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MAPS AND HOMEOMORPHISMS VIA M-OPEN SETS IN NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

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Abstract: In this article, we introduce the concept of NSM-open and NSM-closed mappings in neutrosophic soft topological spaces and study some of their related properties. Further the work is extended to NSM-homeomorphism, NSM-C homeomorphism and $NSMT_{\frac{1}{2}}$-space in neutrosophic soft topological spaces and establish some of their related attributes.

Keywords and Phrases: NSM-open map, NSM-closed map, NSM-homeomorphism, NSM-C homeomorphism, $NSMT_{\frac{1}{2}}$-space.

2020 Mathematics Subject Classification: 03E72, 54A10, 54A40, 54C05.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [21] in 1965. Then Chang [4] introduced the concept of fuzzy topological space in 1968. After that, it was developed into the concept of intuitionistic fuzzy set by Atanassov [2] in 1983 , which gives a degree of membership and a non-membership functions. Coker [6] in 1997 relied on intuitionistic fuzzy set to introduce the concept of intuitionistic fuzzy topological space. Molodtsov [13] initiated the soft set theory as a new mathematical tool in 1999. He successfully applied several directions for the applications of soft set theory in different fields. Shabir and Naz [18] presented soft topological spaces and defined some concepts of soft sets on this space and separation axioms.

The concepts of neutrosophy and neutrosophic set were introduced by Smarandache [16, 19] in 2005. In 2012, Salama and Alblowi [17] defined neutrosophic topological space. Neutrosophic soft sets were first defined by Maji [12] and after this concept was modified by Deli and Broumi [7]. Later neutrosophic soft topological spaces were presented by Bera [3]. Gundaz et al. [5] introduced neutrosophic soft continuity in neutrosophic soft topological spaces. The notion of M-open sets in topological spaces were introduced by El-Maghrabi and Al-Juhani [8] in 2011, kalaiyarasan et al. [11] introduced in fuzzy nano topological spaces and Vadivel et al. [20] investigated in neutrosophic nano topological spaces. Some types of continuous functions and open functions were introduced by Revathi et al. [14, 15] in neutrosophic soft topological spaces and Jeeva et al. [10] introduced neutrosophic soft M-open sets in neutrosophic topological spaces and developed the concepts of neutrosophic soft M-Continuity and M-Irresolute maps.

2. Preliminaries

Definition 2.1. [7] *Let Y be an initial universe, Q be a set of parameters. Let $P(Y)$ denotes the set of all neutrosophic sets of Y. Then a neutrosophic soft set $(\tilde{H}, Q)$ over Y (briefly, NSs) is defined by a set valued function $\tilde{H}$ representing a mapping $\tilde{H} : Q \to P(Y)$, where $\tilde{H}$ is called the approximate function of the neutrosophic soft set $(\tilde{H}, Q)$.*

In other words, the neutrosophic soft set is a parametrized family of some elements of the set $P(Y)$ and hence it can be written as a set of ordered pairs: $(\tilde{H}, Q) = \{(q, \langle y, \mu_{\tilde{H}(q)}(y), \sigma_{\tilde{H}(q)}(y), \nu_{\tilde{H}(q)}(y)\rangle : y \in Y) : q \in Q\}$, where $\mu_{\tilde{H}(q)}(y), \sigma_{\tilde{H}(q)}(y), \nu_{\tilde{H}(q)}(y) \in [0, 1]$ are respectively called the degree of membership function, the degree of indeterminacy function and the degree of non-membership function of $\tilde{H}(q)$. Since the supremum of each μ, σ, ν is 1, the inequality $0 \leq \mu_{\tilde{H}(q)}(y) + \sigma_{\tilde{H}(q)}(y) + \nu_{\tilde{H}(q)}(y) \leq 3$ is obvious.

Definition 2.2. [3, 12] *Let Y be an initial universe & the NSs's $(\tilde{H},Q)$ & $(\tilde{G},Q)$ are in the form $(\tilde{H},Q)=\{(q,\langle y,\mu_{\tilde{H}(q)}(y),\sigma_{\tilde{H}(q)}(y),\nu_{\tilde{H}(q)}(y)\rangle : y\in Y) : q\in Q\}$ & $(\tilde{G},Q)=\{(q,\langle y,\mu_{\tilde{G}(q)}(y),\sigma_{\tilde{G}(q)}(y),\nu_{\tilde{G}(q)}(y)\rangle : y\in Y) : q\in Q\}$, then*

(i) $0_{(Y,Q)}=\{(q,\langle y,0,0,1\rangle : y\in Y) : q\in Q\}$ *and* $1_{(Y,Q)}=\{(q,\langle y,0,0,1\rangle : y\in Y) : q\in Q\}$

(ii) $(\tilde{H},Q)\subseteq(\tilde{G},Q)$ *iff* $\mu_{\tilde{H}(q)}(y)\leq\mu_{\tilde{G}(q)}(y)$, $\sigma_{\tilde{H}(q)}(y)\leq\sigma_{\tilde{G}(q)}(y)$ & $\nu_{\tilde{H}(q)}(y)\geq\nu_{\tilde{G}(q)}(y) : y\in Y : q\in Q$.

(iii) $(\tilde{H},Q)=(\tilde{G},Q)$ *iff* $(\tilde{H},Q)\subseteq(\tilde{G},Q)$ *and* $(\tilde{G},Q)\subseteq(\tilde{H},Q)$.

(iv) $(\tilde{H},Q)^c=\{(q,\langle y,\nu_{\tilde{H}(q)}(y),1-\sigma_{\tilde{H}(q)}(y),\mu_{\tilde{H}(q)}(y)\rangle : y\in Y) : q\in Q\}$.

(v) $(\tilde{H},Q)\cup(\tilde{G},Q)=\{(q,\langle y,\max(\mu_{\tilde{H}(q)}(y),\mu_{\tilde{G}(q)}(y)),\max(\sigma_{\tilde{H}(q)}(y),\sigma_{\tilde{G}(q)}(y)),$ $\min(\nu_{\tilde{H}(q)}(y),\nu_{\tilde{G}(q)}(y))\rangle : y\in Y) : q\in Q\}$.

(vi) $(\tilde{H},Q)\cap(\tilde{G},Q)=\{(q,\langle y,\min(\mu_{\tilde{H}(q)}(y),\mu_{\tilde{G}(q)}(y)),\min(\sigma_{\tilde{H}(q)}(y),\sigma_{\tilde{G}(q)}(y)),$ $\max(\nu_{\tilde{H}(q)}(y),\nu_{\tilde{G}(q)}(y))\rangle : y\in Y) : q\in Q\}$.

Definition 2.3. [3] *A neutrosophic soft topology (briefly, NSt) on an initial universe Y is a family τ of neutrosophic soft subsets $(\tilde{H},Q)$ of Y where Q is a set of parameters, satisfying*

(i) $0_{(Y,Q)}$, $1_{(Y,Q)}\in\tau$.

(ii) $[(\tilde{H},Q)\cap(\tilde{G},Q)]\in\tau$ *for any* $(\tilde{H},Q),(\tilde{G},Q)\in\tau$.

(iii) $\bigcup_{\rho\in A}(\tilde{H},Q)_\rho\in\tau$, $\forall$ $(\tilde{H},Q)_\rho : \rho\in A\subseteq\tau$.

Then (Y,τ,Q) is called a neutrosophic soft topological space (briefly, NSts) in Y. The τ elements are called neutrosophic soft open sets (briefly, NSos) in Y. A NSs $(\tilde{H},Q)$ is called a neutrosophic soft closed set (briefly, NScs) if its complement $(\tilde{H},Q)^c$ is NSos.

Definition 2.4. [1, 3] *Let (Y,τ,Q) be NSts on Y and $(\tilde{H},Q)$ be an NSs on Y, then the neutrosophic soft*

(i) interior of $(\tilde{H},Q)$ (briefly, $NSint(\tilde{H},Q)$) is defined by $NSint((\tilde{H},Q))=\bigcup\{(\tilde{F},Q) : (\tilde{F},Q)\subseteq(\tilde{H},Q)$ and $(\tilde{G},Q)$ is a NSos in $Y\}$.

(ii) *closure of* $(\tilde{H}, Q)$ *(briefly,* $NScl(\tilde{H}, Q)$*) is defined by* $NScl((\tilde{H}, Q)) = \bigcap\{(\tilde{F}, Q) : (\tilde{F}, Q) \supseteq (\tilde{H}, Q)$ *and* $(\tilde{G}, Q)$ *is a* $NScs$ *in* $Y\}$.

(iii) δ *interior of* $(\tilde{H}, Q)$ *(briefly,* $NS\delta int(\tilde{H}, Q)$*) is defined by* $NS\delta int(\tilde{H}, Q) = \bigcup\{(\tilde{F}, Q) : (\tilde{F}, Q) \subseteq (\tilde{H}, Q)$ & $(\tilde{F}, Q)$ *is a* $NSros$ *in* $Y\}$.

(iv) δ *closure of* $(\tilde{H}, Q)$ *(briefly,* $NS\delta cl(\tilde{H}, Q)$*) is defined by* $NS\delta cl(\tilde{H}, Q) = \bigcap\{(\tilde{F}, Q) : (\tilde{H}, Q) \subseteq (\tilde{F}, Q)$ & $(\tilde{F}, Q)$ *is a* $NSrcs$ *in* $Y\}$.

Definition 2.5. [3, 9] *Let* (Y, τ, Q) *be* $NSts$ *on* Y *and* $(\tilde{H}, Q)$ *be a* NSs *on* Y. *Then* $(\tilde{H}, Q)$ *is said to be a neutrosophic soft regular (resp. pre, semi,* α *&* β*) open set (briefly,* $NSros$ *(resp.* $NS\mathcal{P}os$, $NS\mathcal{S}os$, $NS\alpha os$ & $NS\beta os$*)) if* $(\tilde{H}, Q) = NSint(NScl(\tilde{H}, Q))$ *(resp.* $(\tilde{H}, Q) \subseteq NSint(NScl(\tilde{H}, Q))$, $(\tilde{H}, Q) \subseteq NScl(NSint(\tilde{H}, Q))$, $(\tilde{H}, Q) \subseteq NSint(NScl\ (NSint(\tilde{H}, Q)))$ & $(\tilde{H}, Q) \subseteq NScl(NSint(NScl(\tilde{H}, Q))))$*).*

The complement of a $NS\mathcal{P}os$ (resp. $NS\mathcal{S}os$, $NS\alpha os$, $NSros$ & $NS\beta os$) is called a neutrosophic soft pre (resp. semi, α, regular & β) closed set (briefly, $NS\mathcal{P}cs$ (resp. $NS\mathcal{S}cs$, $NS\alpha cs$, $NSrcs$ & $NS\beta cs$)) in Y.

Definition 2.6. [14] *Let* (Y, τ, Q) *be* $NSts$ *on* Y *and* $(\tilde{H}, Q)$ *be a* NSs *on* Y. *Then* $(\tilde{H}, Q)$ *is said to be a neutrosophic soft*

(i) δ*-open set* [1] *(briefly,* $NS\delta os$*) if* $(\tilde{H}, Q) = NS\delta int(\tilde{H}, Q)$.

(ii) δ*-pre open set (briefly,* $NS\delta\mathcal{P}os$*) if* $(\tilde{H}, Q) \subseteq NSint\big(NS\delta cl(\tilde{H}, Q)\big)$.

(iii) δ*-semi open set (briefly,* $NS\delta\mathcal{S}os$*) if* $(\tilde{H}, Q) \subseteq NScl\big(NS\delta int(\tilde{H}, Q)\big)$.

(iv) *e-open set (briefly,* $NSeos$*) if* $(\tilde{H}, Q) \subseteq NScl\big(NS\delta int(\tilde{H}, Q)\big) \cup NSint\big(NS\delta\ cl(\tilde{H}, Q)\big)$.

The complement of a NSe-open set (resp. $NS\delta os$, $NS\delta\mathcal{P}os$ & $NS\delta\mathcal{S}os$) is called a neutrosophic soft e- (resp. δ, δ-pre & δ-semi) closed set (briefly, $NSecs$ (resp. $NS\delta cs$ $NS\delta\mathcal{P}cs$ & $NS\delta\mathcal{S}cs$)) in Y.

Definition 2.7. [10] *Let* (Y, τ, Q) *be* $NSts$ *on* Y *and* $(\tilde{H}, Q)$ *be a* NSs *on* Y. *Then* $(\tilde{H}, Q)$ *is said to be a neutrosophic soft*

(i) θ *interior of* $(\tilde{H}, Q)$ *(briefly,* $NS\theta int(\tilde{H}, Q)$*) is defined by* $NS\theta int(\tilde{H}, Q) = \bigcup\{NSint(\tilde{G}, Q) : (\tilde{G}, Q) \subseteq (\tilde{H}, Q)$ & $(\tilde{G}, Q)$ *is a* $NScs$ *in* $Y\}$.

(ii) θ *closure of* $(\tilde{H}, Q)$ *(briefly,* $NS\theta cl(\tilde{H}, Q)$*) is defined by* $NS\theta cl(\tilde{H}, Q) = \bigcap\{NScl(\tilde{G}, Q) : (\tilde{H}, Q) \subseteq (\tilde{G}, Q)$ & $(\tilde{G}, Q)$ *is a* $NSos$ *in* $Y\}$.

(iii) *θ-open set (briefly, $NS\theta os$) if $(\tilde{H},Q) = NS\theta int(\tilde{H},Q))$.*

(iv) *θ-semi open set (briefly, $NS\theta\mathcal{S}os$) if $(\tilde{H},Q) \subseteq NScl\big(NS\theta int(\tilde{H},Q)\big)$.*

(v) *M-open set (briefly, $NSMos$) if $(\tilde{H},Q) \subseteq NScl\big(NS\theta int(\tilde{H},Q)\big) \cup NSint\big(NS\delta cl(\tilde{H},Q)\big)$.*

The complement of a $NSMos$ (resp. $NS\theta os$ & $NS\theta\mathcal{S}os$) is called a neutrosophic soft M- (resp. θ & θ-semi) closed set (briefly, $NSMcs$ (resp. $NS\theta cs$ & $NS\theta\mathcal{S}cs$)) in Y.

Definition 2.8. [10] *Let (Y,τ,Q) be $NSts$ on Y and $(\tilde{H},Q)$ be a NSs on Y. Then $(\tilde{H},Q)$ is said to be a neutrosophic soft*

(i) *M interior of $(\tilde{H},Q)$ (briefly, $NSMint(\tilde{H},Q)$) is defined by $NSMint(\tilde{H},Q) = \bigcup\{(\tilde{G},Q) : (\tilde{G},Q) \subseteq (\tilde{H},Q)$ & $(\tilde{G},Q)$ is a $NSMos$ in $Y\}$.*

(ii) *M closure of $(\tilde{H},Q)$ (briefly, $NSMcl(\tilde{H},Q)$) is defined by $NSMcl(\tilde{H},Q) = \bigcap\{(\tilde{G},Q) : (\tilde{H},Q) \subseteq (\tilde{G},Q)$ & $(\tilde{H},Q)$ is a $NSMcs$ in $Y\}$.*

Definition 2.9. [10, 14, 15] *Let (Y_1,τ,Q) and (Y_2,σ,Q) be any two $NSts$'s. A map $h : (Y_1,\tau,Q) \to (Y_2,\sigma,Q)$ is said to be neutrosophic soft*

(i) *continuous (resp. M-continuous) (briefly, $NSCts$ (resp. $NSMCts$)) if the inverse image of every $NSos$ in (Y_2,σ,Q) is a $NSos$ (resp. $NSMos$) in (Y_1,τ,Q).*

(ii) *M-irresolute (briefly, $NSMIrr$) map if $h^{-1}(\tilde{G},Q)$ is a $NSMos$ in (Y_1,τ,Q) for every $NSMos$ $(\tilde{G},Q)$ of (Y_2,σ,Q).*

(iii) *e-open (resp. open, δ-semi open & δ-pre open) (briefly, $NSeO$ (resp. NSO, $NS\delta\mathcal{S}O$ & $NS\delta\mathcal{P}O$)) if the image of every neutrosophic soft open set of (Y_1,τ,Q) is $NSeo$ (resp.NSo, $NS\delta\mathcal{S}o$ & $NS\delta\mathcal{P}o$) set in (Y_2,σ,Q).*

(iv) *homeomorphism (briefly $NSHom$) if h and h^{-1} are $NSCts$ mappings.*

3. Neutrosophic Soft M-open Mapping

Definition 3.1. *A mapping $h : (Y_1,\tau,Q) \to (Y_2,\sigma,Q)$ is neutrosophic soft θ-open (resp. $\theta\mathcal{S}$-open & M-open) (briefly, $NS\theta O$ (resp. $NS\theta\mathcal{S}O$ & $NSMO$)) mapping if the image of every $NSos$ in (Y_1,τ,Q) is a $NS\theta os$ (resp. $NS\theta\mathcal{S}os$ & $NSMos$) in (Y_2,σ,Q).*

Theorem 3.1. *The statements are hold but the converse does not true. Every*

(i) *Every $NS\theta O$ mapping is a NSO mapping.*

(ii) *Every $NS\theta O$ mapping is a $NS\theta\mathcal{S}O$ mapping.*

(iii) *Every $NS\theta\mathcal{S}O$ mapping is a $NSMO$ mapping.*

(iv) *Every NSO mapping is a $NS\delta\mathcal{S}O$ mapping.*

(v) *Every NSO mapping is a $NS\delta\mathcal{P}O$ mapping.*

(vi) *Every $NS\delta\mathcal{S}O$ mapping is a $NSeO$ mapping.*

(vii) *Every $NS\delta\mathcal{P}O$ mapping is a $NSMO$ mapping.*

(viii) *Every $NSMO$ mapping is a $NSeO$ mapping.*

Proof. Only (vii) is proven; the others are similar.

(vii) Let $(\tilde{H}, Q)$ be a $NSos$ in Y_1. Since h is $NS\delta\mathcal{P}O$ mapping, $h(\tilde{H}, Q)$ is a $NS\delta\mathcal{P}os$ in Y_2. Since every $NS\delta\mathcal{P}os$ is a $NSMos$ [10], $h(\tilde{H}, Q)$ is a $NSMos$ in Y_2. Hence h is a $NSMO$ mapping.

Example 3.1. Let $U = \{u_1, u_2, u_3\}, V = \{v_1, v_2, v_3\}, Q = \{q_1, q_2\}$ and NSs's $(\tilde{F}_1, Q)$ in U and $(\tilde{G}_1, Q)$ & $(\tilde{G}_2, Q)$ in V are defined as

$$(\tilde{F}_1, q_1) = \{\langle u_1, (0.20, 0.5, 0.80)\rangle, \langle u_2, (0.30, 0.5, 0.70)\rangle, \langle u_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{F}_1, q_2) = \{\langle u_1, (0.21, 0.5, 0.79)\rangle, \langle u_2, (0.31, 0.5, 0.69)\rangle, \langle u_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle v_1, (0.20, 0.5, 0.80)\rangle, \langle v_2, (0.30, 0.5, 0.70)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle v_1, (0.21, 0.5, 0.79)\rangle, \langle v_2, (0.31, 0.5, 0.69)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_2, q_1) = \{\langle v_1, (0.10, 0.5, 0.90)\rangle, \langle v_2, (0.10, 0.5, 0.90)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_2, q_2) = \{\langle v_1, (0.11, 0.5, 0.89)\rangle, \langle v_2, (0.11, 0.5, 0.89)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$

Then we have $\tau = \{0_{(U,Q)}, 1_{(U,Q)}, (\tilde{F}_1, Q)\}$ and $\sigma = \{0_{(V,Q)}, 1_{(V,Q)}, (\tilde{G}_1, Q), (\tilde{G}_2, Q)\}$. Let $h : (U, \tau, Q) \to (V, \sigma, Q)$ be an identity mapping. Then h is NSO mapping in U but not $NS\theta O$ mapping in V.

Example 3.2. Let $U = \{u_1, u_2, u_3\}, V = \{v_1, v_2, v_3\}, Q = \{q_1, q_2\}$ and NSs's

$(\tilde{F}_1, Q)$ in U and $(\tilde{G}_1, Q), (\tilde{G}_2, Q)$ & $(\tilde{G}_3, Q)$ in V are defined as

$$(\tilde{F}_1, q_1) = \{\langle u_1, (0.90, 0.5, 0.10)\rangle, \langle u_2, (0.80, 0.5, 0.20)\rangle, \langle u_3, (0.70, 0.5, 0.30)\rangle\}$$
$$(\tilde{F}_1, q_2) = \{\langle u_1, (0.91, 0.5, 0.09)\rangle, \langle u_2, (0.81, 0.5, 0.19)\rangle, \langle u_3, (0.71, 0.5, 0.29)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle v_1, (0.20, 0.5, 0.80)\rangle, \langle v_2, (0.30, 0.5, 0.70)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle v_1, (0.21, 0.5, 0.79)\rangle, \langle v_2, (0.31, 0.5, 0.69)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_2, q_1) = \{\langle v_1, (0.10, 0.5, 0.90)\rangle, \langle v_2, (0.10, 0.5, 0.90)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_2, q_2) = \{\langle v_1, (0.11, 0.5, 0.89)\rangle, \langle v_2, (0.11, 0.5, 0.89)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_3, q_1) = \{\langle v_1, (0.90, 0.5, 0.10)\rangle, \langle v_2, (0.80, 0.5, 0.20)\rangle, \langle v_3, (0.70, 0.5, 0.30)\rangle\}$$
$$(\tilde{G}_3, q_2) = \{\langle v_1, (0.91, 0.5, 0.09)\rangle, \langle v_2, (0.81, 0.5, 0.19)\rangle, \langle v_3, (0.71, 0.5, 0.29)\rangle\}$$

Then we have $\tau = \{0_{(U,Q)}, 1_{(U,Q)}, (\tilde{F}_1, Q)\}$ and $\sigma = \{0_{(V,Q)}, 1_{(V,Q)}, (\tilde{G}_1, Q), (\tilde{G}_2, Q)\}$. Let $h : (U, \tau, Q) \to (V, \sigma, Q)$ be an identity mapping. Then h is $NS\theta\mathcal{S}O$ mapping in U but not $NS\theta O$ mapping in V.

Example 3.3. Let $U = \{u_1, u_2, u_3\}, V = \{v_1, v_2, v_3\}, Q = \{q_1, q_2\}$ and NSs's $(\tilde{F}_1, Q)$ in U and $(\tilde{G}_1, Q), (\tilde{G}_2, Q)$ & $(\tilde{G}_3, Q)$ in V are defined as

$$(\tilde{F}_1, q_1) = \{\langle u_1, (0.20, 0.5, 0.80)\rangle, \langle u_2, (0.20, 0.5, 0.80)\rangle, \langle u_3, (0.30, 0.5, 0.70)\rangle\}$$
$$(\tilde{F}_1, q_2) = \{\langle u_1, (0.21, 0.5, 0.79)\rangle, \langle u_2, (0.21, 0.5, 0.79)\rangle, \langle u_3, (0.31, 0.5, 0.69)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle v_1, (0.20, 0.5, 0.80)\rangle, \langle v_2, (0.30, 0.5, 0.70)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle v_1, (0.21, 0.5, 0.79)\rangle, \langle v_2, (0.31, 0.5, 0.69)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_2, q_1) = \{\langle v_1, (0.10, 0.5, 0.90)\rangle, \langle v_2, (0.10, 0.5, 0.90)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_2, q_2) = \{\langle v_1, (0.11, 0.5, 0.89)\rangle, \langle v_2, (0.11, 0.5, 0.89)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_3, q_1) = \{\langle v_1, (0.20, 0.5, 0.80)\rangle, \langle v_2, (0.20, 0.5, 0.80)\rangle, \langle v_3, (0.30, 0.5, 0.70)\rangle\}$$
$$(\tilde{G}_3, q_2) = \{\langle v_1, (0.21, 0.5, 0.79)\rangle, \langle v_2, (0.21, 0.5, 0.79)\rangle, \langle v_3, (0.31, 0.5, 0.69)\rangle\}$$

Then we have $\tau = \{0_{(U,Q)}, 1_{(U,Q)}, (\tilde{F}_1, Q)\}$ and $\sigma = \{0_{(V,Q)}, 1_{(V,Q)}, (\tilde{G}_1, Q), (\tilde{G}_2, Q)\}$. Let $h : (U, \tau, Q) \to (V, \sigma, Q)$ be an identity mapping. Then h is $NSMO$ mapping in U but not $NS\theta\mathcal{S}O$ mapping in V.

Example 3.4. Let $U = \{u_1, u_2, u_3\}, V = \{v_1, v_2, v_3\}, Q = \{q_1, q_2\}$ and NSs's

$(\tilde{F}_1, Q)$ in U and $(\tilde{G}_1, Q), (\tilde{G}_2, Q)$ & $(\tilde{G}_3, Q)$ in V are defined as

$$(\tilde{F}_1, q_1) = \{\langle u_1, (0.20, 0.5, 0.80)\rangle, \langle u_2, (0.40, 0.5, 0.60)\rangle, \langle u_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{F}_1, q_2) = \{\langle u_1, (0.21, 0.5, 0.79)\rangle, \langle u_2, (0.41, 0.5, 0.59)\rangle, \langle u_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle v_1, (0.20, 0.5, 0.80)\rangle, \langle v_2, (0.30, 0.5, 0.70)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle v_1, (0.21, 0.5, 0.79)\rangle, \langle v_2, (0.31, 0.5, 0.69)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_2, q_1) = \{\langle v_1, (0.10, 0.5, 0.90)\rangle, \langle v_2, (0.10, 0.5, 0.90)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_2, q_2) = \{\langle v_1, (0.11, 0.5, 0.89)\rangle, \langle v_2, (0.11, 0.5, 0.89)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_3, q_1) = \{\langle v_1, (0.20, 0.5, 0.80)\rangle, \langle v_2, (0.40, 0.5, 0.60)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_3, q_2) = \{\langle v_1, (0.21, 0.5, 0.79)\rangle, \langle v_2, (0.41, 0.5, 0.59)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$

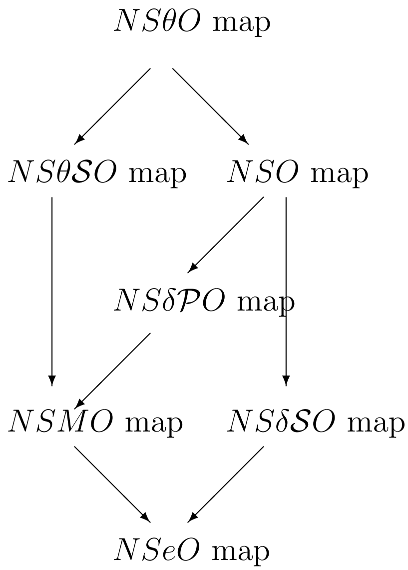
Then we have $\tau = \{0_{(U,Q)}, 1_{(U,Q)}, (\tilde{F}_1, Q)\}$ and $\sigma = \{0_{(V,Q)}, 1_{(V,Q)}, (\tilde{G}_1, Q), (\tilde{G}_2, Q)\}$. Let $h : (U, \tau, Q) \to (V, \sigma, Q)$ be an identity mapping. Then h is $NSeO$ mapping in U but not $NSMO$ mapping in V.

Example 3.5. Let $U = \{u_1, u_2, u_3\}, V = \{v_1, v_2, v_3\}, Q = \{q_1, q_2\}$ and NSs's $(\tilde{F}_1, Q)$ in U and $(\tilde{G}_1, Q), (\tilde{G}_2, Q), (\tilde{G}_3, Q)$, $(\tilde{G}_4, Q)$ & $(\tilde{G}_5, Q)$ in V are defined as

$$(\tilde{F}_1, q_1) = \{\langle u_1, (0.70, 0.5, 0.30)\rangle, \langle u_2, (0.50, 0.5, 0.50)\rangle, \langle u_3, (0.50, 0.5, 0.50)\rangle\}$$
$$(\tilde{F}_1, q_2) = \{\langle u_1, (0.71, 0.5, 0.29)\rangle, \langle u_2, (0.51, 0.5, 0.49)\rangle, \langle u_3, (0.51, 0.5, 0.49)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle v_1, (0.30, 0.5, 0.70)\rangle, \langle v_2, (0.40, 0.5, 0.60)\rangle, \langle v_3, (0.30, 0.5, 0.70)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle v_1, (0.31, 0.5, 0.69)\rangle, \langle v_2, (0.41, 0.5, 0.59)\rangle, \langle v_3, (0.31, 0.5, 0.69)\rangle\}$$
$$(\tilde{G}_2, q_1) = \{\langle v_1, (0.60, 0.5, 0.40)\rangle, \langle v_2, (0.50, 0.5, 0.50)\rangle, \langle v_3, (0.50, 0.5, 0.50)\rangle\}$$
$$(\tilde{G}_2, q_2) = \{\langle v_1, (0.61, 0.5, 0.39)\rangle, \langle v_2, (0.51, 0.5, 0.49)\rangle, \langle v_3, (0.51, 0.5, 0.49)\rangle\}$$
$$(\tilde{G}_3, q_1) = \{\langle v_1, (0.60, 0.5, 0.40)\rangle, \langle v_2, (0.50, 0.5, 0.50)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_3, q_2) = \{\langle v_1, (0.61, 0.5, 0.39)\rangle, \langle v_2, (0.51, 0.5, 0.49)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_4, q_1) = \{\langle v_1, (0.30, 0.5, 0.70)\rangle, \langle v_2, (0.40, 0.5, 0.60)\rangle, \langle v_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{G}_4, q_2) = \{\langle v_1, (0.31, 0.5, 0.69)\rangle, \langle v_2, (0.41, 0.5, 0.59)\rangle, \langle v_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{G}_5, q_1) = \{\langle v_1, (0.70, 0.5, 0.30)\rangle, \langle v_2, (0.50, 0.5, 0.50)\rangle, \langle v_3, (0.50, 0.5, 0.50)\rangle\}$$
$$(\tilde{G}_5, q_2) = \{\langle v_1, (0.71, 0.5, 0.29)\rangle, \langle v_2, (0.51, 0.5, 0.49)\rangle, \langle v_3, (0.51, 0.5, 0.49)\rangle\}$$

Then we have $\tau = \{0_{(U,Q)}, 1_{(U,Q)}, (\tilde{F}_1, Q)\}$ and $\sigma = \{0_{(V,Q)}, 1_{(V,Q)}, (\tilde{G}_1, Q), (\tilde{G}_2, Q), (\tilde{G}_3, Q), (\tilde{G}_4, Q)\}$. Let $h : (U, \tau, Q) \to (V, \sigma, Q)$ be an identity mapping. Then h is $NSMO$ mapping in U but not $NS\delta\mathcal{P}O$ mapping in V.

Remark 3.1. *The following diagram shows the above results.*



Theorem 3.2. *A mapping $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ is NSMO iff for every NSs $(\tilde{H}, Q)$ of (Y_1, τ, Q), $h(NSint(\tilde{H}, Q)) \subseteq NSMint(h(\tilde{H}, Q))$.*

Proof. Necessity: Let h be a $NSMO$ mapping and $(\tilde{H}, Q)$ be a $NSos$ in (Y_1, τ, Q). Now, $NSint(\tilde{H}, Q) \subseteq (\tilde{H}, Q)$ implies $h(NSint(\tilde{H}, Q)) \subseteq h(\tilde{H}, Q)$. Since h is a $NSMO$ mapping, $h(NSint(\tilde{H}, Q))$ is $NSMos$ in (Y_2, σ, Q) such that $h(NSint(\tilde{H}, Q)) \subseteq h(\tilde{H}, Q)$. Therefore $h(NSint(\tilde{H}, Q)) \subseteq NSMint(h(\tilde{H}, Q))$.

Sufficiency: Assume $(\tilde{H}, Q)$ is a $NSos$ of (Y_1, τ, Q). Then $h(\tilde{H}, Q) = h(NSint(\tilde{H}, Q)) \subseteq NSMint(h(\tilde{H}, Q))$. But $NSMint\ (h(\tilde{H}, Q)) \subseteq h(\tilde{H}, Q)$. So $h(\tilde{H}, Q) = NSMint(\tilde{H}, Q)$ which implies $h(\tilde{H}, Q)$ is a $NSMos$ of (Y_2, σ, Q) and hence h is a $NSMO$.

Theorem 3.3. *If $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ is a NSMO mapping, then $NSint(h^{-1}(\tilde{H}, Q)) \subseteq h^{-1}(NSMint(\tilde{H}, Q))$ for every NSs $(\tilde{H}, Q)$ of (Y_2, σ, Q).*

Proof. Let $(\tilde{H}, Q)$ be a NSs of (Y_2, σ, Q). Then $NSint(h^{-1}(\tilde{H}, Q))$ is a $NSos$ in (Y_1, τ, Q). Since h is $NSMO$, $h(NSint(h^{-1}(\tilde{H}, Q)))$ is $NSMO$ in (Y_2, σ, Q) and hence $h(NSint(h^{-1}(\tilde{H}, Q))) \subseteq NSMint(h(h^{-1}(\tilde{H}, Q))) \subseteq NSMint(\tilde{H}, Q)$. Thus $NSint(h^{-1}(\tilde{H}, Q)) \subseteq h^{-1}(NSMint(\tilde{H}, Q))$.

Theorem 3.4. *A mapping $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ is NSMO iff for each NSs $(\tilde{G}, Q)$ of (Y_2, σ, Q) and for each NScs $(\tilde{H}, Q)$ of (Y_1, τ, Q) containing $h^{-1}(\tilde{G}, Q)$, there is a NSMcs $(\tilde{K}, Q)$ of (Y_2, σ, Q) such that $(\tilde{G}, Q) \subseteq (\tilde{H}, Q)$ and $h^{-1}(\tilde{K}, Q) \subseteq$*

$(\tilde{H},Q)$.

Proof. Necessity: Assume h is a $NSMO$ mapping. Let $(\tilde{G},Q)$ be the $NScs$ of (Y_2,σ,Q) and $(\tilde{H},Q)$ is a $NScs$ of (Y_1,τ,Q) such that $h^{-1}(\tilde{G},Q) \subseteq (\tilde{H},Q)$. Then $(\tilde{K},Q) = (h^{-1}(\tilde{H},Q)^c)^c$ is $NSMcs$ of (Y_2,σ,Q) such that $h^{-1}(\tilde{K},Q) \subseteq (\tilde{H},Q)$.

Sufficiency: Assume $(\tilde{H},Q)$ is a $NSos$ of (Y_1,τ,Q). Then $h^{-1}((h(\tilde{H},Q))^c) \subseteq (\tilde{H},Q)^c$ and $(\tilde{H},Q)^c$ is $NScs$ in (Y_1,τ,Q). By hypothesis, there is a $NSMcs$ $(\tilde{G},Q)$ of (Y_2,σ,Q) such that $(h(\tilde{H},Q))^c \subseteq (\tilde{G},Q)$ and $h^{-1}(\tilde{G},Q) \subseteq (\tilde{H},Q)^c$. Therefore $(\tilde{H},Q) \subseteq (h^{-1}(\tilde{G},Q))^c$. Hence $(\tilde{G},Q)^c \subseteq h(\tilde{H},Q) \subseteq h((h^{-1}(\tilde{G},Q))^c) \subseteq (\tilde{G},Q)^c$ which implies $h(\tilde{H},Q) = (\tilde{G},Q)^c$. Since $(\tilde{G},Q)^c$ is $NSMos$ of (Y_2,σ,Q), $h(\tilde{H},Q)$ is $NSMO$ in (Y_2,σ,Q) and thus h is $NSMO$ mapping.

Theorem 3.5. *A mapping $h : (Y_1,\tau,Q) \rightarrow (Y_2,\sigma,Q)$ is $NSMO$ iff $h^{-1}(NSMcl(\tilde{G},Q)) \subseteq NScl(h^{-1}(\tilde{G},Q))$ for every NSs $(\tilde{G},Q)$ of (Y_2,σ,Q).*

Proof. Necessity: Assume h is a $NSMO$ mapping. For any NSs $(\tilde{G},Q)$ of (Y_2,σ,Q), $h^{-1}(\tilde{G},Q) \subseteq NScl(h^{-1}(\tilde{G},Q))$. Therefore by Theorem 3.4, there exists a $NSMcs$ $(\tilde{H},Q)$ in (Y_2,σ,Q) such that $(\tilde{G},Q) \subseteq (\tilde{H},Q)$ and $h^{-1}(\tilde{H},Q) \subseteq NScl(h^{-1}(\tilde{G},Q))$. Therefore we obtain that $h^{-1}(NSMcl(\tilde{G},Q)) \subseteq h^{-1}(\tilde{H},Q) \subseteq NScl(h^{-1}(\tilde{G},Q))$.

Sufficiency: Assume $(\tilde{G},Q)$ is a NSs of (Y_2,σ,Q) and $(\tilde{H},Q)$ is a $NScs$ of (Y_1,τ,Q) containing $h^{-1}(\tilde{G},Q)$. Put $(\tilde{K},Q) = NScl(\tilde{G},Q)$, then $(\tilde{G},Q) \subseteq (\tilde{K},Q)$ and $(\tilde{K},Q)$ is $NSMc$ and $h^{-1}(\tilde{K},Q) \subsetneq NScl(h^{-1}(\tilde{G},Q)) \subseteq (\tilde{H},Q)$. Then by Theorem 3.4, h is $NSMO$ mapping.

Theorem 3.6. *If $h : (Y_1,\tau,Q) \rightarrow (Y_2,\sigma,Q)$ and $g : (Y_2,\sigma,Q) \rightarrow (Y_3,\rho,Q)$ be two neutrosophic soft mappings and $g \circ h : (Y_1,\tau,Q) \rightarrow (Y_3,\rho,Q)$ is $NSMO$. If $g : (Y_2,\sigma,Q) \rightarrow (Y_3,\rho,Q)$ is $NSMIrr$, then $h : (Y_1,\tau,Q) \rightarrow (Y_2,\sigma,Q)$ is $NSMO$ mapping.*

Proof. Let $(\tilde{H},Q)$ be a $NSos$ in (Y_1,τ,Q). Then $g\circ h(\tilde{H},Q)$ is $NSMos$ of (Y_3,ρ,Q) because $g\circ h$ is $NSMO$ mapping. Since g is $NSMIrr$ and $g\circ h(\tilde{H},Q)$ is $NSMos$ of (Y_3,ρ,Q), $g^{-1}(g\circ h(\tilde{H},Q)) = h(\tilde{H},Q)$ is $NSMos$ in (Y_2,σ,Q). Hence h is $NSMO$ mapping.

Theorem 3.7. *If $h : (Y_1,\tau,Q) \rightarrow (Y_2,\sigma,Q)$ is NSO and $g : (Y_2,\sigma,Q) \rightarrow (Y_3,\rho,Q)$ is $NSMO$ mappings, then $g \circ h : (Y_1,\tau,Q) \rightarrow (Y_3,\rho,Q)$ is $NSMO$.*

Proof. Let $(\tilde{H},Q)$ be a $NSos$ in (Y_1,τ,Q). Then $h(\tilde{H},Q)$ is a $NSos$ of (Y_2,σ,Q) because h is a NSO mapping. Since g is $NSMO$, $g(h(\tilde{H},Q)) = (g\circ h)(\tilde{H},Q)$ is a $NSMos$ of (Y_3,ρ,Q). Hence $g\circ h$ is $NSMO$ mapping.

4. Neutrosophic Soft M-closed Mapping

Definition 4.1. *A mapping $h : (Y_1,\tau,Q) \rightarrow (Y_2,\sigma,Q)$ is neutrosophic soft θ-closed* *(resp. $\theta\mathcal{S}$-closed & M-closed) (briefly, $NS\theta C$ (resp. $NS\theta\mathcal{S}C$ & $NSMC$)) mapping if the image of every $NScs$ in (Y_1, τ, Q) is a $NS\theta cs$ (resp. $NS\theta\mathcal{S}cs$ & $NSMcs$) in (Y_2, σ, Q).*

Theorem 4.1. *The statements are hold but the converse does not true. Every*

(i) Every $NS\theta C$ mapping is a NSC mapping.

(ii) Every $NS\theta C$ mapping is a $NS\theta\mathcal{S}C$ mapping.

(iii) Every $NS\theta\mathcal{S}C$ mapping is a $NSMC$ mapping.

(iv) Every NSC mapping is a $NS\delta\mathcal{S}C$ mapping.

(v) Every NSC mapping is a $NS\delta\mathcal{P}C$ mapping.

(vi) Every $NS\delta\mathcal{S}C$ mapping is a $NSeC$ mapping.

(vii) Every $NS\delta\mathcal{P}C$ mapping is a $NSMC$ mapping.

(viii) Every $NSMC$ mapping is a $NSeC$ mapping.

Proof. Only (vii) is proven; the others are similar.

(vii) Let $(\tilde{H}, Q)$ be a $NScs$ in Y_1. Since h is $NS\delta\mathcal{P}C$ mapping, $h(\tilde{H}, Q)$ is a $NS\delta\mathcal{P}cs$ in Y_2. Since every $NS\delta\mathcal{P}cs$ is a $NSMcs$ [10], $h(\tilde{H}, Q)$ is a $NSMcs$ in Y_2. Hence h is a $NSMC$ mapping.

Example 4.1. In Example 3.1, $(\tilde{F}_1, Q)^c$ is NSC mapping in U but not $NS\theta C$ mapping in V.

Example 4.2. In Example 3.2, $(\tilde{F}_1, Q)^c$ is $NS\theta\mathcal{S}C$ mapping in U but not $NS\theta C$ mapping in V.

Example 4.3. In Example 3.3, $(\tilde{F}_1, Q)^c$ is $NSMC$ mapping in U but not $NS\theta\mathcal{S}C$ mapping in V.

Example 4.4. In Example 3.4, $(\tilde{F}_1, Q)^c$ is $NSeC$ mapping in U but not $NSMC$ mapping in V.

Example 4.5. In Example 3.5, $(\tilde{F}_1, Q)^c$ is $NSMC$ mapping in U but not $NS\delta\mathcal{P}C$ mapping in V.

Theorem 4.2. *A mapping $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ is $NSMC$ iff for each NSs $(\tilde{G}, Q)$ of (Y_2, σ, Q) and for each $NSos$ $(\tilde{H}, Q)$ of (Y_1, τ, Q) containing $h^{-1}(\tilde{G}, Q)$, there is a $NSMos$ $(\tilde{K}, Q)$ of (Y_2, σ, Q) such that $(\tilde{G}, Q) \subseteq (\tilde{K}, Q)$ and $h^{-1}(\tilde{K}, Q) \subseteq (\tilde{H}, Q)$.*

Proof. Necessity: Assume h is a $NSMC$ mapping. Let $(\tilde{G}, Q)$ be the $NScs$ of (Y_2, σ, Q) and $(\tilde{H}, Q)$ is a $NSos$ of (Y_1, τ, Q) such that $h^{-1}(\tilde{G}, Q) \subseteq (\tilde{H}, Q)$. Then $(\tilde{K}, Q) = Y_2 - h^{-1}((\tilde{H}, Q)^c)$ is $NSMos$ of (Y_2, σ, Q) such that $h^{-1}(\tilde{K}, Q) \subseteq (\tilde{H}, Q)$.
Sufficiency: Assume $(\tilde{H}, Q)$ is a $NScs$ of (Y_1, τ, Q). Then $(h(\tilde{H}, Q))^c$ is a NSs of (Y_2, σ, Q) and $(\tilde{H}, Q)^c$ is $NSos$ in (Y_1, τ, Q) such that $h^{-1}((h(\tilde{H}, Q))^c) \subseteq (\tilde{H}, Q)^c$. By hypothesis, there is a $NSMos$ $(\tilde{K}, Q)$ of (Y_2, σ, Q) such that $(h(\tilde{H}, Q))^c \subseteq (\tilde{K}, Q)$ and $h^{-1}(\tilde{K}, Q) \subseteq (\tilde{H}, Q)^c$. Therefore $(\tilde{H}, Q) \subseteq (h^{-1}(\tilde{K}, Q))^c$. Hence $(\tilde{K}, Q)^c \subseteq h(\tilde{K}, Q) \subseteq h((h^{-1}(\tilde{K}, Q))^c) \subseteq (\tilde{K}, Q)^c$ which implies $h(\tilde{H}, Q) = (\tilde{K}, Q)^c$. Since $(\tilde{K}, Q)^c$ is $NSMcs$ of (Y_2, σ, Q), $h(\tilde{H}, Q)$ is $NSMc$ in (Y_2, σ, Q) and thus h is $NSMC$ mapping.

Theorem 4.3. *If $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ is NSC and $g : (Y_2, \sigma, Q) \to (Y_3, \rho, Q)$ is $NSMC$. Then $g \circ h : (Y_1, \tau, Q) \to (Y_3, \rho, Q)$ is $NSMC$.*
Proof. Let $(\tilde{H}, Q)$ be a $NScs$ in (Y_1, τ, Q). Then $h(\tilde{H}, Q)$ is $NScs$ of (Y_2, σ, Q) because h is NSC mapping. Now $(g \circ h)(\tilde{H}, Q) = g(h(\tilde{H}, Q))$ is $NSMcs$ in (Y_3, ρ, Q) because g is $NSMC$ mapping. Thus $g \circ h$ is $NSMC$ mapping.

Theorem 4.4. *If $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ is $NSMC$ map, then $NSMcl(h(\tilde{H}, Q)) \subseteq h(NScl(\tilde{H}, Q))$.*
Proof. Obvious.

Theorem 4.5. *Let $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ and $g : (Y_2, \sigma, Q) \to (Y_3, \rho, Q)$ are $NSMC$ mappings. If every $NSMcs$ of (Y_2, σ, Q) is $NScs$, then $g \circ h : (Y_1, \tau, Q) \to (Y_3, \rho, Q)$ is $NSMC$.*
Proof. Let $(\tilde{H}, Q)$ be a $NScs$ in (Y_1, τ, Q). Then $h(\tilde{H}, Q)$ is $NSMcs$ of (Y_2, σ, Q) because h is $NSMC$ mapping. By hypothesis, $h(\tilde{H}, Q)$ is $NScs$ of (Y_2, σ, Q). Now $g(h(\tilde{H}, Q)) = (g \circ h)(\tilde{H}, Q)$ is $NSMcs$ in (Y_3, ρ, Q) because g is $NSMC$ mapping. Thus $g \circ h$ is $NSMC$ mapping.

Theorem 4.6. *Let $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ be a bijective mapping. Then the statements are equivalent:*

(i) h is a $NSMO$ mapping.

(ii) h is a $NSMC$ mapping.

(iii) h^{-1} is $NSMCts$ mapping.

Proof. (i) $\Rightarrow$ (ii): Let us assume that h is a $NSMO$ mapping. By definition, $(\tilde{H}, Q)$ is a $NSos$ in (Y_1, τ, Q), then $h(\tilde{H}, Q)$ is a $NSMos$ in (Y_2, σ, Q). Here, $(\tilde{H}, Q)$ is $NScs$ in (Y_1, τ, Q). Then $Y_1 - (\tilde{H}, Q)$ is a $NSos$ in (Y_1, τ, Q). By assumption, $h(Y_1 - (\tilde{H}, Q))$ is a $NSMos$ in (Y_2, σ, Q). Hence, $Y_2 - h(Y_1 - (\tilde{H}, Q))$ is a $NSMcs$ in (Y_2, σ, Q). Therefore, h is a $NSMC$ mapping.

(ii) $\Rightarrow$ (iii): Let $(\tilde{H}, Q)$ be a $NScs$ in (Y_1, τ, Q) By (ii), $h(\tilde{H}, Q)$ is a $NSMcs$ in (Y_2, σ, Q). Hence, $h(\tilde{H}, Q) = (h^{-1})^{-1}(\tilde{H}, Q)$. So h^{-1} is a $NSMcs$ in (Y_2, σ, Q). Hence, h^{-1} is $NSMCts$.

(iii) $\Rightarrow$ (i): Let $(\tilde{H}, Q)$ be a $NSos$ in (Y_1, τ, Q). By (iii), $(h^{-1})^{-1}(\tilde{H}, Q) = h(\tilde{H}, Q)$ is a $NSMO$ mapping.

5. Neutrosophic Soft M-homeomorphism

Definition 5.1. *A bijection $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ is called a neutrosophic soft M-homeomorphism (briefly $NSMHom$) if h and h^{-1} are $NSMCts$.*

Theorem 5.1. *Each $NSHom$ is a $NSMHom$. But not conversely.*
Proof. Let h be $NSHom$, then h and h^{-1} are $NSCts$. But every $NSCts$ function is $NSMCts$. Hence, h and h^{-1} are $NSMCts$. Therefore, h is a $NSMHom$.

Example 5.1. Let $U = \{u_1, u_2, u_3\}, V = \{v_1, v_2, v_3\}, Q = \{q_1, q_2\}$ and NSs's $(\tilde{F}_1, Q), (\tilde{F}_2, Q)$ & $(\tilde{F}_3, Q)$ in U and $(\tilde{G}_1, Q)$ in V are defined as

$$(\tilde{F}_1, q_1) = \{\langle u_1, (0.20, 0.5, 0.80)\rangle, \langle u_2, (0.30, 0.5, 0.70)\rangle, \langle u_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{F}_1, q_2) = \{\langle u_1, (0.21, 0.5, 0.79)\rangle, \langle u_2, (0.31, 0.5, 0.69)\rangle, \langle u_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{F}_2, q_1) = \{\langle u_1, (0.10, 0.5, 0.90)\rangle, \langle u_2, (0.10, 0.5, 0.90)\rangle, \langle u_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{F}_2, q_2) = \{\langle u_1, (0.11, 0.5, 0.89)\rangle, \langle u_2, (0.11, 0.5, 0.89)\rangle, \langle u_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{F}_3, q_1) = \{\langle u_1, (0.20, 0.5, 0.80)\rangle, \langle u_2, (0.20, 0.5, 0.80)\rangle, \langle u_3, (0.30, 0.5, 0.70)\rangle\}$$
$$(\tilde{F}_3, q_2) = \{\langle u_1, (0.21, 0.5, 0.79)\rangle, \langle u_2, (0.21, 0.5, 0.79)\rangle, \langle u_3, (0.31, 0.5, 0.69)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle v_1, (0.20, 0.5, 0.80)\rangle, \langle v_2, (0.20, 0.5, 0.80)\rangle, \langle v_3, (0.30, 0.5, 0.70)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle v_1, (0.21, 0.5, 0.79)\rangle, \langle v_2, (0.21, 0.5, 0.79)\rangle, \langle v_3, (0.31, 0.5, 0.69)\rangle\}$$

Then we have $\tau = \{0_{(U,Q)}, 1_{(U,Q)}, (\tilde{F}_1, Q), (\tilde{F}_2, Q)\}$ and $\sigma = \{0_{(V,Q)}, 1_{(V,Q)}, (\tilde{G}_1, Q)\}$. Let $h : (U, \tau, Q) \to (V, \sigma, Q)$ be an identity mapping. Then h is $NSMHom$ but not $NSHom$.

Theorem 5.2. *Let $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ be a bijective mapping. If h is $NSMCts$, then the statements are equivalent:*

(i) h is a $NSMC$ mapping.

(ii) h is a $NSMO$ mapping.

(iii) h^{-1} is a $NSMHom$.

Proof. (i) $\Rightarrow$ (ii) : Assume that h is a bijective mapping and a $NSMC$ mapping. Hence, h^{-1} is a $NSMCts$ mapping. We know that each $NSos$ in (Y_1, τ, Q) is a $NSMos$ in (Y_2, σ, Q). Hence, h is a $NSMO$ mapping.

(ii) $\Rightarrow$ (iii) : Let h be a bijective and NSO mapping. Further, h^{-1} is a $NSMCts$ mapping. Hence, h and h^{-1} are $NSMCts$. Therefore, h is a $NSMHom$.

(iii) $\Rightarrow$ (i): Let h be a $NSMHom$. Then h and h^{-1} are $NSMCts$. Since each $NScs$ in (Y_1, τ, Q) is a $NSMcs$ in (Y_2, σ, Q), h is a $NSMC$ mapping.

Definition 5.2. A $NSts$ (Y_1, τ, Q) is said to be a neutrosophic soft $MT_{\frac{1}{2}}$ (briefly, $NSMT_{\frac{1}{2}}$)-space if every $NSMcs$ is $NScs$ in (Y_1, τ, Q).

Theorem 5.3. *Let $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ be a $NSMHom$. Then h is a $NSHom$ if (Y_1, τ, Q) and (Y_2, σ, Q) are $NSMT_{\frac{1}{2}}$-space.*

Proof. Assume that $(\tilde{G}, Q)$ is a $NScs$ in (Y_2, σ, Q). Then $h^{-1}(\tilde{G}, Q)$ is a $NSMcs$ in (Y_1, τ, Q). Since (Y_1, τ, Q) is an $NSMT_{\frac{1}{2}}$-space, $h^{-1}(\tilde{G}, Q)$ is a $NScs$ in (Y_1, τ, Q). Therefore, h is $NSCts$. By hypothesis, h^{-1} is $NSMCts$. Let $(\tilde{H}, Q)$ be a $NScs$ in (Y_1, τ, Q). Then, $(h^{-1})^{-1}(\tilde{H}, Q) = h(\tilde{H}, Q)$ is a $NScs$ in (Y_2, σ, Q), by presumption. Since (Y_2, σ, Q) is a $NSMT_{\frac{1}{2}}$-space, $h(\tilde{H}, Q)$ is a $NScs$ in (Y_2, σ, Q). Hence, h^{-1} is $NSCts$. Hence, h is a $NSHom$.

Theorem 5.4. *Let $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ be a $NSts$. Then the statements are equivalent if (Y_2, σ, Q) is a $NSMT_{\frac{1}{2}}$-space:*

(i) h is $NSMC$ mapping.

(ii) If $(\tilde{H}, Q)$ is a $NSos$ in (Y_1, τ, Q), then $h(\tilde{H}, Q)$ is $NSMos$ in (Y_2, σ, Q).

(iii) $h(NSint(\tilde{H}, Q)) \subseteq NScl(NSint(h(\tilde{H}, Q)))$ for every NSs $(\tilde{H}, Q)$ in (Y_1, τ, Q).

Proof. (i) $\Rightarrow$ (ii): Obvious.

(ii) $\Rightarrow$ (iii): Let $(\tilde{H}, Q)$ be a NSs in (Y_1, τ, Q). Then, $NSint(\tilde{H}, Q)$ is a $NSos$ in (Y_1, τ, Q). Then, $h(NSint(\tilde{H}, Q))$ is a $NSMos$ in (Y_2, σ, Q). Since (Y_2, σ, Q) is a $NSMT_{\frac{1}{2}}$-space, so $h(NSint(\tilde{H}, Q))$ is a $NSos$ in (Y_2, σ, Q). Therefore, $h(NSint(\tilde{H}, Q)) = NSint(h(NSint(\tilde{H}, Q))) \subseteq NScl(NSint(h(\tilde{H}, Q)))$.

(iii) $\Rightarrow$ (i): Let $(\tilde{H}, Q)$ be a $NScs$ in (Y_1, τ, Q). Then, $(\tilde{H}, Q)^c$ is a $NSos$ in (Y_1, τ, Q). From, $h(NSint(\tilde{H}, Q)^c) \subseteq NScl(NSint(h(\tilde{H}, Q)^c))$, $h((\tilde{H}, Q)^c) \subseteq NScl(NSint(h(\tilde{H}, Q)^c))$. Therefore, $h((\tilde{H}, Q)^c)$ is $NSMos$ in (Y_2, σ, Q). Therefore, $h(\tilde{H}, Q)$ is a $NSMcs$ in (Y_1, τ, Q). Hence, h is a NSC mapping.

Theorem 5.5. *Let $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ and $g : (Y_2, \sigma, Q) \to (Y_3, \rho, Q)$ be $NSMC$, where (Y_1, τ, Q) and (Y_3, ρ, Q) are two $NSts$'s and (Y_2, σ, Q) a $NSMT_{\frac{1}{2}}$-space, then the composition $g \circ h$ is $NSMC$.*

Proof. Let $(\tilde{H}, Q)$ be a $NScs$ in (Y_1, τ, Q). Since h is $NSMc$ and $h(\tilde{H}, Q)$ is a $NSMcs$ in (Y_2, σ, Q), by assumption, $h(\tilde{H}, Q)$ is a $NScs$ in (Y_2, σ, Q). Since g is $NSMc$, then $g(h(\tilde{H}, Q))$ is $NSMc$ in (Y_3, ρ, Q) and $g(h(\tilde{H}, Q)) = (g \circ h)(\tilde{H}, Q)$. Therefore, $g \circ h$ is $NSMC$.

Theorem 5.6. *Let $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ and $g : (Y_2, \sigma, Q) \to (Y_3, \rho, Q)$ be two $NSts$'s, then the statements are hold:*

(i) If $g \circ h$ is $NSMO$ and h is $NSCts$, then g is $NSMO$.

(ii) If $g \circ h$ is NSO and g is $NSMCts$, then h is $NSMO$.

Proof. Obvious.

6. Neutrosophic Soft M-C Homeomorphism

Definition 6.1. *A bijection $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ is called a neutrosophic soft M-C homeomorphism (briefly, $NSMCHom$) if h and h^{-1} are $NSMIrr$ mappings.*

Theorem 6.1. *Each $NSMCHom$ is a $NSMHom$. But not conversely.*
Proof. Let us assume that $(\tilde{G}, Q)$ is a $NScs$ in (Y_2, σ, Q). This shows that $(\tilde{G}, Q)$ is a $NSMcs$ in (Y_2, σ, Q). By assumption, $h^{-1}(\tilde{G}, Q)$ is a $NSMcs$ in (Y_1, τ, Q). Hence, h is a $NSMCts$ mapping. Hence, h and h^{-1} are $NSMCts$ mappings. Hence h is a $NSMHom$.

Example 6.1. Let $U = \{u_1, u_2, u_3\}, V = \{v_1, v_2, v_3\}, Q = \{q_1, q_2\}$ and NSs's $(\tilde{F}_1, Q), (\tilde{F}_2, Q)$ & $(\tilde{F}_3, Q)$ in U and $(\tilde{G}_1, Q)$ in V are defined as

$$(\tilde{F}_1, q_1) = \{\langle u_1, (0.20, 0.5, 0.80)\rangle, \langle u_2, (0.30, 0.5, 0.70)\rangle, \langle u_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{F}_1, q_2) = \{\langle u_1, (0.21, 0.5, 0.79)\rangle, \langle u_2, (0.31, 0.5, 0.69)\rangle, \langle u_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{F}_2, q_1) = \{\langle u_1, (0.10, 0.5, 0.90)\rangle, \langle u_2, (0.10, 0.5, 0.90)\rangle, \langle u_3, (0.40, 0.5, 0.60)\rangle\}$$
$$(\tilde{F}_2, q_2) = \{\langle u_1, (0.11, 0.5, 0.89)\rangle, \langle u_2, (0.11, 0.5, 0.89)\rangle, \langle u_3, (0.41, 0.5, 0.59)\rangle\}$$
$$(\tilde{F}_3, q_1) = \{\langle u_1, (0.20, 0.5, 0.80)\rangle, \langle u_2, (0.20, 0.5, 0.80)\rangle, \langle u_3, (0.30, 0.5, 0.70)\rangle\}$$
$$(\tilde{F}_3, q_2) = \{\langle u_1, (0.21, 0.5, 0.79)\rangle, \langle u_2, (0.21, 0.5, 0.79)\rangle, \langle u_3, (0.31, 0.5, 0.69)\rangle\}$$
$$(\tilde{G}_1, q_1) = \{\langle v_1, (0.20, 0.5, 0.80)\rangle, \langle v_2, (0.20, 0.5, 0.80)\rangle, \langle v_3, (0.30, 0.5, 0.70)\rangle\}$$
$$(\tilde{G}_1, q_2) = \{\langle v_1, (0.21, 0.5, 0.79)\rangle, \langle v_2, (0.21, 0.5, 0.79)\rangle, \langle v_3, (0.31, 0.5, 0.69)\rangle\}$$

Then we have $\tau = \{0_{(U,Q)}, 1_{(U,Q)}, (\tilde{F}_1, Q), (\tilde{F}_2, Q)\}$ and $\sigma = \{0_{(V,Q)}, 1_{(V,Q)}, (\tilde{G}_1, Q)\}$. Let $h : (U, \tau, Q) \to (V, \sigma, Q)$ be an identity mapping. Then h is $NSMHom$ but not $NSMCHom$.

Theorem 6.2. *If $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ is a $NSMCHom$, then $NSMcl(h^{-1}(\tilde{G}, Q)) \subseteq h^{-1}(NScl(\tilde{G}, Q))$ for each NSs $(\tilde{G}, Q)$ in (Y_2, σ, Q).*

Proof. Let $(\tilde{G}, Q)$ be a NSs in (Y_2, σ, Q). Then, $NScl(\tilde{G}, Q)$ is a $NScs$ in (Y_2, σ, Q), and every $NScs$ is a $NSMcs$ in (Y_2, σ, Q). Assume h is $NSMIrr$ and $h^{-1}(NScl(\tilde{H}, Q))$ is a $NSMcs$ in (Y_1, τ, Q). Then, $NScl(h^{-1}(NScl(\tilde{G}, Q))) = h^{-1}(NScl(\tilde{G}, Q))$. Here, $NSMcl(h^{-1}(\tilde{G}, Q)) \subseteq NSMcl(h^{-1}(NScl(\tilde{H}, Q))) = h^{-1}(NScl(\tilde{G}, Q))$. Therefore, $NSMcl(h^{-1}(\tilde{G}, Q)) \subseteq h^{-1}(NScl(\tilde{G}, Q))$ for every NSs $(\tilde{G}, Q)$ in (Y_2, σ, Q).

Theorem 6.3. *Let $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ be a $NSMCHom$. Then $NSMcl(h^{-1}(\tilde{G}, Q)) = h^{-1}(NSMcl(\tilde{G}, Q))$ for each NSs $(\tilde{G}, Q)$ in (Y_2, σ, Q).*

Proof. Since h is a $NSMCHom$, h is a $NSMIrr$ mapping. Let $(\tilde{G}, Q)$ be a NSs in (Y_2, σ, Q). Clearly, $NSMcl(\tilde{G}, Q)$ is a $NSMcs$ in (Y_2, σ, Q). Then $NSMcl(\tilde{G}, Q)$ is a $NSMcs$ in (Y_2, σ, Q). Since $h^{-1}(\tilde{G}, Q) \subseteq h^{-1}(NSMcl(\tilde{G}, Q))$, then $NSMcl(h^{-1}(\tilde{G}, Q)) \subseteq NSMcl(h^{-1}(NSMcl(\tilde{G}, Q))) = h^{-1}(NSMcl(\tilde{G}, Q))$. Therefore, $NSMcl(h^{-1}(\tilde{G}, Q)) \subseteq h^{-1}(NSMcl(\tilde{G}, Q))$. Let h be a $NSMCHom$. h^{-1} is a $NSMIrr$ mapping. Let us consider NSs $h^{-1}(\tilde{G}, Q)$ in (Y_1, τ, Q), which implies $NSMcl(h^{-1}(\tilde{G}, Q))$ is a $NSMcs$ in (Y_1, τ, Q). Hence, $NSMcl(h^{-1}(\tilde{G}, Q))$ is a $NSMcs$ in (Y_1, τ, Q). This implies that $(h^{-1})^{-1}(NSMcl(h^{-1}(\tilde{G}, Q))) = h(NSMcl(h^{-1}(\tilde{G}, Q)))$ is a $NSMcs$ in (Y_2, σ, Q). This proves $(\tilde{G}, Q) = (h^{-1})^{-1}(h^{-1}(\tilde{G}, Q)) \subseteq (h^{-1})^{-1}(NSMcl(h^{-1}(\tilde{G}, Q))) = h(NSMcl(h^{-1}(\tilde{G}, Q)))$. Therefore, $NSMcl(\tilde{G}, Q) \subseteq NSMcl(h(NSMcl(h^{-1}(\tilde{G}, Q)))) = h(NSMcl(h^{-1}(\tilde{G}, Q)))$, since h^{-1} is a $NSMIrr$ mapping. Hence, $h^{-1}(NSMcl(\tilde{G}, Q)) \subseteq h^{-1}(h(NSMcl(h^{-1}(\tilde{G}, Q)))) = NSMcl(h^{-1}(\tilde{G}, Q))$. That is, $h^{-1}(NSMcl(\tilde{G}, Q)) \subseteq NSMcl(h^{-1}(\tilde{G}, Q))$. Hence, $NSMcl(h^{-1}(\tilde{G}, Q)) = h^{-1}(NSMcl(\tilde{G}, Q))$.

Theorem 6.4. *If $h : (Y_1, \tau, Q) \to (Y_2, \sigma, Q)$ and $g : (Y_2, \sigma, Q) \to (Y_3, \rho, Q)$ are $NSMCHom$'s, then $g \circ h$ is a $NSMCHom$.*

Proof. Let h and g be two $NSMCHom$'s. Assume $(\tilde{G}, Q)$ is a $NSMcs$ in (Y_3, ρ, Q). Then, $g^{-1}(\tilde{G}, Q)$ is a $NSMcs$ in (Y_2, σ, Q). Then, by hypothesis, $h^{-1}(g^{-1}(\tilde{G}, Q))$ is a $NSMcs$ in (Y_1, τ, Q). Hence, $g \circ h$ is a $NSMIrr$ mapping. Now, let $(\tilde{H}, Q)$ be a $NSMcs$ in (Y_1, τ, Q). Then, by presumption, $h(\tilde{H}, Q)$ is a $NSMcs$ in (Y_2, σ, Q). Then, by hypothesis, $g(h(\tilde{H}, Q))$ is a $NSMcs$ in (Y_3, ρ, Q). This implies that $g \circ h$ is a $NSMIrr$ mapping. Hence, $g \circ h$ is a $NSMCHom$.

7. Conclusion

In this paper, the concepts of $NSMO$ and a $NSMC$ mappings in $NSts$ were discussed. Furthermore, the work was extended to include $NSHom$, $NSMHom$ and $NSMT_{\frac{1}{2}}$-space. In addition, the study demonstrated $NSMCHom$ and derived some of its related characteristics. In future, the research is to be investigate on neutrosophic soft M-compactness, neutrosophic soft M-connectedness and neutrosophic soft contra M-continuous functions.

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