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On Hypergeometric Series and Continued Fractions

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Abstract: In this paper, we have established continued fractions representations for the ratio of Hypergeometric Series, Ordinary and Basic both.
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Introduction

Since the times of Euler and Gauss continued fractions have been playing a very important role in Number Theory and Classical Analysis. Significant contributions to the theory of continued fraction expansions were made by Ramanujan. In Chapter 12 of his second note book [17] and also in his "lost" notebook [18], Ramanujan recorded a number of continued fraction identities. This part of Ramanujan's work has been treated and developed consequently by several authors including Andrews [3], Hirschhorn [14], Carlitz [9], Gorden [13], Al-Salam and Ismail [2], Ramanathan [15][16], Denis [10][11][12], Bhargava and Adiga[5][6], Bhargava, Adiga and Somashekara [7][8], Adiga and Somashekara [1], Verma, Denis and Srinivasa Rao [20], Singh [19] and Bhagirathi [4].

1. Notations and Definitions

For α, real or complex, we define

$$[\alpha]_n = \alpha(\alpha+1)(\alpha+2)...(\alpha+n+1) \quad n > 0$$

$$[\alpha]_0 = 1$$

and

$$[\alpha_1\alpha_2\alpha_3...\alpha_r]_n = [\alpha_1]_n[\alpha_2]_n[\alpha_3]_n...[\alpha_r]_n$$

The ordinary hypergeometric series is defined as,

$$ {}_rF_s\left[\begin{array}{l} a_1, a_2, ..., a_r; z \\ b_1, b_2, ..., b_s \end{array}\right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2, ..., a_r]_n z^n}{[b_1, b_2, ..., b_s]_n n!}, \tag{1.1}$$

where $r \leq s$ or $r = s+1$ and $|z| < 1$ for the convergence of the series.
We also define q-shifted factorial as; for α and q real or complex ($|q| < 1$) we define

$$[\alpha; q]_n = (1-\alpha)(1-\alpha q)...(1-\alpha q^{n-1}), \quad n > 0$$

$$[\alpha; q]_0 = 1,$$

$$[\alpha; q]_\infty = \prod_{n=0}^{\infty}(1-\alpha q^n)$$

and

$$[\alpha_1, \alpha_2, ..., \alpha_r; q]_n = [\alpha_1; q]_n, [\alpha_2; q]_n, ..., [\alpha_r; q]_n.$$

From above notations, we define the generalized basic hypergeometric series by

$${}_r\Phi_s\left[\begin{array}{l} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s \end{array}\right] = \sum_{n=0}^{\infty} \frac{[a_1, a_2, ..., a_r; q]_n z^n}{[b_1, b_2, ..., b_s; q]_n}, \tag{1.2}$$

where max. $(|z|, |q|) < 1$ for the convergence of the series (1.2).

2. Main Results

In this section we shall establish following results,

$$\frac{{}_2F_1[\alpha+i, \beta+i+1; \gamma+i; x]}{{}_2F_1[\alpha+i+1, \beta+i; \gamma+i; x]}$$

$$= 1 - \cfrac{(\beta-\alpha)x/(\gamma+i)}{1 - \cfrac{(\gamma-\beta)(\alpha+i+1)x/(\gamma+i)(\gamma+i+1)}{1 - \cfrac{(\gamma-\alpha)(\beta+i+1)x/(\gamma+i+1)(\gamma+i+2)}{1 - \cfrac{(\gamma-\beta+1)(\alpha+i+2)x/(\gamma+i+2)(\gamma+i+3)}{1 - \cfrac{(\gamma-\alpha+1)(\beta+i+2)x/(\gamma+i+3)(\gamma+i+4)}{1 - \ddots}}}}} \tag{2.1}$$

and

$$\frac{{}_2\Phi_1[\alpha q^i, \beta q^{i+1}; \gamma q^i; x]}{{}_2\Phi_1[\alpha q^{i+1}, \beta q^i; \gamma q^i; x]}$$

$$= 1 - \cfrac{(\alpha-\beta)x/(1-\gamma q^i)}{1 - \cfrac{(\beta-\gamma)(1-\alpha q^{i+1})x/(1-\gamma q^i)(1-\gamma q^{i+1})}{1 - \cfrac{(\alpha-\gamma)(1-\beta q^{i+1})x/(1-\gamma q^{i+1})(1-\gamma q^{i+2})}{1 - \cfrac{(\beta-\gamma q)(1-\alpha q^{i+2})x/(1-\gamma q^{i+2})(1-\gamma q^{i+3})}{1 - \cfrac{(\alpha-\gamma q)(1-\beta q^{i+2})x/(1-\gamma q^{i+3})(1-\gamma q^{i+4})}{1 - \ddots}}}}} \tag{2.2}$$

Proof of (2.1);

$$ {}_2F_1[\alpha+i+1,\beta+i;\gamma+i;x]- {}_2F_1[\alpha+i,\beta+i+1;\gamma+i;x] $$
$$ =\frac{(\beta-\alpha)x}{(\gamma+i)}\ {}_2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x] \tag{2.3} $$

Now, we have

$$ \frac{{}_2F_1[\alpha+i,\beta+i+1;\gamma+i;x]}{{}_2F_1[\alpha+i+1,\beta+i;\gamma+i;x]}=1-\frac{(\beta-\alpha)x/(\gamma+i)}{\frac{{}_2F_1[\alpha+i+1,\beta+i;\gamma+i;x]}{{}_2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x]}} \tag{2.4} $$

Again,

$$ {}_2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x]- {}_2F_1[\alpha+i+1,\beta+i;\gamma+i;x] $$
$$ =\frac{(\gamma-\beta)(\alpha+i+1)x}{(\gamma+i)(\gamma+i+1)}\ {}_2F_1[\alpha+i+2,\beta+i+1;\gamma+i+2;x], \tag{2.5} $$

which gives

$$ \frac{{}_2F_1[\alpha+i+1,\beta+i;\gamma+i;x]}{{}_2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x]}=1-\frac{(\gamma-\beta)(\alpha+i+1)x/(\gamma+i)(\gamma+i+1)}{\frac{{}_2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x]}{{}_2F_1[\alpha+i+2,\beta+i+1;\gamma+i+2;x]}}. \tag{2.6} $$

From (2.4) and (2.6) we get,

$$ \frac{{}_2F_1[\alpha+i,\beta+i+1;\gamma+i;x]}{{}_2F_1[\alpha+i+1,\beta+i;\gamma+i;x]} $$
$$ =1-\frac{(\beta-\alpha)x/(\gamma+i)}{1-\frac{(\gamma-\beta)(\alpha+i+1)x/(\gamma+i)(\gamma+i+1)}{\frac{{}_2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x]}{{}_2F_1[\alpha+i+2,\beta+i+1;\gamma+i+2;x]}}} \tag{2.7} $$

Again,

$$ {}_2F_1[\alpha+i+2,\beta+i+1;\gamma+i+2;x]- {}_2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x] $$
$$ =\frac{(\gamma-\alpha)(\beta+i+1)x}{(\gamma+i+1)(\gamma+i+2)}\ {}_2F_1[\alpha+i+2,\beta+i+2;\gamma+i+3;x], $$

which gives

$$\frac{{}_2F_1[\alpha+i+1,\beta+i+1;\gamma+i+1;x]}{{}_2F_1[\alpha+i+2,\beta+i+1;\gamma+i+2;x]}$$
$$=1-\cfrac{(\gamma-\alpha)(\beta+i+1)x/(\gamma+i+1)(\gamma+i+2)}{\cfrac{{}_2F_1[\alpha+i+2,\beta+i+1;\gamma+i+2;x]}{{}_2F_1[\alpha+i+2,\beta+i+2;\gamma+i+3;x]}}. \tag{2.8}$$

From (2.7) and (2.8) we get,

$$\frac{{}_2F_1[\alpha+i,\beta+i+1;\gamma+i;x]}{{}_2F_1[\alpha+i+1,\beta+i;\gamma+i;x]}$$
$$=1-\cfrac{(\beta-\alpha)x/(\gamma+i)}{1-\cfrac{(\gamma-\beta)(\alpha+i+1)x/(\gamma+i)(\gamma+i+1)}{1-\cfrac{(\gamma-\alpha)(\beta+i+1)x/(\gamma+i+1)(\gamma+i+2)}{\cfrac{{}_2F_1[\alpha+i+2,\beta+i+1;\gamma+i+2;x]}{{}_2F_1[\alpha+i+2,\beta+i+2;\gamma+i+3;x]}}}} \tag{2.9}$$

Proceeding in this way, we find (2.1).

Proof of (2.2)

$${}_2\Phi_1[\alpha q^{i+1},\beta q^i;\gamma q^i;x]-\ {}_2\Phi_1[\alpha q^i,\beta q^{i+1};\gamma q^i;x]$$
$$=\frac{(\alpha-\beta)x}{(1-\gamma q^i)}\ {}_2\Phi_1[\alpha q^{i+1},\beta q^{i+1};\gamma q^{i+1};x].$$

which gives

$$\frac{{}_2\Phi_1[\alpha q^i,\beta q^{i+1};\gamma q^i;x]}{{}_2\Phi_1[\alpha q^{i+1},\beta q^i;\gamma q^i;x]}=1-\cfrac{(\alpha-\beta)x/(1-\gamma q^i)}{\cfrac{{}_2\Phi_1[\alpha q^{i+1},\beta q^i;\gamma q^i;x]}{{}_2\Phi_1[\alpha q^{i+1},\beta q^{i+1};\gamma q^{i+1};x]}} \tag{2.10}$$

Again,

$${}_2\Phi_1[\alpha q^{i+1},\beta q^{i+1};\gamma q^{i+1};x]-\ {}_2\Phi_1[\alpha q^{i+1},\beta q^i;\gamma q^i;x]$$
$$=\frac{(\beta-\gamma)(1-\alpha q^{i+1})x}{(1-\gamma q^i)(1-\gamma q^{i+1})}\ {}_2\Phi_1[\alpha q^{i+2},\beta q^{i+1};\gamma q^{i+2};x].$$

which gives

$$\frac{{}_2\Phi_1[\alpha q^{i+1},\beta q^i;\gamma q^i;x]}{{}_2\Phi_1[\alpha q^{i+1},\beta q^{i+1};\gamma q^{i+1};x]}$$

$$= 1 - \cfrac{(\beta-\gamma)(1-\alpha q^{i+1})x/(1-\gamma q^{i})(1-\gamma q^{i+1})}{\cfrac{{}_2\Phi_1[\alpha q^{i+1}, \beta q^{i+1}; \gamma q^{i+1}; x]}{{}_2\Phi_1[\alpha q^{i+2}, \beta q^{i+1}; \gamma q^{i+2}; x]}}. \tag{2.11}$$

From (2.10) and (2.11) we get,

$$\frac{{}_2\Phi_1[\alpha q^{i}, \beta q^{i+1}; \gamma q^{i}; x]}{{}_2\Phi_1[\alpha q^{i+1}, \beta q^{i}; \gamma q^{i}; x]}$$

$$= 1 - \cfrac{(\alpha-\beta)x/(1-\gamma q^{i})}{1 - \cfrac{(\beta-\gamma)(1-\alpha q^{i+1})x/(1-\gamma q^{i})(1-\gamma q^{i+1})}{\cfrac{{}_2\Phi_1[\alpha q^{i+1}, \beta q^{i+1}; \gamma q^{i+1}; x]}{{}_2\Phi_1[\alpha q^{i+2}, \beta q^{i+1}; \gamma q^{i+2}; x]}}}. \tag{2.12}$$

Again,

$${}_2\Phi_1[\alpha q^{i+2}, \beta q^{i+1}; \gamma q^{i+2}; x] - {}_2\Phi_1[\alpha q^{i+1}, \beta q^{i+1}; \gamma q^{i+1}; x]$$
$$= \frac{(\alpha-\gamma)(1-\beta q^{i+1})x}{(1-\gamma q^{i+1})(1-\gamma q^{i+2})}\ {}_2\Phi_1[\alpha q^{i+2}, \beta q^{i+2}; \gamma q^{i+3}; x].$$

which gives

$$\frac{{}_2\Phi_1[\alpha q^{i+1}, \beta q^{i+1}; \gamma q^{i+1}; x]}{{}_2\Phi_1[\alpha q^{i+2}, \beta q^{i+1}; \gamma q^{i+2}; x]}$$

$$= 1 - \cfrac{(\alpha-\gamma)(1-\beta q^{i+1})x/(1-\gamma q^{i+1})(1-\gamma q^{i+2})}{\cfrac{{}_2\Phi_1[\alpha q^{i+2}, \beta q^{i+1}; \gamma q^{i+2}; x]}{{}_2\Phi_1[\alpha q^{i+2}, \beta q^{i+2}; \gamma q^{i+3}; x]}}. \tag{2.13}$$

From (2.12) and (2.13) we get,

$$\frac{{}_2\Phi_1[\alpha q^{i}, \beta q^{i+1}; \gamma q^{i}; x]}{{}_2\Phi_1[\alpha q^{i+1}, \beta q^{i}; \gamma q^{i}; x]}$$

$$= 1 - \cfrac{(\alpha-\beta)x/(1-\gamma q^{i})}{1 - \cfrac{(\beta-\gamma)(1-\alpha q^{i+1})x/(1-\gamma q^{i})(1-\gamma q^{i+1})}{1 - \cfrac{(\alpha-\gamma)(1-\beta q^{i+1})x/(1-\gamma q^{i+1})(1-\gamma q^{i+2})}{\cfrac{{}_2\Phi_1[\alpha q^{i+2}, \beta q^{i+1}; \gamma q^{i+2}; x]}{{}_2\Phi_1[\alpha q^{i+2}, \beta q^{i+2}; \gamma q^{i+3}; x]}}}}. \tag{2.14}$$

Proceeding in this way, we find (2.2)

3. Special Cases

1. Putting i=0 in (2.1) we obtain

$$\frac{{}_2F_1[\alpha, \beta+1; \gamma; x]}{{}_2F_1[\alpha+1, \beta; \gamma; x]}$$

$$= 1 - \cfrac{(\beta-\alpha)x/\gamma}{1 - \cfrac{(\gamma-\beta)(\alpha+1)x/\gamma(\gamma+1)}{1 - \cfrac{(\gamma-\alpha)(\beta+1)x/(\gamma+1)(\gamma+2)}{1 - \cfrac{(\gamma-\beta+1)(\alpha+2)x/(\gamma+2)(\gamma+3)}{1 - \cfrac{(\gamma-\alpha+1)(\beta+2)x/(\gamma+3)(\gamma+4)}{1 - \ddots}}}}} \tag{3.1}$$

2. Putting $\beta = 0$ in (3.1) we obtain

$${}_2F_1[\alpha, 1; \gamma; x] = 1 + \cfrac{\alpha x/\gamma}{1 - \cfrac{\gamma(\alpha+1)x/\gamma(\gamma+1)}{1 - \cfrac{1.(\gamma-\alpha)x/(\gamma+1)(\gamma+2)}{1 - \cfrac{(\gamma+1)(\alpha+2)x/(\gamma+2)(\gamma+3)}{1 - \cfrac{2(\gamma-\alpha+1)x/(\gamma+3)(\gamma+4)}{1 - \ddots}}}}} \tag{3.2}$$

3. Putting $\gamma = 1$ in (3.2) we get,

$${}_2F_1[\alpha, 1; 1; x] = (1-x)^{-\alpha} = 1 + \cfrac{\alpha x/1}{1 - \cfrac{1.(\alpha+1)x/1.2}{1 - \cfrac{1.(1-\alpha)x/2.3}{1 - \cfrac{2.(\alpha+2)x/3.4}{1 - \cfrac{2(2-\alpha)x/4.5}{1 - \ddots}}}}} \tag{3.3}$$

4. Putting $\alpha = 1/2, \gamma = 3/2$ and $x = -x^2$ in (3.2) we get

$${}_2F_1\left[\frac{1}{2}, 1; \frac{3}{2}; -x^2\right] = \frac{1}{x}tan^{-1}x = 1 - \cfrac{x^2/3}{1 + \cfrac{9x^2/15}{1 + \cfrac{4x^2/35}{1 + \cfrac{25x^2/63}{1 + \ddots}}}} \tag{3.4}$$

and putting x =1 in (3.4) we obtain

$$tan^{-1}1 = \frac{\pi}{4} = 1 - \cfrac{1}{3 + \cfrac{9}{5 + \cfrac{4}{7 + \cfrac{25}{9 + \ddots}}}} \tag{3.5}$$

5. Putting $\alpha = 1/2, \gamma = 3/2, x = x^2$ and $|x| < 1$ in (3.2) we get,

$$ {}_2F_1\left[\frac{1}{2}, 1; \frac{3}{2}; x^2\right] = \frac{1}{2x} log\left(\frac{1+x}{1-x}\right) = 1 + \cfrac{x^2/3}{1 - \cfrac{9x^2/15}{1 - \cfrac{4x^2/35}{1 - \cfrac{25x^2/63}{1 - \ddots}}}} \tag{3.6} $$

6. Putting $\alpha = 1, \gamma = 2$ and $x = -x$ in (3.2) we get,

$$ {}_2F_1[1, 1; 2; -x] = log(1+x) = 1 - \cfrac{1.x/2}{1 + \cfrac{2.2x/2.3}{1 + \cfrac{1.1x/3.4}{1 + \cfrac{3.3x/4.5}{1 + \cfrac{2.2x/5.6}{1 + \ddots}}}}} \tag{3.7} $$

7. Putting $\alpha = 1/3$ and $x = -1$ in (3.3) we get

$$ 2^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{2}} = 1 - \cfrac{1}{3 + \cfrac{2}{1 + \cfrac{1}{9 + \cfrac{7}{2 + \ddots}}}} \tag{3.8} $$

8. Putting $\alpha = 1/3, \beta = -1/2, \gamma = 3/2$ and $x = x^2$ in (3.1) we get,

$$ \frac{{}_2F_1\left[\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; x^2\right]}{{}_2F_1\left[\frac{3}{2}, -\frac{1}{2}; \frac{3}{2}; x^2\right]} = \frac{(1-x^2)^{1/2}}{x} sin^{-1}x = 1 + \cfrac{x^2/\frac{3}{2}}{1 - \cfrac{2x^2/\frac{5}{2}}{1 - \cfrac{1.\frac{x^2}{2}/\frac{5}{2}\frac{7}{2}}{1 - \ddots}}} \tag{3.9} $$

and putting $x = \frac{1}{\sqrt{2}}$ in (3.9) we get

$$sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4} = 1 + \cfrac{1}{3 - \cfrac{6}{5 - \cfrac{1}{5 - \ddots}}} \tag{3.10}$$

9. Putting $\alpha = 0, \gamma = 1$ and $x = x/\beta$ in (3.1) we get

$$\lim_{\beta\to\infty} \frac{1}{{}_2F_1[1,\beta;1;x/\beta]} = e^{-x} = 1 - \cfrac{x}{1 + \cfrac{x}{2 + \cfrac{x}{3 + \ddots}}} \tag{3.11}$$

10. Putting i=0 in (2.2) we get,

$$\frac{{}_2F_1[\alpha,\beta q;\gamma;x]}{{}_2F_1[\alpha q,\beta;\gamma;x]}$$

$$= 1 - \cfrac{(\alpha-\beta)x/(1-\gamma)}{1 - \cfrac{(\beta-\gamma)(1-\alpha q)x/(1-\gamma)(1-\gamma q)}{1 - \cfrac{(\alpha-\gamma)(1-\beta q)x/(1-\gamma q)(1-\gamma q^2)}{1 - \cfrac{(\beta-\gamma q)(1-\alpha q^2)x/(1-\gamma q^2)(1-\gamma q^3)}{1 - \cfrac{(\alpha-\gamma q)(1-\beta q^2)x/(1-\gamma q^3)(1-\gamma q^4)}{1 - \ddots}}}}} \tag{3.12}$$

11. Putting $\beta = 1$ in (3.12) we get,

$${}_2F_1[\alpha,q;\gamma;x] = 1 - \cfrac{(\alpha-1)x/(1-\gamma)}{1 - \cfrac{(1-\gamma)(1-\alpha q)x/(1-\gamma)(1-\gamma q)}{1 - \cfrac{(\alpha-\gamma)(1-q)x/(1-\gamma q)(1-\gamma q^2)}{1 - \cfrac{(1-\gamma q)(1-\alpha q^2)x/(1-\gamma q^2)(1-\gamma q^3)}{1 - \cfrac{(\alpha-\gamma q)(1-q^2)x/(1-\gamma q^3)(1-\gamma q^4)}{1 - \ddots}}}}} \tag{3.13}$$

12. Putting $\gamma = q$ in (3.13) we get,

$${}_2F_1[\alpha,q;q;x] = \frac{[\alpha x;q]_\infty}{[x;q]_\infty}$$

$$= 1 - \cfrac{(\alpha-1)x/(1-q)}{1 - \cfrac{(1-q)(1-\alpha q)x/(1-q)(1-q^2)}{1 - \cfrac{(\alpha-q)(1-q)x/(1-q^2)(1-q^3)}{1 - \cfrac{(1-q^2)(1-\alpha q^2)x/(1-q^3)(1-q^4)}{1 - \cfrac{(\alpha-q^2)(1-q^2)x/(1-q^4)(1-q^5)}{1 - \ddots}}}}} \tag{3.14}$$

13. Replacing x by x/α and taking $\alpha \to \infty$ in (3.14) we get

$$\sum_{n=0}^{\infty} \frac{(-1)^n q^{n(n-1)/2}}{(q;q)_n} x^n = 1 - \frac{x/(1-q)}{1+} \frac{xq/(1-q^2)}{1+} \frac{x/(1+q)(1+q^2)}{1+} ... \tag{3.15}$$

14. If we set $\alpha = 0$ and $|x| < 1$ in (3.14) we get

$${}_1\Phi_0[0, -; q; x] = \sum_{n=0}^{\infty} \frac{x^n}{(q;q)_n} = \frac{1}{[x;q]_n} = 1 + \frac{x/(1-q)}{1-} \frac{x/(1-q^2)}{1+} ... \tag{3.16}$$

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