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INTUITIONISTIC FUZZY ASPECTS OF MULTIPLICATION N-GROUPS

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Abstract: Intuitionistic fuzzy (IF) sets were first put forward by K. Atanassov as a generalised notation for fuzzy sets in 1983. The concepts of intuitionistic fuzzy near rings, intuitionistic fuzzy N-groups, intuitionistic fuzzy N-subgroups, intuitionistic fuzzy ideals of N-group are described by P. Saikia, L. K. Barthakur and H. K. Saikia in [10, 11]. Fuzzy distributive modules are studied by Sh. B. Semeein and I. M. A. Hadi in [4]. Intuitionistic fuzzy multiplication modules are studied by P. K. Sharma in [13]. We extend the notion of intuitionistic fuzzy multiplication modules to intuitionistic fuzzy multiplication N-groups. Here, we define intuitionistic fuzzy multiplication N-group and some basic definitions that are needed in this sequel. The relations ship of multiplication N-groups, intuitionistic fuzzy multiplication N-groups, DN-groups and intuitionistic fuzzy DN-groups are also studied.

Keywords and Phrases: Near rings, N- groups, DN-groups, multiplication DN-

groups, intuitionistic fuzzy multiplication N-groups.

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1. Preliminaries

In this study, E is taken into account as the unitary left N-group and N is thought of as a zero symmetric commutative right near ring with unity. The basic concepts used in this paper are available in [2, 11]. We defined some of the definitions and results that are needed in this sequel. Throughout this paper $\wedge$ and $\vee$ denote maximum and minimum in the unit interval $[0, 1]$ and in general $\gamma, \lambda \in [0, 1]$ with $\gamma + \lambda \leq 1$. Here, we use IF to mean intuitionistic fuzzy and the symbols $\leq_N, \leq_{IFN}, \trianglelefteq_N$, $\triangleleft$ and $\triangleleft_{IF}$ are used to mean N-subgroup, intuitionistic fuzzy N-subgroup, normal N-group, ideal and intuitionistic fuzzy ideal respectively.

Definition 1.1. [10] *If the following standards are satisfied, a nonempty set N combined with the binary operations "$+$" and "." is referred to as right near ring.*
i. $(N, +)$ is a group(not necessarily abelian).
ii. $(N, .)$ is a semi group.
iii. $(p + b)c = pc + bc, \forall p, b, c \in N$.

Definition 1.2. [10] *An additive group $(E, +)$ is referred to be a left N-group, if $\exists$ a map $N \times E \to E, (n, u) \to nu$ in which the following standards are satisfied-*
i. $(m + n)u = mu + nu$.
ii. $(mn)u = m(nu)$.

It is to be noted that N is itself an N-group over itself. If for $1 \in N$ such that $1.u = u \;\forall\; u \in E$, then E is called an unitary N-group.

Definition 1.3. [10] *In the event that A is a subgroup of $(E, +)$ and $NA \subseteq A$ for any $A \subseteq E$, then A is referred to as an N-subgroup of E.*

Definition 1.4. [10] *If F is a normal subgroup of $(E, +)$ with $na \in F$, $\forall\; n \in N, a \in F$, then F is referred to be a normal N-subgroup of E.*

Definition 1.5. [6] *If D is a normal subgroup of $(E, +)$ such that $n(a+e)-ne \in D$, $\forall\; n \in N, a \in D, e \in E$, then D is referred to as an ideal of E.*

Definition 1.6. [11] *The object $A =< \phi_A, \psi_A >= \{< s, \phi_A(s), \psi_A(s) > | s \in S\}$ is referred to as an intuitionistic fuzzy (IF) set on a non empty set S, where ϕ_A and ψ_A are fuzzy subset of S such that $0 \leq \phi_A(s) + \psi_A(s) \leq 1$.*

Some Operations on IF Sets-[11]
Let $M =< \phi_M, \psi_M >$ and $B =< \phi_B, \psi_B >$ be IF sets on S. Then
i. $M \subseteq B \Leftrightarrow \phi_M \leq \phi_B, \psi_M \geq \psi_B$.

ii. $M = B \Leftrightarrow \phi_M = \phi_B, \psi_M = \psi_B$.
iii. $M^c = \bar{M} =< \psi_M, \phi_M >$.
iv. $M \cup B =< \phi_M \vee \phi_B, \psi_M \wedge \psi_B >$.
v. $M \cap B =< \phi_M \wedge \phi_B, \psi_M \vee \psi_B >$.
vi. $M + B =< \phi_{M+B}, \psi_{M+B} >$, where $\phi_{M+B}(x) = \vee\{\phi_M(p) \wedge \phi_B(m) : p, m \in S, x = p + m\}$ and $\psi_{M+B}(x) = \wedge\{\psi_M(p) \vee \psi_B(m) : p, m \in S, x = p + m\}$, $\forall\ x \in S$.
vii. $M.B = MB =< \phi_{MB}, \psi_{MB} >$, where $\phi_{MB}(x) = \vee\{\phi_M(p) \wedge \phi_B(m) : p, m \in S, x = pm\}$ and $\psi_{MB}(x) = \wedge\{\psi_M(p) \vee \psi_B(m) : p, m \in S, x = pm\}$, $\forall\ x \in S$.
Note that if M and H are IF sets on S, then $M^c, M \cup H, M \cap H, M + H, MH$, are all IF sets on S.

Definition 1.7. [12] *Let $P =< \phi_P, \psi_P >$ be an IF sets in E. Then (γ, λ)-cut of P is referred by-*
${}^{(\gamma,\lambda)}P = \{m \in E : \phi_P(m) \geq \gamma, \psi_P(m) \leq \lambda\}$.

${}^{(\gamma,\lambda)}E =\{ {}^{(\gamma,\lambda)}A : A =< \phi_A, \psi_A > \leq_{IFN} E\}$.

Note that ${}^{(\gamma,\lambda)}P, {}^{(\gamma,\lambda)}E \subseteq E$.

Definition 1.8. *If $A \leq_N E$, then $(A : E) = \{n \in N : nE \subseteq A\}$.*

Lemma 1.1. *If $Z \leq_N E$, then $(Z : E)E \subseteq Z$.*
Proof. Let $n \in (Z : E)$. Then $nE \subseteq Z$ and so $(Z : E)E \subseteq Z$.

Definition 1.9. [12] *An IF sets $A =< \phi_A, \psi_A >$ in N is called IF N-subgroup of N $(A \leq_{IFN} N)$ if*
i. $\phi_A(p-m) \geq \phi_A(p) \wedge \phi_A(m)$. ii. $\phi_A(np) \geq \phi_A(p)$. iii. $\psi_A(p-m) \leq \psi_A(p) \vee \psi_A(m)$. iv. $\psi_A(np) \leq \psi_A(p)$, $\forall\ p, m, n \in N$.

Definition 1.10. *An IF sets $A =< \phi_A, \psi_A >$ in E is called IF N-subgroup of E $(A \leq_{IFN} E)$ if*
i. $\phi_A(p-m) \geq \phi_A(p) \wedge \phi_A(m)$. ii. $\phi_A(np) \geq \phi_A(p)$. iii. $\psi_A(p-m) \leq \psi_A(p) \vee \psi_A(m)$. iv. $\psi_A(np) \leq \psi_A(p)$, $\forall\ p, m \in E, n \in N$.

Definition 1.11. [12] *An IF sets $A =< \phi_A, \psi_A >$ in N is called IF ideal of N $(A \lhd_{IF} N)$ if*
i. $\phi_A(p-m) \geq \phi_A(p) \wedge \phi_A(m)$. ii. $\phi_A(np) \geq \phi_A(p)$. iii. $\phi_A(m+p-m) \geq \phi_A(p)$. iv. $\phi_A(n(p+m)-np) \geq \phi_A(m)$. v. $\psi_A(p-m) \leq \psi_A(p) \vee \psi_A(m)$. vi. $\psi_A(np) \leq \psi_A(p)$. vii. $\psi_A(m+p-m) \leq \psi_A(p)$. viii. $\psi_A(n(p+m)-np) \leq \psi_A(m)$, $\forall\ p, m, n \in N$.

Definition 1.12. [11] *An IF sets $A =< \phi_A, \psi_A >$ in E is called IF ideal of E $(A \lhd_{IF} E)$ if*
i. $\phi_A(p-m) \geq \phi_A(p) \wedge \phi_A(m)$. ii. $\phi_A(np) \geq \phi_A(p)$. iii. $\phi_A(m+p-m) \geq \phi_A(p)$. iv. $\phi_A(n(p+m)-np) \geq \phi_A(m)$. v. $\psi_A(p-m) \leq \psi_A(p) \vee \psi_A(m)$. vi. $\psi_A(np) \leq \psi_A(p)$.

vii. $\psi_A(m+p-m) \leq \psi_A(p)$. *viii.* $\psi_A(n(p+m)-np) \leq \psi_A(m)$, $\forall\, p, m \in E, n \in N$.

Proposition 1.1. *If* $L =< \phi_L, \psi_L > \leq_{IFN} E$, *then* ${}^{(\gamma,\lambda)}L \leq_N E$.

Proof. By definition, ${}^{(\gamma,\lambda)}L$ is a subset of E. For any $n \in N$, $s, y \in {}^{(\gamma,\lambda)}L$ we have, $\phi_L(s), \phi_L(y) \geq \gamma$ and $\psi_L(s), \psi_L(y) \leq \lambda$. $\therefore \phi_L(ns) \geq \phi_L(s) \geq \gamma$ [since L is an IF N-subgroup] and $\psi_L(ns) \leq \psi_L(s) \leq \lambda$. Also, $ns \in E$. $\therefore ns \in {}^{(\gamma,\lambda)}L$. Also, $s - y \in E$ such that $\phi_L(s-y) \geq \phi_L(s) \wedge \phi_L(y) \geq \gamma \wedge \gamma = \gamma$ and $\psi_L(s-y) \leq \psi_L(s) \vee \psi_L(y) \leq \lambda \vee \lambda = \lambda$. So, $s - y \in {}^{(\gamma,\lambda)}L$. This shows that ${}^{(\gamma,\lambda)}L$ is an N-subgroup of E.

2. Intuitionistic Fuzzy Multiplication N-groups

Definition 2.1. *An IF point* $d_{(\gamma,\lambda)}$ *of a nonempty set* K, *is predicted as* $\{d_{(\gamma,\lambda)}\} =< \phi_{d_{(\gamma,\lambda)}}, \psi_{d_{(\gamma,\lambda)}} >$,
where $d \in K$, $\phi_{d_{(\gamma,\lambda)}}(h) = \begin{cases} \gamma, & if h = d \\ 0, & otherwise \end{cases}$ *and* $\psi_{d_{(\gamma,\lambda)}}(h) = \begin{cases} \lambda, & if h = d \\ 1, & otherwise. \end{cases}$

Note that if for any IF set A, $\{d_{(\gamma,\lambda)}\} \subseteq A$, then it is predicted as $d_{(\gamma,\lambda)} \in A$.

Definition 2.2. *If* $Y \subseteq X$ *(non empty), then characteristic function of* Y *is a IF set on* X *and defined by* $\chi Y =< \phi_{\chi Y}, \psi_{\chi Y} >$, *where*
$\phi_{\chi Y}(s) = \begin{cases} 1, & if s \in Y \\ 0, & otherwise \end{cases}$ *and* $\psi_{\chi Y}(s) = \begin{cases} 0, & if s \in Y \\ 1, & otherwise. \end{cases}$

Definition 2.3. *Let* K, B, C *be IF sets on* E *and* C *be IF set on* N. *Then*
$(K : B) = \{D : D$ *is IF set on* N *such that* $DB \subseteq K\}$ *i.e* $(K : B) =< \phi_{(K:B)}, \psi_{(K:B)} >$, *where* $\phi_{(K:B)}(n) = \{\phi_D(n) : D$ *is an IF set on* N *such that* $DB \subseteq K\}$ *and* $\psi_{(K:B)}(n) = \{\psi_D(n) : D$ *is an IF set on* N *such that* $DB \subseteq K\}$
$(K : C) = \{F : F$ *is IF set on* E *such that* $CF \subseteq K\}$ *i.e* $(K : C) =< \phi_{(K:C)}, \psi_{(K:C)} >$, *where* $\phi_{(K:C)}(n) = \{\phi_F(n) : F$ *is an IF set on* E *such that* $CF \subseteq K\}$ *and* $\psi_{(K:C)}(n) = \{\psi_F(n) : F$ *is an IF set on* E *such that* $CF \subseteq K\}$.
If $K \leq_{IFN} E$, *then* $(K : \chi E) = \{D : D \leq_{IFN} N$ *such that* $D\chi E \subseteq K\}$.

Lemma 2.1. *Let* Z, K *be IF sets on* E *and* C *be IF set on* N. *Then*
(i)$(Z : K)K \subseteq Z$.
(ii) $C(Z : C) \subseteq Z$.
(iii) $CK \subseteq Z \Leftrightarrow C \subseteq (Z : K) \Leftrightarrow K \subseteq (Z : C)$.

Proof. (i) Let $Z =< \phi_Z, \psi_Z >, K =< \phi_K, \psi_K >$ be IF sets on E and $C =< \phi_C, \psi_C >$ be IF set on N. Then $(Z : K)K =< \phi_{(Z:K)K}, \psi_{(Z:K)K} >$, where $\phi_{(Z:K)K}(x) = \vee\{\phi_{(Z:K)}(n) \wedge \phi_K(y) : x = ny, n \in N, y \in E\}$ and $\psi_{(Z:K)K}(x) = \wedge\{\psi_{(Z:K)}(n) \vee \psi_K(y) : x = ny, n \in N, y \in E\}$. But $\phi_{(Z:K)}(n) = \{\phi_D(n) : D$ is an IF set on N such that $DK \subseteq Z\}$. $\therefore \phi_{(Z:K)K}(x) = \vee\{\phi_D(n) \wedge \phi_K(y) : D$ is an IF set on N such that $DK \subseteq Z, x = ny, n \in N, y \in E\} \leq \vee\{\phi_{DK}(x) : D$ is a IF set on N such that $DK \subseteq Z\} \leq \phi_Z(x), \forall x \in E$. Similarly, $\psi_{(Z:K)K}(x) \geq \psi_Z(x)$. Thus

$(Z:K)K \subseteq Z$.
(ii) We have, $C(Z:C) =< \phi_{C(Z:C)}, \psi_{C(Z:C)} >$, where $\phi_{C(Z:C)}(x) = \vee\{\phi_C(n) \wedge \phi_{(Z:C)}(y) : x = ny, n \in N, y \in E\}$ and $\psi_C(x) = \wedge\{\psi_C(n) \vee \psi_{(Z:C)}(y) : x = ny, n \in N, y \in E\}$. But $\phi_{(Z:C)}(n) = \{\phi_D(n) : D$ is an IF set on E such that $CD \subseteq Z\}$. $\therefore \phi_{C(Z:C)}(x) = \vee\{\phi_C(n) \wedge \phi_D(y) : D$ is an IF set on E such that $CD \subseteq Z, x = ny, n \in N, y \in E\} \leq \vee\{\phi_{CD}(x) : D$ is a IF set on E such that $CD \subseteq Z\} \leq \phi_Z(x)$. Similarly, $\psi_{C(Z:C)}(x) \geq \psi_Z$. Thus $C(Z:C) \subseteq Z$.
(iii) It is clear from the definition.

Definition 2.4. *An N-group E is called IF multiplication N-group iff for each $A \leq_{IFN} E$, $\exists$ $C \lhd_{IF} N$ such that $A = C.\chi E$. We denote it by $A = C\chi E$.*

Lemma 2.2. *If $Z =< \phi_Z, \psi_Z > \leq_{IFN} E$, then $(Z:\chi E) \lhd_{IF} N$.*
Proof. Let $Z =< \phi_Z, \psi_Z > \leq_{IFN} E$. Then for any $u, h, n \in N$ we have, $\phi_{(Z:\chi E)}(u-h) = \{\phi_D(u-h) : D \leq_{IFN} N$ such that $D\chi E \subseteq Z\} \geq \{\phi_D(u) \wedge \phi_D(h) : D \leq_{IFN} N$ such that $D\chi E \subseteq Z\} = \phi_{(Z:\chi E)}(u) \wedge \phi_{(Z:\chi E)}(h)$ $\therefore \phi_{(Z:\chi E)}(u-h) \geq \phi_{(Z:\chi E)}(u) \wedge \phi_{(Z:\chi E)}(h)$. Similarly, $\psi_{(Z:\chi E)}(u-h) \leq \psi_{(Z:\chi E)}(u) \wedge \psi_{(Z:\chi E)}(h)$. Now, $\phi_{(Z:\chi E)}(nu) = \{\phi_D(nu) : D \leq_{IFN} N$ such that $D\chi E \subseteq Z\} \geq \{\phi_D(u) : D \leq_{IFN} N$ such that $D\chi E \subseteq Z\} = \phi_{(Z:\chi E)}(u)$. $\therefore \phi_{(Z:\chi E)}(nu) \geq \phi_{(Z:\chi E)}(u)$. Similarly, $\psi_{(Z:\chi E)}(nu) \leq \psi_{(Z:\chi E)}(u)$. Since N is commutative, $h + u - h = u$ and so $\phi_{(Z:\chi E)}(h+u-h) = \phi_{(Z:\chi E)}(u)$ and $\psi_{(Z:\chi E)}(h+u-h) = \psi_{(Z:\chi E)}(u)$. Again, since N is commutative, $n(u+h) - nu = nh$ and so $\phi_{(Z:\chi E)}(n(u+h) - nu = nh) \geq \phi_{(Z:\chi E)}(nh) \geq \phi_{(Z:\chi E)}(h)$. Similarly, $\psi_{(Z:\chi E)}(n(u+h) - nu = nh) \leq \psi_{(Z:\chi E)}(nh) \leq \psi_{(Z:\chi E)}(h)$. Thus the result.

Theorem 2.1. *E is IF multiplication N-group iff for each $u \in E$ $\exists$ an IF ideal C of N such that $\{u_{(\gamma,\lambda)}\} = C\chi E$.*
Proof. Let us suppose, for each $u \in E$ $\exists$ an IF ideal C of N such that $\{u_{(\gamma,\lambda)}\} = C\chi E$. Let $A =< \phi_A, \psi_A > \leq_{IFN} E$. Choose $\gamma, \lambda \in [0,1]$ such that $\gamma + \lambda \leq 1$ with $\phi_A(u) = \gamma, \psi_A(u) = \lambda$. Now, for any $u \in E$ we have, $u_{(\gamma,\lambda)}(u) =< \phi_{u_{(\gamma,\lambda)}}(u), \psi_{u_{(\gamma,\lambda)}}(u) >=< \gamma, \lambda >=< \phi_A(u), \psi_A(u) >= A(u)$. $\therefore \{u_{(\gamma,\lambda)}\} = A$ $\Rightarrow \{u_{(\gamma,\lambda)}\} \subseteq A \Rightarrow C\chi E \subseteq A \Rightarrow C \subseteq (A:\chi E)$ [using **lemma 2.1**]. Also, $\phi_A(u) = \gamma = \phi_{u_{(\gamma,\lambda)}}(u) = \phi_{C\chi E}(u) = \vee\{\phi_C(n) \wedge \phi_{\chi E}(u') : n \in N, u' \in E, u = nu'\} \leq \vee\{\phi_{(A:\chi E)}(n) \wedge \phi_{\chi E}(u') : n \in N, u' \in E, u = nu'\} = \vee\{\phi_{(A:\chi E)E}(nu') : n \in N, u' \in E, u = nu'\} = \{\phi_{(A:\chi E)\chi E}(u)\}$. $\therefore \phi_A(u) \leq \{\phi_{(A:\chi E)\chi E}(u)\}$, for all $u \in E$. Similarly, $\psi_A(u) \geq \{\psi_{(A:\chi E)\chi E}(u)\}$, for all $u \in E$. $\therefore A \subseteq (A:\chi E)\chi E$. But by **lemma 2.1**, $(A:\chi E)\chi E \subseteq A$. $\therefore A = (A:\chi E)\chi E$. Also, by **lemma 2.2**, $(A:\chi E)$ is an IF ideal of N. Thus E is an IF multiplication N-group.
Conversely, let E be an IF multiplication N-group. Let $A =< \phi_A, \psi_A > \leq_{IFN} E$ and $u \in E$ and $\gamma, \lambda \in [0,1]$ such that $\gamma + \lambda \leq 1$ with $\phi_A(u) = \gamma, \psi_A(u) = \lambda$. Since

E is multiplication N-group, $\exists$ IF ideal C of N such that $A = C\chi E$. As above we have, $\{u_{(\gamma,\lambda)}\} = A$. Thus $\{u_{(\gamma,\lambda)}\} = C\chi E$.

Proposition 2.1. *If $A =< \phi_A, \psi_A >$ be an IF set on E, then $(A : \chi E) = \{z_{(\gamma,\lambda)} : z \in ({}^{(\gamma,\lambda)}A : E), z \in N\}$.*

Proof. Let $z \in N$ and D be IF set on N. We can choose $\gamma, \lambda \in [0,1], \gamma + \lambda \leq 1$ with $\phi_D(z) = \gamma, \psi_D(z) = \lambda$. Then $\phi_{z_{(\gamma,\lambda)}}(z) = \gamma = \phi_D(z), \psi_{z_{(\gamma,\lambda)}}(z) = \lambda = \psi_D(z)$. $\therefore \{z_{(\gamma,\lambda)}\} = D$. Let $D\chi E \subseteq A \Rightarrow D \subseteq (A : \chi E)$[using **lemma 2.1**] $\Rightarrow \{z_{(\gamma,\lambda)}\} \subseteq (A : \chi E) \Rightarrow \{z_{(\gamma,\lambda)}\}\chi E \subseteq A$. Again, let $\{z_{(\gamma,\lambda)}\}\chi E \subseteq A$ $\Rightarrow D\chi E \subseteq A$. $\therefore \{z_{(\gamma,\lambda)}\}\chi E \subseteq A \Leftrightarrow D\chi E \subseteq A$. $\therefore \{D : D$ is IF set on N such that $D\chi E \subseteq A\} = \{z_{(\gamma,\lambda)} : z \in N, \{z_{(\gamma,\lambda)}\}\chi E \subseteq A\}$. $\therefore (A : \chi E) = \{z_{(\gamma,\lambda)} : z \in N, \{z_{(\gamma,\lambda)}\}\chi E \subseteq A\}$. Now, for each $u \in E$ we have, $\phi_{z_{(\gamma,\lambda)}\chi E}(u) =$

$$\begin{cases} \vee\{\phi_{z_{(\gamma,\lambda)}}(z) \wedge \phi_{\chi E}(u')\}, & u = zu', u' \in E \\ 0, & otherwise \end{cases}$$

Since $\phi_{z_{(\gamma,\lambda)}}(z) = \gamma$ and $\phi_{\chi E}(u') = 1$, therefore

$$\phi_{z_{(\gamma,\lambda)}\chi E}(u) = \begin{cases} \vee\{\gamma \wedge 1\}, & u = zu', u' \in E \\ 0, & otherwise \end{cases} = \begin{cases} \gamma, & u = zu', u' \in E \\ 0, & otherwise \end{cases}$$

Similarly, $\psi_{z_{(\gamma,\lambda)}\chi E}(u) = \begin{cases} \lambda, & u = zu', u' \in E \\ 1, & otherwise \end{cases}$

Now, $\{z_{(\gamma,\lambda)}\}\chi E \subseteq A \Rightarrow \phi_{z_{(\gamma,\lambda)}\chi E}(u) \leq \phi_A(u)$ and $\psi_{z_{(\gamma,\lambda)}\chi E}(u) \geq \psi_A(u)$, for $u \in E$ $\Rightarrow \phi_A(zu') \geq \gamma$ and $\psi_A(zu') \leq \lambda$, for $z \in N, u' \in E$. $\therefore (A : \chi E) = \{z_{(\gamma,\lambda)} : z \in N, \phi_A(zu') \geq \gamma$ and $\psi_A(zu') \leq \lambda, u' \in E\} = \{z_{(\gamma,\lambda)} : z \in N, zu' \in {}^{(\gamma,\lambda)}A, u' \in E\} = \{z_{(\gamma,\lambda)} : z \in N, zE \subseteq {}^{(\gamma,\lambda)}A\} = \{z_{(\gamma,\lambda)} : z \in N, z \in ({}^{(\gamma,\lambda)}A : E)\}$.

Lemma 2.3. *If $z \in E$, then $z_{(\gamma,\lambda)} \in \chi E$.*

Proof. For $y \in E$ we get, $\{z_{(\gamma,\lambda)}\} =< \phi_{z_{(\gamma,\lambda)}}, \psi_{z_{(\gamma,\lambda)}} >$,

where $\phi_{z_{(\gamma,\lambda)}}(y) = \begin{cases} \gamma, & if y = z \\ 0, & otherwise \end{cases}$ and $\psi_{z_{(\gamma,\lambda)}}(y) = \begin{cases} \lambda, & if y = z \\ 0, & otherwise \end{cases}$

$$\therefore z_{(\gamma,\lambda)}(y) = \begin{cases} < \gamma, \lambda >, & if y = z \\ < 0, 1 >, & otherwise \end{cases} \text{ and } \chi E(y) = \begin{cases} < 1, 0 >, & if y \in E \\ < 0, 1 >, & otherwise \end{cases}$$

Since $0 \leq \gamma, \lambda \leq 1$, we get $\{z_{(\gamma,\lambda)}\} \subseteq \chi E$ and so $z_{(\gamma,\lambda)} \in \chi E$.

Lemma 2.4. *If $A \leq_N E$, then $(A : E) \lhd N$.*

Proof. Since $(A : E) = \{u \in N\colon uE \subseteq A\}$, $(A : E) \subseteq N$. Now, $u_1, u_2 \in (A : E)$ and $u \in N \Rightarrow u_1E \subseteq A, u_2E \subseteq A$. Now, for any $e \in E$ we have, $(u_1 - u_2)e = u_1e - u_2e$. Since $A \leq_N E$, $u_1 - u_2 \in A$. $\therefore (u_1 - u_2)e \in A \Rightarrow (u_1 - u_2)E \subseteq A$ $\Rightarrow (u_1 - u_2) \in (A : E)$. Since N is commutative $(uu_1)e = (u_1u)e = u_1(ue) \in u_1E \subseteq A$[since $ue \in E$]. $\therefore (uu_1)E \subseteq A \Rightarrow uu_1 \in (A : E)$. Since N is commutative $u_1u \in (A : E)$. This proves the result.

Proposition 2.2. *E is an multiplication N-group iff every $Z \leq_N E$ is structured like $Z = (Z : E)E$.*

Proof. Let $n \in (Z : E)$. Then $nE \subseteq Z$ and $n \in N \Rightarrow (Z : E)E \subseteq Z$. Since E is a multiplication N-subgroup, $Z = IE$, for some $I \lhd N$. Now, $IE = Z \Rightarrow IE \subseteq Z \Rightarrow I \subseteq (Z : E)$. Again, $(Z : E) \subseteq N$ and $Z \subseteq IE \Rightarrow Z \subseteq (Z : E)E$. $\therefore Z = (Z : E)E$. Conversely, let $Z = (Z : E)E$. Since by **lemma 2.4**, $(Z : E) \lhd N$, therefore Z is multiplication N-subgroup.

Proposition 2.3. *If E is an multiplication N-group, then for every $K =< \phi_K, \psi_K > \leq_{IFN} E$, ${}^{(\gamma,\lambda)}K = ({}^{(\gamma,\lambda)}K : E)E$.*

Proof. Since $K =< \phi_K, \psi_K > \leq_{IFN} E$, by **proposition 1.1**, ${}^{(\gamma,\lambda)}K \leq_N E$. Since E is multiplication, ${}^{(\gamma,\lambda)}K = ({}^{(\gamma,\lambda)}K : E)E$.

Lemma 2.5. *Given a non-empty set K, if $z_{(\gamma,\lambda)} \in K$, then $z \in {}^{(\gamma,\lambda)}K$.*

Proof. $z_{(\gamma,\lambda)} \in K \Rightarrow \{z_{(\gamma,\lambda)}\} \subseteq K$. $\therefore \phi_{z_{(\gamma,\lambda)}} \leq \phi_K, \psi_{z_{(\gamma,\lambda)}} \geq \psi_K$. $\therefore \phi_K(z) \geq \phi_{z_{(\gamma,\lambda)}}(z) = \gamma$ and $\psi_K(z) \leq \psi_{z_{(\gamma,\lambda)}}(z) = \lambda$. $\therefore z \in {}^{(\gamma,\lambda)}K$.

Lemma 2.6. *If $u \in E, s \in N$, then $(su)_{(\gamma,\lambda)} = s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}$.*

Proof. For any $l \in E$ we have, $\{(su)_{(\gamma,\lambda)}\}(l) =< \phi_{(su)_{(\gamma,\lambda)}}(l), \psi_{(su)_{(\gamma,\lambda)}}(l) >$

$$= \begin{cases} < \gamma, \lambda >, & if l = su \\ < 0, 1 >, & otherwise \end{cases} \text{ and } \{s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}\}(l) =< \phi_{s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}}(l), \psi_{s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}}(l) >.$$

Now, $\phi_{s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}}(l) = \vee\{\phi_{s_{(\gamma,\lambda)}(s')} \wedge \psi_{u_{(\gamma,\lambda)}(u')}, l = s'u', s' \in N, u' \in E\}$. If $s = s', u = u'$, then $\phi_{s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}}(l) = \gamma$. Similarly, if $l = su$ then $\psi_{s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}}(l) = \lambda$. Again if $s \neq s', l \neq l'$ then $\phi_{s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}}(l) = 0$ and $\psi_{s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}}(l) = 1$. $\therefore \{s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}\}(l) =$

$$\begin{cases} < \gamma, \lambda >, & if l = su \\ < 0, 1 >, & otherwise \end{cases} \therefore (su)_{(\gamma,\lambda)} = s_{(\gamma,\lambda)}u_{(\gamma,\lambda)}$$

Lemma 2.7. *If $B \leq_N E$, then $\chi B \leq_{IFN} E$.*

Proof. Let $u, z \in E$ and $n \in N$. We have, $\chi B =< \phi_{\chi B}, \psi_{\chi B} >$. If $u, z \in B$, then $u - z \in B$[since B is subgroup of $(E, +)$]. So, $\phi_{\chi B}(u) = 1, \phi_{\chi B}(z) = 1, \phi_{\chi B}(u - z) = 1$. $\therefore \phi_{\chi B}(u - z) = 1 \wedge 1 = \phi_{\chi B}(u) \wedge \phi_{\chi B}(z)$. If $u, z \notin B$, then either $u - z \in B$ or $u - z \notin B$. If $u - z \in B$, then $\phi_{\chi B}(u) = 0, \phi_{\chi B}(z) = 0, \phi_{\chi B}(u - z) = 1$ and so $\phi_{\chi B}(u - z) > \phi_{\chi B}(u) \wedge \phi_{\chi B}(z)$. If $u - z \notin B$, then $\phi_{\chi B}(u) = 0, \phi_{\chi B}(z) = 0, \phi_{\chi B}(u - z) = 0$. $\therefore \phi_{\chi B}(u - z) = 0 \wedge 0 = \phi_{\chi B}(u) \wedge \phi_{\chi B}(z)$. If $u \in B$ but $z \notin B$, then $u - z \notin B$ and so $\phi_{\chi B}(u) = 1, \phi_{\chi B}(z) = 0, \phi_{\chi B}(u - z) = 0$. $\therefore \phi_{\chi B}(u - z) = 1 \wedge 0 = \phi_{\chi B}(u) \wedge \phi_{\chi B}(z)$. Again if $u \notin B$ but $z \in B$, then $u - z \notin B$ and so $\phi_{\chi B}(u) = 0, \phi_{\chi B}(z) = 1, \phi_{\chi B}(u - z) = 0$. $\therefore \phi_{\chi B}(u - z) = 0 \wedge 1 = \phi_{\chi B}(u) \wedge \phi_{\chi B}(z)$. $\therefore \phi_{\chi B}(u - z) \geq \phi_{\chi B}(u) \wedge \phi_{\chi B}(z)$, for $u, z \in E$. Similarly, $\psi_{\chi B}(u - z) \leq \psi_{\chi B}(u) \vee \psi_{\chi B}(z)$, for $u, z \in E$. Now, if $u \in B$, then $nu \in B$ and so $\phi_{\chi B}(u) = 1, \phi_{\chi B}(nu) = 1$. $\therefore \phi_{\chi B}(nu) = \phi_{\chi B}(u)$, if $u \in B$. Also, if

$u \notin B$, then either $nu \in B$ or $nu \notin B$. So, if $u \notin B$ and $nu \in B$, then $\phi_{\chi B}(u) = 0, \phi_{\chi B}(nu) = 1 \therefore \phi_{\chi B}(nu) > \phi_{\chi B}(u)$ and if $u \notin B$ and $nu \notin B$, then $\phi_{\chi B}(u) = 0, \phi_{\chi B}(nu) = 0$. $\therefore \phi_{\chi B}(nu) = \phi_{\chi B}(u)$. Thus $\phi_{\chi B}(nu) \geq \phi_{\chi B}(u)$, for $u \in E$. Similarly, $\psi_{\chi B}(nu) \leq \psi_{\chi B}(u)$, for $u \in E$. Thus the result.

Theorem 2.2. *E be an IF multiplication N-group iff for every $A \leq_{IFN} E$, $A = (A : \chi E)\chi E$.*

Proof. By **lemma 2.1** we get, $(A : \chi E)\chi E \subseteq A$. So, it is sufficient to show that $A \subseteq (A : \chi E)\chi E$. Since E is an IF multiplication N-group, $\exists$ an IF ideal C of N such that $A = C\chi E$. Now, $A = C\chi E \Rightarrow C\chi E \subseteq A \Rightarrow C \subseteq (A : \chi E) \Rightarrow C\chi E \subseteq (A : \chi E)\chi E \Rightarrow A \subseteq (A : \chi E)\chi E$. $\therefore A = (A : \chi E)\chi E$. Conversely, suppose $A = (A : \chi E)\chi E$. Since by **lemma 2.2**, $(A : \chi E)$ is an IF ideal of N, by definition A is an IF multiplication N-group.

Theorem 2.3. *E is a multiplication N-group iff E is an IF multiplication N-group.*

Proof. Let E be a multiplication N-group and $A =< \phi_A, \psi_a > \leq_{IFN} E$. By **lemma 2.1**, $(A : \chi E)\chi E \subseteq A$. Since by **lemma 2.2**, $(A : \chi E)$ is an IF ideal of N, it is sufficient to show that $A \subseteq (A : \chi E)\chi E$. For $u \in E$, we can choose $\gamma, \lambda \in [0, 1], \gamma + \lambda \leq 1$ with $\phi_A(u) = \gamma, \psi_A(u) = \lambda$. Then $u \in {}^{(\gamma,\lambda)}A$. Since E is a multiplication N-group, by **proposition 2.3**, ${}^{(\gamma,\lambda)}A = ({}^{(\gamma,\lambda)}A : E)E$. $\therefore u = nu'$, for some $n \in ({}^{(\gamma,\lambda)}A : E), u' \in E$. By **proposition 2.1**, $n \in ({}^{(\gamma,\lambda)}A : E) \Rightarrow n_{(\gamma,\lambda)} \in (A : \chi E)$. Since $u' \in E$, by **lemma 2.3**, $u'_{(\gamma,\lambda)} \in \chi E$. So by **lemma 2.6**, $u_{(\gamma,\lambda)} = (nu')_{(\gamma,\lambda)} = n_{(\gamma,\lambda)}u'_{(\gamma,\lambda)} \Rightarrow u_{(\gamma,\lambda)} \in (A : \chi E)\chi E \Rightarrow u \in {}^{(\gamma,\lambda)}\{(A : \chi E)\chi E\}$[by **lemma 2.5**] $\Rightarrow \phi_{(A:\chi E)\chi E}(u) \geq \gamma = \phi_A(u), \psi_{(A:\chi E)\chi E}(u) \leq \lambda = \psi_A(u)$. $\therefore A \subseteq (A : \chi E)\chi E$. Thus $A = (A : \chi E)\chi E$. Thus E is an IF multiplication N-group.

Conversely, let E be IF multiplication N-group. Let $B \leq_N E$. Then $(B : E)E \subseteq B$ by **lemma 1.1**. To show $B \subseteq (B : E)E$. Now, we define an IF set P on E by,
$\phi_P(x) = \begin{cases} 1, & if x \in B \\ 0, & otherwise \end{cases}$ and $\psi_P(x) = \begin{cases} 0, & if x \in B \\ 1, & otherwise. \end{cases}$

Then $P = \chi B$ and ${}^{(\gamma,\lambda)}P = B$ with $\gamma, \lambda \in (0, 1], \gamma+\lambda \leq 1$. By **lemma 2.7**, $P = \chi B \leq_{IFN} E$. Since E is an IF multiplication N-group, by **theorem 2.2**, $P = (P : \chi E)\chi E$. Let $b \in B$. Then $\phi_P(b) = \phi_{(P:\chi E)\chi E}(b) = 1$ and $\psi_P(b) = \psi_{(P:\chi E)\chi E}(b) = 0$ [by assumption of P]. But $\phi_{(P:\chi E)\chi E}(b) = \vee\{\phi_{(P:\chi E)}(n') \wedge \phi_{\chi E}(u') : b = n'u'$, for some $n' \in N, u' \in E\} = \vee\{\phi_{(P:\chi E)}(n') : b = n'u'$, for some $n' \in N, u' \in E\}$[since $\phi_{\chi E}(u') = 1] = \vee\{\phi_{n_{\gamma,\lambda}}(n') : nE \subseteq {}^{(\gamma,\lambda)}P = B, b = n'u'$, for some $n' \in N, u' \in E$ with $\gamma, \lambda \in (0, 1], \gamma + \lambda \leq 1\}$ [by **proposition 2.1**] $= \vee\{\phi_{n_{\gamma,\lambda}}(n') : n \in (B : E), b = n'u'$, for some $n' \in N, u' \in E$ with $\gamma, \lambda \in (0, 1], \gamma + \lambda \leq 1\}$. Similarly,

$\psi_{(P:\chi E)\chi E}(b) = \wedge\{\psi_{n_{\gamma,\lambda}}(n') : n \in (B : E), b = n'u'$, for some $n' \in N, u' \in E$ with $\gamma, \lambda \in (0,1], \gamma + \lambda \leq 1\}$. Let us consider $S = \{n : n \in (B : E), b \in nE\}$. If S is empty, then for each $b \in tE$, we have $t \notin (B : E)$ when $t \in N$. Then $\phi_{(P:\chi E)\chi E}(b) = \vee\{\phi_{n_{\gamma,\lambda}}(t) : n \in (B : E), b \in tE, t \in N$, with $\gamma, \lambda \in (0,1], \gamma+\lambda \leq 1\}$. Since $n \in (B : E), t \notin (B : E), n \neq t$ and so $\phi_{n_{\gamma,\lambda}}(t) = 0$. $\therefore \phi_{(P:\chi E)\chi E}(b) = 0$. Similarly, $\psi_{(P:\chi E)\chi E}(b) = 1$. These are contradictions. So we can conclude that S is non-empty. Thus $\exists\ n \in N$ such that $b \in nE$ and $n \in (B : E)$. $\therefore b \in nE \Rightarrow b \in (B : E)E$. But $b \in B$. $\therefore B \subseteq (B : E)E$. Thus $B = (B : E)E$. Hence E is a multiplication N-group.

Definition 2.5. *Let* $A =< \phi_A, \psi_A > \leq_{IFN} E$*, then* ${}^{(\gamma,\lambda)}A \leq_N {}^{(\gamma,\lambda)}E$ *if* $m-y, nm \in {}^{(\gamma,\lambda)}A$*, for any* $m, y \in {}^{(\gamma,\lambda)}A$ *and* $n \in N$.

Theorem 2.4. *An IF multiplication N-group is an IF DN-group.*

Proof. Let $F, K, C \leq_{IFN} E$. Since E is an IF multiplication N-group, $F = (F : \chi E)\chi E, K = (K : \chi E)\chi E, C = (C : \chi E)\chi E$. Let $u \in E$.

Now, $\phi_F(u) = \phi_{(F:\chi E)\chi E}(u) = \vee\{\phi_{(F:\chi E)}(n) \wedge \phi_{\chi E}(e) : u = ne, n \in N, e \in E\} = \vee\{\phi_{(F:\chi E)}(n) : u \in nE, n \in N\}$ [since $\phi_{\chi E}(e) = 1$]. Similarly, $\psi_F(u) = \wedge\{\psi_{(F:\chi E)}(n) : u \in nE, n \in N\}$. But by **proposition 2.1**, $(F : \chi E) = \{n_{(\gamma,\lambda)} : \gamma, \lambda \in [0,1], \gamma+\lambda \leq 1, nE \subseteq^{(\gamma,\lambda)} F\}$. $\therefore \phi_F(u) = \vee\{\phi_{n_{(\gamma,\lambda)}}(n) : u \in nE \subseteq^{(\gamma,\lambda)} F, n \in N\} = \gamma$, where $u \in^{(\gamma,\lambda)} F$. Similarly, $\psi_F(u) = \lambda$, where $u \in^{(\gamma,\lambda)} F$. Now, we define

$$\phi_F(u) = \begin{cases} \gamma, & u \in X \\ 0, & u \notin X \end{cases}, \psi_F(u) = \begin{cases} \lambda, & u \in X \\ 1, & u \notin X \end{cases}, \phi_K(u) = \begin{cases} \gamma, & u \in Y \\ 0, & u \notin Y \end{cases},$$

$$\psi_K(u) = \begin{cases} \lambda, & u \in Y \\ 1, & u \notin Y \end{cases}, \phi_C(u) = \begin{cases} \gamma, & u \in Z \\ 0, & u \notin Z \end{cases}, \psi_C(u) = \begin{cases} \lambda, & u \in Z \\ 1, & u \notin Z \end{cases}$$

with $\gamma, \lambda \in (0,1]$. Then, for $u \in X$, $\phi_F(u) = \gamma, \psi_F(u) = \lambda$ and so $u \in {}^{(\gamma,\lambda)}F$. Also, if $u \in {}^{(\gamma,\lambda)}F$, then either $u \in X$ or $u \notin X$. If $u \notin X$, then $\phi_F(u) = 0 \geq \gamma$ and $\psi_F(u) = 1 \leq \lambda$-which is a contradiction to the fact that $\gamma, \lambda \in (0,1]$. Thus ${}^{(\gamma,\lambda)}F = X$.

Similarly, ${}^{(\gamma,\lambda)}K = Y, {}^{(\gamma,\lambda)}C = Z$ with $\gamma, \lambda \in (0,1]$ and so X, Y, Z are subsets of E. Now, for any $u \in X \cap Y$, $(F + K)(u) =< \phi_{F+K}(u), \psi_{F+K}(u) >$, where $\phi_{F+K}(u) = \vee\{\phi_F(y) \wedge \phi_K(z) : y, z \in X \cap Y$ and $u = y + z \in X \cap Y\}$ and $\psi_{F+K}(u) = \wedge\{\psi_F(y) \vee \psi_K(z) : y, z \in X \cap Y$ and $u = y+z \in X \cap Y\}$. $\therefore \phi_{F+K}(u) = \gamma$ and $\psi_{F+K}(u) = \lambda$, where $u \in X \cap Y$[since $\phi_F(u) = \gamma$ and $\psi_F(u) = \lambda$ for all $u \in X$ and $\phi_K(u) = \gamma$ and $\psi_K(u) = \lambda$ for all $u \in Y$].

Thus $(F + K)(u) =< \gamma, \lambda >$, where $u \in X \cap Y$. Also, $(F \cap K)(u) =< \phi_F(u) \wedge \phi_K(u), \psi_F(u) \vee \psi_K(u) >=< \gamma, \lambda >$, if $u \in X \cap Y$. If $u \in Z$, then $u \in X \cap Y \cap Z$ and $((F + K) \cap C)(u) =< \gamma, \lambda > \cap < \gamma, \lambda >=< \gamma, \lambda >$. If $u \notin Z$, then $u \notin X \cap Y \cap Z$ and $((F + K) \cap C)(u) =< \gamma, \lambda > \cap < 0, 1 >=< 0, 1 >$. Again,

$((F \cap C) + (K \cap C))(u) =< \gamma, \lambda > + < \gamma, \lambda >=< \gamma, \lambda >$, where $u \in X \cap Y \cap Z$[since $(F + F)(u) = F(u)$ for all $u \in X \subseteq E$]. If $u \notin Z$ and $u \in X \cap Y$, then $u \notin X \cap Y \cap Z$ and $((F \cap C) + (K \cap C))(u) =< \phi_F(u) \wedge \phi_C(u), \psi_F(u) \vee \psi_C(u) > + < \phi_K(u) \wedge \phi_C(u), \psi_K(u) \vee \psi_C(u) >=< 0, 1 > + < 0, 1 >=< 0, 1 >$. So, we can conclude that $((F + K) \cap C)(u) = ((F \cap C) + (K \cap C))(u)$, for all $u \in E$. Thus $(F + K) \cap C = (F \cap C) + (K \cap C)$ and hence E is an IF DN-group.

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