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FUZZY $\mathrm{T}_{\tilde{g}}$-SPACES

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Abstract: In this paper, we introduce and study the concepts of fuzzy $T\tilde{g}$-spaces, fuzzy $gT_{\tilde{g}}$-spaces and their properties are obtain. The relations between $T_{\tilde{g}}$-spaces, fuzzy $gT_{\tilde{g}}$-spaces and other fuzzy spaces are given. Also many suitable examples are given.

Keywords and Phrases: Fuzzy $\tilde{g}$-closed sets, fuzzy $\mathrm{T}_{\tilde{g}}$-spaces and fuzzy $gT_{\tilde{g}}$-spaces.

2020 Mathematics Subject Classification: 54A05, 54A10, 54C08, 54C10.

1. Introduction

In the year 1965, Zadeh [18] introduced and studied the concept of a fuzzy subset. Following research in this area and related activities, many branches of science and engineering have found applications. Chang [5] introduced and studied fuzzy. Many researchers have contributed to the development of fuzzy topological spaces, for example Azad [1], Wong [16], Tirpahy [14], Dutta [6] and Sarma [10] and so on. The concepts of image and inverse image of a fuzzy set under a function are included, as are the properties demonstrated by Chang [5] and Warren [15].

The basic concepts and results on fuzzy topological spaces from the work of Chang [5], Wong [16] and Malghan [8] are extended.

2. Preliminaries

Definition 2.1. *A fuzzy subset A of a fuzzy topological space (X, τ) is called:*

1. *fuzzy semi-open set [1] if $A \leq cl(int(A))$.*
2. *fuzzy α-open set [4] if $A \leq int(cl(int(A)))$.*
3. *fuzzy semi-preopen set [13] if $A \leq cl(int(cl(A)))$.*
4. *fuzzy regular open set [1] if $A = int(cl(A))$.*

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

The operators namely, fuzzy semi-closure [17], fuzzy α-closure [9], fuzzy semi-preclosure [17] in (X, F_τ).

Definition 2.2. *A fuzzy subset A of a fuzzy topological space (X, τ) is called:*

1. *a fuzzy generalized semi-closed (briefly fgs-closed) set [7] if $scl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X, τ). The complement of fgs-closed set is called fgs-open set;*
2. *a fuzzy α-generalized closed (briefly fαg-closed) set [11] if $\alpha cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X, τ). The complement of fαg-closed set is called fαg-open set;*
3. *a fuzzy ω-closed set (briefly $f\omega$-closed) [12] if $cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open in (X, τ). The complement of $f\omega$-closed set is called $f\omega$-open set;*
4. *a fuzzy $\tilde{g}$-closed set (shortly denotes $f\tilde{g}$-closed) [3] if $cl(H) \leq U$ whenever $H \leq U$ and U is fsg-open.*

The complements of the above mentioned fuzzy closed sets are called their respective fuzzy open sets.

Definition 2.3. *A fts (X, τ) is called fuzzy $T_{1/2}$-space [2] if every fg-closed set in it is fuzzy closed.*

3. On Fuzzy $T_{\tilde{g}}$-spaces

We introduce the following definitions.

Definition 3.1. *A fuzzy topological space (X, F_τ) is said to be a*

1. *fuzzy $T\omega$-space if each $f\omega$-closed set in it is fuzzy closed.*
2. *fuzzy $T\tilde{g}$-space if each $f\tilde{g}$-closed set in it is fuzzy closed.*
3. *fuzzy αT_b-space if each $f\alpha g$-closed set in it is fuzzy closed.*
4. *fuzzy T_b-space if each fgs-closed set in it is fuzzy closed.*

Example 3.2. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ where η is a fuzzy set in X defined by $\eta(m) = 0.5, \eta(n) = 0.4$. Then (X, F_τ) is a fuzzy topological space. This verifies that (X, F_τ) is a fuzzy $T\omega$ space.

Example 3.3. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ where η is a fuzzy set in X defined by $\eta(m) = 0.5, \eta(n) = 0.5$. Then (X, F_τ) is a fuzzy topological space. This verifies that (X, F_τ) is a fuzzy $T\tilde{g}$ space.

Example 3.4. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, \mu, 1_X\}$ where η, μ are fuzzy sets in X defined by $\eta(m) = 0.4, \eta(n) = 0.4, \mu(m) = 0.6, \mu(n) = 0.6$. Then (X, F_τ) is a fuzzy topological space. This verifies (X, F_τ) is a fuzzy αT_b-space.

Example 3.5. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ where η is a fuzzy set in X defined by $\eta(m) = 0.5, \eta(n) = 0.5$. This verifies (X, F_τ) is a fuzzy topological space. This verifies that (X, F_τ) is a fuzzy T_b-space.

Example 3.6. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = 1$ and $\alpha(n) = 0$. Then (X, F_τ) is a fuzzy topological space. This verifies that (X, F_τ) is not a fuzzy $T_{\tilde{g}}$-space.

4. Properties of Fuzzy $T_{\tilde{g}}$-spaces

Proposition 4.1. *A fuzzy $T_{1/2}$-space is fuzzy $T_{\tilde{g}}$-space.*
Proof. If H is a $f\tilde{g}$-closed set of (X, F_τ). Each $f\tilde{g}$-closed set is fg-closed. Since (X, F_τ) is a fuzzy $T_{1/2}$-space, then H is fuzzy closed. Which verifies that (X, F_τ) is a fuzzy $T_{\tilde{g}}$-Space.

Remark 4.2. *The following example shows that the converse of Proposition 4.1 is not true in general.*

Example 4.3. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = 0.5, \alpha(n) = 0.5$. Then (X, F_τ) is a fuzzy topological space. This verifies that (X, F_τ) is a fuzzy $T_{\tilde{g}}$-space but not fuzzy $T_{1/2}$-space.

Proposition 4.4. *A fuzzy T_ω-space is fuzzy $T_{\tilde{g}}$-space.*
Proof. If H is a $f\tilde{g}$-closed set of (X, F_τ). Each $f\tilde{g}$-closed set is $f\omega$-closed. Since (X, F_τ) is a fuzzy T_ω-space, then H is fuzzy closed. This verifies that (X, F_τ) is a

fuzzy $T_{\tilde{g}}$-space.

Remark 4.5. *The following example shows that the converse of Proposition 4.4 is not true in general.*

Example 4.6. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = 0.5, \alpha(n) = 0.5$. Then (X, F_τ) is a fuzzy topological space. This verifies that (X, F_τ) is fuzzy $T_{\tilde{g}}$-space but not a fuzzy T_ω-space.

Proposition 4.7. *A fuzzy αT_b-space is fuzzy $T_{\tilde{g}}$-space.*
Proof. If H is a $f\tilde{g}$-closed set of (X, F_τ) . Each $f\tilde{g}$-closed set is $f\alpha$g-closed. Since (X, F_τ) is a fuzzy αT_b-space, then H is fuzzy closed. This verifies that (X, F_τ) is fuzzy $\mathrm{T}_{\ddot{g}}$-space.

Remark 4.8. *The following example shows that the converse of Proposition 4.7 is not true in general.*

Example 4.9. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = 0.5, \alpha(n) = 0.5$. Then (X, F_τ) is a fuzzy topological space. This verifies that (X, F_τ) is fuzzy $T_{\tilde{g}}$-space but not a fuzzy αT_b-space.

5. Fuzzy $gT_{\tilde{g}}$-spaces

Definition 5.1. *A fuzzy topological space (X, F_τ) is said be a fuzzy $gT_{\tilde{g}}$-space if each fg-closed set in $f\tilde{g}$-closed.*

Example 5.2. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = \alpha(n) = 0.5$. Then (X, F_τ) is a fuzzy topological space. This verifies that (X, F_τ) is a fuzzy $gT_{\tilde{g}}$-space.

Example 5.3. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = 1$ and $\alpha(n) = 0$. It follows that (X, F_τ) is not a fuzzy $gT_{\tilde{g}}$-space.

Proposition 5.4. *A fuzzy $T_{1/2}$-space is fuzzy $gT_{\tilde{g}}$-space.*
Proof. Let H be any fg-closed set of (X, F_τ). Since (X, F_τ) is a fuzzy $T_{1/2}$ space, H is fuzzy closed. Hence (X, F_τ) is a fuzzy $gT_{\tilde{g}}$-space.

Remark 5.5. *The following example shows that the converse of Proposition 5.4 is not true in general.*

Example 5.6. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = 0.5, \alpha(n) = 0.5$. Thus (X, F_τ) is a fuzzy $gT_{\tilde{g}}$-space but not fuzzy $T_{1/2}$-space.

Remark 5.7. *The following example shows that the concepts of fuzzy $T\tilde{g}$-space* *and the concepts of fuzzy $gT_{\tilde{g}}$-space are independent.*

Example 5.8.

1. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = 0.6$ and $\alpha(n) = 0.6$. Then (X, F_τ) is a fuzzy topological space. This verifies that (X, F_τ) is fuzzy $gT_{\tilde{g}}$-space but not a fuzzy $T_{\tilde{g}}$-space.

2. Consider $X = \{m, n\}$ with $F_\tau = \{0_X, \alpha, 1_X\}$ where α is a fuzzy set in X defined by $\alpha(m) = 0.5, \alpha(n) = 0.5$. Thus (X, F_τ) is a fuzzy $T_{\tilde{g}}$-space but not fuzzy $gT_{\tilde{g}}$-space.

Theorem 5.9. *A fuzzy topological space (X, F_τ), then the following statements are equivalent.*

1. *(X, F_τ) is a fuzzy $T_{1/2}$-space*

2. *(X, F_τ) is fuzzy $T_{\tilde{g}}$-space and fuzzy $gT_{\tilde{g}}$-space.*

Proof. (1) $\Rightarrow$ (2): It follows from the Proposition 4.1 and the Proposition 5.4.

(2) $\Rightarrow$ (1): Assume that X is both a fuzzy $T_{\tilde{g}}$-space and fuzzy $gT_{\tilde{g}}$-space. Let H be a fg-closed set of (X, F_τ). Since (X, F_τ) is fuzzy $gT_{\tilde{g}}$-space, H is a $f\tilde{g}$-closed set of (X, F_τ). Since (X, F_τ) is a fuzzy $T_{\tilde{g}}$-space, H is a fuzzy closed set of (X, F_τ). Therefore (X, F_τ) is a fuzzy $T_{1/2}$-space.

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