = \frac{1}{5}$ and $s = 1$. As a result, the unique common fixed point is $5/6.(5/6 = A5/6 = B5/6)$. All probable instances are shown in Table 3 below for verification.

S.N.	Case	$\|Bx-By\|$	$\leq$	$\|Ax-Ay\|$	$\|Bx-Ax\|$	$\|By-Ay\|$
1	$x,y \in [0,1/2]$	$\|7/8-7/8\| = 0$	$\leq$	$\|1-1\| = 0$	$\|7/8-1\| = 1/8$	$\|7/8-1\| = 1/8$
2	$x \in [0,1/2], y \in (1/2,1)$	$\|7/8-5/6\| = 1/24$	$\leq$	$\|1-5/6\| = 1/6$	$\|7/8-1\| = 1/8$	$\|5/6-5/6\| = 0$
3	$x \in [0,1/2], y = 1$	$\|7/8-5/6\| = 1/24$	$\leq$	$\|1-7/8\| = 1/8$	$\|7/8-1\| = 1/8$	$\|5/6-7/8\| = 1/24$
4	$x \in (1/2,1), y \in [0,1/2]$	$\|5/6-7/8\| = 1/24$	$\leq$	$\|5/6-1\| = 1/6$	$\|5/6-5/6\| = 0$	$\|7/8–1\| = 1/8$
5	$x,y \in (1/2,1)$	$\|5/6-5/6\| = 0$	$\leq$	$\|5/6-5/6\| = 0$	$\|5/6-5/6\| = 0$	$\|5/6-5/6\| = 0$
6	$x \in (1/2,1), y = 1$	$\|5/6-5/6\| = 0$	$\leq$	$\|5/6-7/8\| = 1/24$	$\|5/6-5/6\| = 0$	$\|5/6-7/8\| = 1/24$
7	$x = 1, y \in [0,1/2]$	$\|5/6-7/8\| = 1/24$	$\leq$	$\|7/8-1\| = 1/8$	$\|5/6-7/8\| = 1/24$	$\|7/8-1\| = 1/8$
8	$x = 1, y \in (1/2,1)$	$\|5/6-5/6\| = 0$	$\leq$	$\|7/8-5/6\| = 1/24$	$\|5/6-7/8\| = 1/24$	$\|5/6-5/6\| = 0$
9	$x = 1, y = 1$	$\|5/6-5/6\| = 0$	$\leq$	$\|7/8-7/8\| = 0$	$\|5/6-7/8\| = 1/24$	$\|5/6-7/8\| = 1/24$

Table 3: Various values of x, y

Example 2.5. Check Theorem 2.1 for map $m_v^b : M \times M \to \mathbb{R}^+$ and $M = [0,1]$ satisfy the following

$$m_v^b(x,y) = \begin{cases} \frac{x}{2}; & y = 0 \\ 0; & x = y \\ \frac{y}{2}; & x = 0 \\ \frac{x+y}{x+y+1}; & \text{else} \end{cases}$$

And $B, A : [0,1] \to [0,1]$ be such that

$$\begin{aligned} Bx &= 6/7 \quad x \in [0,1/2] \\ &= 5/6 \quad x \in (1/2,1] \\ Ax &= 4/5 \quad x \in [0,1/2] \\ &= 5/6 \quad x \in (1/2,1) \\ &= 6/7 \quad x = 1 \end{aligned}$$

Solution. Clearly, the pair (M, m_v^b) is an M_v^b- metric space for any $v \in \mathbb{N}$ and any $s \geq 1$. We cannot select values for α, β, γ with $\alpha + \beta + \gamma < \frac{1}{s}$; $\alpha \geq 0, \beta \geq 0, \gamma \geq 0$; such that $m_v^b(Bx, By) \leq \alpha[m_v^b(Ax, Ay)] + \beta[m_v^b(Bx, Ax)] + \gamma[m_v^b(By, Ay)]$. If $x \in (0,1/2]$ and $y \in (1/2,1)$ then $71/113 \leq \alpha(49/79) + \beta(58/93)$. If $x \in (0,1/2]$ and $y = 1$ then $71/113 \leq \alpha(58/93) + \beta(58/93) + \gamma(71/113)$. The mapping B is not A-weak contraction. Consequently, Theorem 2.1's requirements are not confirmed. Despite the fact that the unique common fixed point is $5/6, (5/6 = A5/6 = B5/6)$, Theorem 2.1 does not apply to this situation.

3. Second Result

As the second result, motivated by Rangamma et al [8], we show the following theorem.

Theorem 3.1. *Let (W, m_v^b) be an M_v^b-complete metric space, and assume that $K, L, M,$ and $N : W \to W$ be four self-mappings of W that satisfy either one of the* *following inequality conditions for any $x, y \in W$ and $\alpha, \beta, \gamma \geq 0$ with $\alpha+\beta+\gamma < \frac{1}{s}$.*

$$m_v^b(Kx, Ly) \leq \alpha m_v^b(Mx, Ny) + \beta m_v^b(Mx, Kx) + \gamma m_v^b(Ny, Ly). \tag{3.1}$$

$$m_v^b(Kx, Ky) \leq \alpha m_v^b(Mx, My) + \beta m_v^b(Mx, Kx) + \gamma m_v^b(My, Ky). \tag{3.2}$$

$$m_v^b(Lx, Ly) \leq \alpha m_v^b(Nx, Ny) + \beta m_v^b(Nx, Lx) + \gamma m_v^b(Ny, Ly). \tag{3.3}$$

If $K(W) \subseteq N(W), L(W) \subseteq M(W)$ and one of $K(W), N(W), L(W), M(W)$ is a complete subspace of W, then the pairs (K, M) and (L, N) have a unique point of coincidence in W. Additionally, if the pairs (K, M) and (L, N) are weakly compatible then K, L, M, N have a unique common fixed point in W.

Proof. Let x_0 be any random point in W. Construct a sequence $\{y_n\}$ that is applicable to every $n \geq 0$ and $y_{2n} = Kx_{2n} = Nx_{2n+1}$ and $y_{2n+1} = Lx_{2n+1} = Mx_{2n+2}.\{K(W) \subseteq N(W), L(W) \subseteq M(W)\}$.

$$\begin{aligned} m_v^b(y_{2n+1}, y_{2n+1}) &= m_v^b(Kx_{2n+1}, Lx_{2n+1}) \\ &\leq \alpha m_v^b(Mx_{2n+1}, Nx_{2n+1}) + \beta m_v^b(Mx_{2n+1}, Kx_{2n+1}) + \gamma m_v^b(Nx_{2n+1}, Lx_{2n+1}) \\ &\leq \alpha m_v^b(y_{2n}, y_{2n}) + \beta m_v^b(y_{2n}, y_{2n+1}) + \gamma m_v^b(y_{2n}, y_{2n+1}) \\ m_v^b(y_{2n+1}, y_{2n+1}) &= m_v^b(Kx_{2n+1}, Kx_{2n+1}) \\ &\leq \alpha m_v^b(Mx_{2n+1}, Mx_{2n+1}) + \beta m_v^b(Mx_{2n+1}, Kx_{2n+1}) + \gamma m_v^b(Mx_{2n+1}, KLx_{2n+1}) \\ &\leq \alpha m_v^b(y_{2n}, y_{2n}) + \beta m_v^b(y_{2n}, y_{2n+1}) + \gamma m_v^b(y_{2n}, y_{2n+1}) \\ m_v^b(y_{2n+1}, y_{2n+1}) &= m_v^b(Lx_{2n+1}, Lx_{2n+1}) \\ &\leq \alpha m_v^b(Nx_{2n+1}, Nx_{2n+1}) + \beta m_v^b(Nx_{2n+1}, Lx_{2n+1}) + \gamma m_v^b(Nx_{2n+1}, Lx_{2n+1}) \\ &\leq \alpha m_v^b(y_{2n}, y_{2n}) + \beta m_v^b(y_{2n}, y_{2n+1}) + \gamma m_v^b(y_{2n}, y_{2n+1}) \end{aligned}$$

We reached the same findings using the three inequalities presented in the theorem for y_{2n+1} and y_{2n+1}. Likewise, for y_{2n} and y_{2n+1}. So, we would just take into account the first inequality (3.1).

$$\begin{aligned} m_v^b(y_{2n}, y_{2n+1}) &= m_v^b(Kx_{2n}, Lx_{2n+1}) \\ &\leq \alpha m_v^b(Mx_{2n}, Nx_{2n+1}) + \beta m_v^b(Mx_{2n}, Kx_{2n}) + \gamma m_v^b(Nx_{2n+1}, Lx_{2n+1}) \\ &\leq \alpha m_v^b(y_{2n-1}, y_{2n}) + \beta m_v^b(y_{2n-1}, y_{2n}) + \gamma m_v^b(y_{2n}, y_{2n+1}) \\ (1-\gamma) m_v^b(y_{2n}, y_{2n+1}) &\leq (\alpha+\beta) m_v^b(y_{2n-1}, y_{2n}) \\ m_v^b(y_{2n}, y_{2n+1}) &\leq \frac{\alpha+\beta}{1-\gamma} m_v^b(y_{2n-1}, y_{2n}) \end{aligned}$$

$$m_v^b(y_{2n}, y_{2n+1}) \leq \mu m_v^b(y_{2n-1}, y_{2n}) \quad \text{where} \quad \mu = \frac{\alpha+\beta}{1-\gamma} < 1.$$

Apply limit $p, q \to \infty$ to any p, q to get $\lim_{p,q\to\infty}[m_v^b(y_p, y_q) - m_{vy_p,y_q}^b] = 0$ and $\lim_{p,q\to\infty}[M_{vy_p,y_q}^b - m_{vy_p,y_q}^b] = 0$. The definition 1.6 gives m_v^b- Cauchy sequence $\{y_n\}$ in (W, m_v^b). Given that W is an M_v^b- complete metric space, every Cauchy sequence must be convergent and have a finite limit. Assume g be the limit.

If $M(W)$ is a complete subspace of W, then $g \in M(W)$ exists such that $\lim_{n\to\infty} y_{2n+1} = \lim_{n\to\infty} Mx_{2n+2} = g$. There is always an x in W where $g = Mx$ be found. To establish $Kx = g$, which implies that $m_v^b(Kx, g) = m_v^b(g, g)$ OR $m_v^b(Kx, g) = m_v^b(Kx, Kx)$, we can demonstrate $m_v^b(Kx, g) - m_{vKx,g}^b = 0$. So, $Kx = g$ and $g = Kx = Mx$. Consequently, x is the coincidence point in (M, K) and g is the point of coincidence in W. Considering that $K(W) \subseteq N(W)$, there must be some x' in N where $Nx' = g$. It is evident that $Lx' = g$ and $g = Lx' = Nx'$ from definitions. Then x' is the coincidence point in (L, N) and g is the point of coincidence in $W.g = Mx = Kx = Nx' = Lx'$. The pairings (M, K) and (L, N) be weakly compatible mappings. Take $g1$ and $g2$ be two points of coincidence in W such that $Kg = KMx = MKx = Mg = g1$ and $Lg = LNx' = NLx' = Ng = g2$. Simple math indicates that $m_v^b(g1, g2) < m_v^b(g1, g2)$ if $g1 \neq g2$. A contradiction exists here. $g1 = g2$ is therefore true, and the coincidence point is unique.

Furthermore, $Kg = KMx = MKx = Mg = g1 = Lg = LNx' = NLx' = Ng = g2$ indicates that $Kg = Mg = Lg = Ng = g1 = g2$ and $g = Kx = Mx = Nx' = Lx'$. The point of coincidence is unique. So, $g = g1 = g2$. As a result, $g = Kg = Mg = Lg = Ng$ is obtained. The common fixed point of K, L, M, N is then g. If h is another common fixed point of K, L, M, N, then $h = Kh = Mh = Lh = Nh$. Let $g \neq h$. We can see from the definitions that it is a contradiction, hence $g = h$. The unique common fixed point of K, L, M, and N is "g".

If $K(W), N(W)$, or $L(W)$ is a complete subspace of W, then (K, M) and (L, N) have a unique point of coincidence in W. If the pairs (K, M) and (L, N) are weakly compatible, then K, L, M, N have a unique common fixed point in W. Therefore, g is the unique common fixed point of K, L, M, N. Theorem's proof is now complete.

Corollary 3.2. Banach contraction principle

Let (W, m_v^b) be an M_v^b-complete metric space, and assume that K, L, M, and $N : W \to W$ be four self-mappings of W that satisfy either one of the following inequality conditions for any $x, y \in W$ and $0 \leq \alpha < \frac{1}{s}$; $s \geq 1$.

$$m_v^b(Kx, Ly) \leq \alpha m_v^b(Mx, Ny)$$

$$m_v^b(Kx, Ky) \leq \alpha m_v^b(Mx, My)$$

If $K(W) \subseteq N(W), L(W) \subseteq M(W)$ and one of $K(W), N(W), L(W), M(W)$ is a *complete subspace of W, then the pairs (K, M) and (L, N) have a unique point of coincidence in W. Additionally, if the pairs (K, M) and (L, N) are weakly compatible then K, L, M, N have a unique common fixed point in W.*

Proof. The proof is obtained by inserting $\beta = 0$ and $\gamma = 0$ in Theorem 3.1.

Corollary 3.3. Kannan contraction principle

Let (W, m_v^b) be an M_v^b-complete metric space, and assume that K, L, M, and $N : W \to W$ be four self-mappings of W that satisfy either one of the following inequality conditions for any $x, y \in W$ and $\beta, \gamma \geq 0$ with $\beta + \gamma < \frac{1}{s}$; $s \geq 1$.

$$m_v^b(Kx, Ly) \leq \beta m_v^b(Mx, Kx) + \gamma m_v^b(Ny, Ly).$$
$$m_v^b(Kx, Ky) \leq \beta m_v^b(Mx, Kx) + \gamma m_v^b(My, Ky).$$

If $K(W) \subseteq N(W), L(W) \subseteq M(W)$ and one of $K(W), N(W), L(W), M(W)$ is a complete subspace of W, then the pairs (K, M) and (L, N) have a unique point of coincidence in W. Additionally, if the pairs (K, M) and (L, N) are weakly compatible then K, L, M, N have a unique common fixed point in W.

Proof. We get the proof by changing $\alpha = 0$ in Theorem 3.1.

Example 3.4. Find common fixed point (use Theorem 3.1). Let $m_v^b : Q \times Q \to \mathbb{R}^+, Q = [0, 1]$ is defined by $m_v^b(x, y) = |x - y|$. Let the self-maps be $K, L, M, N : [0, 1] \to [0, 1]$, where

$$\begin{aligned}
Kx &= 7/8 && x \in [0, 1/2] \\
&= 5/6 && x \in (1/2, 1] \\
Lx &= 6/7 && x \in [0, 1/2] \\
&= 5/6 && x \in (1/2, 1] \\
Mx &= 4/5 && x \in [0, 1/2] \\
&= 5/6 && x \in (1/2, 1) \\
&= 6/7 && x = 1 \\
Nx &= 1 && x \in [0, 1/2] \\
&= 5/6 && x \in (1/2, 1) \\
&= 7/8 && x = 1
\end{aligned}$$

Solution. The map $m_v^b : Q \times Q \to \mathbb{R}^+$ is a complete metric space for any s, v. Pick $s = 1$ and $v = 2$. There are several alternative choices for α, β, γ with $\alpha + \beta + \gamma < \frac{1}{s}$; $\alpha \geq 0; \beta \geq 0; \gamma \geq 0$; such as $(\alpha = 0.1, \beta = 0.6, \gamma = 0.1)$ or $(\alpha = 0.2, \beta = 0.5, \gamma = 0.1)$. Especially if $\alpha = \beta = \gamma$ then $\alpha + \beta + \gamma = 3\alpha < \frac{1}{s}$.

Select $\alpha = \frac{1}{4}$, $s = 1$ and in the case where $x \in [0, 1/2]; y = 1$;

$$\begin{aligned} m_v^b(Kx, Ly) &\leq \alpha[m_v^b(Mx, Ny)] + \beta[m_v^b(Mx, Kx)] + \gamma[m_v^b(Ny, Ly)]. \\ |7/8 - 5/6| &\leq (|4/5 - 7/8| + |4/5 - 7/8| + |7/8 - 5/6|)\alpha \\ 1/24 &\leq (3/40 + 3/40 + 1/24)1/4 \end{aligned}$$

Note: $m_v^b(x, x) = 0$.
Multiple cases of x and y are presented in Table 4, for reference.

S.N.	Case	$\lvert Kx - Ly\rvert$	$\leq$	$\lvert Mx - Ny\rvert$	$\lvert Mx - Kx\rvert$	$\lvert Ny - Ly\rvert$
1	$x, y \in [0, 1/2]$	$\lvert 7/8 - 6/7\rvert = 1/56$	$\leq$	$\lvert 4/5 - 1\rvert = 1/5$	$\lvert 4/5 - 7/8\rvert = 3/40$	$\lvert 1 - 6/7\rvert = 1/7$
2	$x \in [0, 1/2], y \in (1/2, 1)$	$\lvert 7/8 - 5/6\rvert = 1/24$	$\leq$	$\lvert 4/5 - 5/6\rvert = 1/30$	$\lvert 4/5 - 7/8\rvert = 3/40$	$\lvert 5/6 - 5/6\rvert = 0$
3	$x \in [0, 1/2], y = 1$	$\lvert 7/8 - 5/6\rvert = 1/24$	$\leq$	$\lvert 4/5 - 7/8\rvert = 3/40$	$\lvert 4/5 - 7/8\rvert = 3/40$	$\lvert 7/8 - 5/6\rvert = 1/24$
4	$x \in (1/2, 1), y \in [0, 1/2]$	$\lvert 5/6 - 6/7\rvert = 1/42$	$\leq$	$\lvert 5/6 - 1\rvert = 1/6$	$\lvert 5/6 - 5/6\rvert = 0$	$\lvert 1 - 6/7\rvert = 1/7$
5	$x, y \in (1/2, 1)$	$\lvert 5/6 - 5/6\rvert = 0$	$\leq$	$\lvert 5/6 - 5/6\rvert = 0$	$\lvert 5/6 - 5/6\rvert = 0$	$\lvert 5/6 - 5/6\rvert = 0$
6	$x \in (1/2, 1), y = 1$	$\lvert 5/6 - 5/6\rvert = 0$	$\leq$	$\lvert 5/6 - 7/8\rvert = 1/24$	$\lvert 5/6 - 5/6\rvert = 0$	$\lvert 7/8 - 5/6\rvert = 1/24$
7	$x = 1, y \in [0, 1/2]$	$\lvert 5/6 - 6/7\rvert = 1/42$	$\leq$	$\lvert 6/7 - 1\rvert = 1/7$	$\lvert 6/7 - 5/6\rvert = 1/42$	$\lvert 1 - 6/7\rvert = 1/7$
8	$x = 1, y \in (1/2, 1)$	$\lvert 5/6 - 5/6\rvert = 0$	$\leq$	$\lvert 6/7 - 5/6\rvert = 1/42$	$\lvert 6/7 - 5/6\rvert = 1/42$	$\lvert 5/6 - 5/6\rvert = 0$
9	$x = 1, y = 1$	$\lvert 5/6 - 5/6\rvert = 0$	$\leq$	$\lvert 6/7 - 7/8\rvert = 1/56$	$\lvert 6/7 - 5/6\rvert = 1/42$	$\lvert 7/8 - 5/6\rvert = 1/24$

Table 4: Different cases of x, y

This pair (Q, m_v^b) is an M_v^b -metric space for $v = 2 \in \mathbb{N}; \alpha = \beta = \gamma = \frac{1}{4}$ and $s = 1$. Simple calculations demonstrate that (K, M) and (L, N) are compatible.
$x \in [0, 1/2] KMx = K4/5 = 5/6; MKx = M7/8 = 5/6$.
$x \in (1/2, 1) KMx = K5/6 = 5/6; MKx = M5/6 = 5/6$.
$x = 1; KMx = K6/7 = 5/6; MKx = M5/6 = 5/6$.
$x \in [0, 1/2]; LNx = L1 = 5/6; NLx = N6/7 = 5/6$.
$x \in (1/2, 1); LNx = L5/6 = 5/6; NLx = N5/6 = 5/6$.
$x = 1; LNx = L7/8 = 5/6; NLx = N5/6 = 5/6$.
$(5/6 = K5/6 = L5/6 = M5/6 = N5/6)$
All of Theorem's 3.1 requirements are satisfied. As a result, the unique common fixed point is "5/6".

Example 3.5. The distance function $m_v^b(x, y) = \max\{|x|, |y|\}$ for any x, y in $W = [0, 1]$. Let $K, L, M, N : W \to W$ be four self-mappings defined by $Kx = x/4; Lx = x/8; Nx = x/2; Mx = x$. To find unique fixed point, apply Theorem 3.1.

Solution.
Note: Take notice that $m_v^b(x, x) = x$ and $m_v^b(x, x) \neq 0$.
(W, m_v^b) is an M_v^b- complete metric space for $v = 2 \in \mathbb{N}$ and $s \geq 1$ is obvious. It is clear that $K(W) \subseteq N(W)$ and $L(W) \subseteq M(W)$. $KMx = Kx = x/4$ and $MKx = Mx/4 = x/4 (K, M)$ are compatible. $LNx = Lx/2 = x/16$, and $NLx = Nx/8 = x/16 (L, N)$ are compatible. When $\alpha = \frac{1}{2}$; $\beta = \gamma = 0$ the inequality below

is correct. $m_v^b(Kx, Ly) \leq \alpha m_v^b(Mx, Ny) + \beta m_v^b(Mx, Kx) + \gamma m_v^b(Ny, Ly)$.
All of Theorem's properties are met, and there is a unique common fixed point for K, L, M, N, which is $x = 0$. Theorem 3.1 has been proven, and a common point exists and is unique.

Example 3.6. If $W = [0, 1]$, is defined as $m_v^b(x, y) = |x - y|$. Let $K, L, M, N : W \to W$ be four self mappings defined as $Kx = (x/2)^4; Lx = (x/2)^8; Nx = (x/2)^2; Mx = (x/2)^1$. Show that Theorem 3.1 is true.
Solution. Note: $m_v^b(x, x) = 0$.
(W, m_v^b) is an M_v^b- complete metric space for any $v \in \mathbb{N}$ and $s \geq 1$. $K(W) \subseteq N(W)$ and $L(W) \subseteq M(W)$. $KMx = K(x/2) = (x/4)^4$, and $MKx = M(x/2)^4 = (x/2)^4/2$ implies that (K, M) are NOT-compatible. $LNx = L(x/2)^2 = (x/2)^{16}/2^8$, and $NLx = N(x/2)^8 = (x/2)^{16}/4$ implies that (L, N) are NOT-compatible. When $\alpha = 1/2, \beta = \gamma = 0$, the inequality is accurate. $m_v^b(Kx, Ly) \leq \alpha m_v^b(Mx, Ny) + \beta m_v^b(Mx, Kx) + \gamma m_v^b(Ny, Ly)$. Unique common fixed point for K, L, M, N is $x = 0$. None of Theorem 3.1's criteria are true. Despite the existence and uniqueness of the common fixed point, Theorem 3.1 cannot be applied.

Example 3.7. If $W = [0, 1]$ and a map defined as $m_v^b(x, y) = |x - y|$, then (W, m_v^b) is a complete metric space for any $v \in N$ and $s \geq 1$. Assume $K, L, M, N : W \to W$ be the four self-mappings defined as in the following:
$Kx = x^4; Lx = x^8, Nx = x^2, Mx = x^1$. Can we produce a singular fixed point under Theorem 3.1?.
Solution. Simple math indicates that $K(W) \subseteq N(W)$ and $L(W) \subseteq M(W)$. $KMx = Kx = x^4, MKx = Mx^4 = x^4$, and (K, M) are compatible. $LNx = Lx^2 = x^{16}; NLx = Nx^8 = x^{16}; (L, N)$ are compatible. A specific sample point $x = 1, y = 0$ violates inequality $m_v^b(Kx, Ly) \leq \alpha m_v^b(Mx, Ny) + \beta m_v^b(Mx, Kx) + \gamma m_v^b(Ny, Ly)$. As a contradiction for the value of α, we arrive at $1 \leq \alpha$. Although Theorem 3.1 cannot be proved and the point is not unique, we can clearly detect two common fixed points $x = 0$ and $x = 1$, for K, L, M, N.

Remark 3.8. *Theorem 2.1 and Theorem 3.1 requirements are necessary, but not sufficient. If the theorem's prerequisites are fulfilled, a unique fixed point must exists. A common fixed point may or may not exist if the prerequisites of the theorem are not fulfilled.*

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