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SOMEWHAT gp-CONTINUOUS AND SOMEWHAT gp-IRRESOLUTE FUNCTIONS

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Abstract: The main object of this paper is to study the basic properties of somewhat gp-continuous functions using the concept of gp-open sets in topological spaces. Also, the concept of somewhat gp-open functions and somewhat gp-irresolute functions with some counter examples are explained in this paper. Further, the author establishes the relationship between the new classes of functions with other classes of functions by giving examples, counterexamples, properties and characterizations.

Keywords and Phrases: Somewhat continuous, somewhat gp-continuous, somewhat gpr-continuous functions.

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1. Introduction

In the literature, many topologists have focused their research in the direction of investigating different types of continuous functions in topological spaces. gp-closed sets and their properties were introduced by H. Maki et al. [6]. Gentry and Hoyle [5] studied somewhat continuous functions in topological spaces and continued by [1], [2], [3], [4], [8], [10].

This paper deals with the study of new weaker class of functions called somewhat gp-continuous and some characterizations and somewhat gp-open and somewhat gp-closed functions. Lastly, somewhat gp-irresolute maps are defined and discussed some of their basic properties.

2. Preliminaries

Throughout this paper (P, τ), (Q, σ) and (R, μ) (or simply P, Q and R) always mean topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1. [6] *S is gp-closed, if pcl(S) $\subset U$ whenever $S \subset U$ and U is open in P.*

We have referred the following research papers [1], [5] and [9] for the properties of somewhat gpr-continuous (sw.gpr-c), somewhat continuous(sw.c) and somewhat-open(sw-O) and gp-continuous and gp-irresolute maps respectively.

3. Somewhat gp-continuous Functions

This section contains a new weaker forms of functions called somewhat gp-continuous functions and their properties.

Definition 3.1. *A function $k : (P, \tau) \to (Q, \sigma)$ is somewhat gp-continuous (sw.gp-c) if for each $M \in \sigma$ and $k^{-1}(M) \neq \phi$, there exists $N \in$ gp-O(P) with $N \subset k^{-1}(M)$.*

Example 3.1. Let $P = Q = \{m_1, m_2\}$, $\tau = \{P, \phi\}$ and $\sigma = \{Q, \phi, \{m_1\}\}$. Let $k : P \to Q$ be a function defined as $k(m_1) = m_2$ and $k(m_2) = m_1$. Consider, $M = \{m_1\} \in \sigma$ with $m_1 \neq \phi$, there exists $N = \{m_2\}$ with $k^{-1}(\{m_1\}) = \{m_2\} \neq \phi$ and $\{m_2\} \subset K^{-1}(\{m_1\})$ and so k is sw.gp-c.

Theorem 3.1. *Every sw.c function is sw.gp-c.*
Proof. Let $k : P \to Q$ be sw.c and $M \in O(Q)$ with $k^{-1}(M) \neq \phi$. Since k is sw.c, $\exists\ N \in O(P)$ with $N \neq \phi$ and $N \subset k^{-1}(M)$. Since every open set is gp-open, then $N \subset k^{-1}(M)$ and so k is sw.gp-c.

Example 3.2. Let $P = Q = \{m_1, m_2, m_3\}$, $\tau = \{P, \phi, \{m_1\}, \{m_2, m_3\}\}$, $\sigma = \{Q, \phi, \{m_1\}\}$. Then $k : P \to Q$ defined as $k(m_1) = m_3, k(m_2) = m_1, k(m_3) = m_2$. Then k is sw.gp-c but not sw.c. The open set $M = \{m_1\} \in \sigma$, then there does not exist any open set M such that $M \subset k^{-1}(\{m_1\}) = \{m_2\}$.

Theorem 3.2. *Every sw.gp-c function is sw.gpr-c.*
Proof. Proof is obvious.

Theorem 3.3. *Let $k_1 : (P, \tau) \to (Q, \sigma)$ be sw.gp-c and $k_2 : (Q, \sigma) \to (R, \eta)$ be continuous. Then $k_2 \circ k_1 : P \to R$ is sw.gp-c.*
Proof. Let $M \in \eta$ with $k_1^{-1}(M) \neq \phi$. As $M \in \eta$ and $k_2$is continuous, $k_2^{-1}(m) \in \sigma$. As k_1 is sw.gp-c, there exists $N \in gp$-O(Q) with $N \neq \phi$ and $N \subset k_1^{-1}(k_2^{-1}(M))$. That is $N \subset (k_2 \circ k_1)^{-1}(M)$ and so $k_2 \circ k_1$ is sw.gp-c.

Remark 3.1. *Composition of two sw.gp-c functions need not be sw.gp-c.*

Example 3.3. Let $P = Q = R = \{m_1, m_2, m_3\}$. Let $\tau = \{P, \phi, \{m_2, m_3\}\}$, $\sigma = \{Q, \phi, \{m_1\}, \{m_2, m_3\}\}$ and $\eta = \{R, \phi, \{m_1\}, \{m_1, m_2\}\}$. Let $k_1 : P \to Q$ defined as $k_1(m_1) = m_3, k_1(m_2) = m_1, k_1(m_3) = m_2$. Then, k_1 is sw.gp-c. Let $k_2 : Q \to R$ be the identity function and k_2 is sw.gp-c. But, the composition $k_2 \circ k_1 : P \to R$ is not sw.gp-c. Consider the open set $M = \{m_1, m_2\} \in \eta$, then $k_1^{-1}(k_2^{-1}(M)) = \{m_2, m_3\}$. Thus, there does not exists any $N \in gp$-O(P) with $N \subset k_1^{-1}(M)$.

Corollary 3.1. *Let $k_1 : P \to Q$ be sw.gp-c and $k_2 : Q \to R$ be pre-continuous. Then $k_2 \circ k_1 : P \to Q$ is sw.gp-c.*

Corollary 3.2. *Let $k_1 : P \to Q$ be sw.gp-c and $k_2 : Q \to R$ be pre-irresolute. Then $k_2 \circ k_1 : P \to Q$ is sw.gp-c.*

Definition 3.2. *Let $M_1 \subset P$. Then M is dense in P, if there exists no proper gp-closed set C_1 in P with $M_1 \subset C_1 \subset P$.*

Theorem 3.4. *The following properties are equivalent for a surjective function $k : P \to Q$:*
(i) k is sw.gp-c
(ii) If $A \in C(Q)$ with $k^{-1}(A) \neq P$, then there exists a proper gp-closed set D of P such that $k^{-1}(A) \subset D$.
(iii) If M is gp-dense set of P, then k(M) is dense set of Q.
Proof. $(i) \to (ii)$ Let $A \in C(Q)$ with $k^{-1}(A) \neq P$, so $Q \setminus A \in O(Q)$ with $k^{-1}(Q \setminus A) = P \setminus k^{-1}(A) \neq \phi$. From (i), there exists $V \in gp$-O(P) such that $V \neq \phi$ and $V \subset k^{-1}(Q \setminus A) = P \setminus k^{-1}(A)$, $k^{-1}(A) \subset P \setminus V$ and $P \setminus V \in gp$-C(P). Thus (ii) holds.
$(ii) \to (iii)$ Let M be gp-dense set in P. We have to show that k(M) is dense in Q. Assume that M is not gp-dense in Q, then exists a proper closed set A in Q with $k(M) \subset A \subset Q$, so $k^{-1}(A) \neq P$. From (ii), there exists a proper gp-closed set D

such that $k^{-1}(A) = D$ with $M \subset k^{-1}(A) \subset D \subset P$, which is contradiction to the assumption. Thus k(M) is dense in Q.
$(iii) \to (ii)$ Let the property (ii) is not true, that is $\exists\ A \in C(Y)$ with $k^{-1}(A) \neq P$. But there does not exists proper gp-closed set D with $k^{-1}(A) \subset D$, implies that $k^{-1}(A)$ is gp-dense in P. From (iii), $k(k^{-1}(A)) = A$ must be dense in Q, which is contradiction to the assumption.
$(ii) \to (i)$ Let $U \in O(Q)$ with $k^{-1}(U) \neq \phi$. Then $Q \setminus U \in C(Q)$ and $k^{-1}(Q \setminus U) = P \setminus k^{-1}(U) \neq P$. From (ii), there exists a proper gp-closed set D such that $k^{-1}(Q \setminus U) \subset D$, that is $k^{-1}(U) \subset P \setminus D$ where $P \setminus D \in gp$-O(P) and $P \setminus D \neq \phi$.

Theorem 3.5. *Let $A \in O(P)$ and $k : (A, \tau_A) \to (Q, \sigma)$ be sw.gp-c with k(A) is dense in Q. Then any extension K of k is sw.gp-c.*
Proof. Let $U \in O(Q)$ with $k^{-1}(U) \neq \phi$. As $k(A) \subset Q$ which is dense and $U \cap k(A) \neq \phi$. Thus $K^{-1}(U) \cap A \neq \phi$ and so $k^{-1}(U) \cap A \neq \phi$. From hypothesis, there exists $V \in gp$-O(P) with $V \neq \phi$ and $V \subset k^{-1}(U) \subset K^{-1}(U)$ and so K is sw.gp-c.

Theorem 3.6. *Let $k : P \to Q$ be any function with $P = A_1 \cup B_1$, where $A_1, B_1 \in O(P)$. The restriction functions $k_{|A_1}$ and $k_{|B_1}$ are sw.gp-c, then k is sw.gp-c.*
Proof. Let $U \in O(Q)$ with $k^{-1}(U) \neq \phi$. Then $k^{-1}_{|A_1}(U) \neq \phi$ or $k^{-1}_{|B_1}(U) \neq \phi$ or both $k^{-1}_{|A_1}(U), k^{-1}_{|B_1}(U) \neq \phi$.
Case I. Let $k^{-1}_{|A_1}(U) \neq \phi$. As k is sw.gp-c, there exists $V \in gp$-O(A) with $V \neq \phi$ and $V \subset (k_{|A_1})^{-1}(U) \subset k^{-1}(U)$. As $V \in gp$-O(A) and $V \in O(P)$, then $V \in gp$-O(P). Hence k is sw.gp-c.
Case II. Let $(k_{|B_1})^{-1}(U) \neq \phi$. Rest of the proof is similar to case I.
Case III. Let $(k_{|A_1})^{-1}(U) \neq \phi$ and $(k_{|B_1})^{-1}(U) \neq \phi$. Follows from case I and case II.
Hence we can observe that k is sw.gp-c.

Definition 3.3. *A space P is said to be gp-separable, if there exists a countable subset B_1 of P which is gp-dense in P.*

Theorem 3.7. *Let k be sw.gp-c from P onto Q with P is gp-separable. Then Q is separable.*
Proof. Let $k : P \to Q$ be sw.gp-c with P is gp-separable. Then, there exists a countable subset B_1 of P, which is gp-dense in P. From Theorem 3.4, $k(B_1)$ is dense in Q. As B_1 is countable, $k(B_1)$ is also countable, which is dense in Q. Hence Q is separable.

4. Weakly Equivalent Topologies

Definition 4.1. [5] *If P is a set and τ and σ are topologies on P. Then τ is said to be weakly equivalent to σ provided if $U \in \tau$ and $U \neq \phi$, there exists $V \in O(P)$ such that $V \neq \phi$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \phi$. Then there exists $V \in O(P)$ such that $V \neq \phi$ and $V \subset U$.*

Definition 4.2. *If P is a set and τ and σ are topologies on X. Then τ is said to be gp-weakly equivalent (gp-w.e) to σ provided if $U \in \tau$ and $U \neq \phi$, there exists $V \in gp - O(P)$ such that $V \neq \phi$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \phi$. Then there exists $V \in gp - O(P)$ such that $V \neq \phi$ and $V \subset U$.*

Theorem 4.1. *Let $k : (P, \tau) \to (Q, \sigma)$ be sw-c and τ^* be a topology for P, which is gp-w.e to τ. Then $k : (P, \tau^*) \to (Q, \sigma)$ is sw.gp-c.*
Proof. Let $U \in O(Q)$ with $k^{-1}(U) \neq \phi$. As k is sw.gp-c, there exists $V \in O(P)$ with $V \neq \phi$ and $V \subset k^{-1}(U)$. Since V is open and $V \neq \phi$, then τ is gp-w.e to τ^*. From definition 4.2, there exists $W \in gp\text{-O}(P, \tau^*)$ such that $W \neq \phi$ and $W \subset V \subset k^{-1}(U)$. So $V \subset k^{-1}(U)$. Hence $k : (P, \tau^*) \to (Q, \sigma)$ is sw.gp-c.

Theorem 4.2. *Let $k : (P, \tau) \to (Q, \sigma)$ be sw.gp-c surjective function and σ^* be a topology for Q, which is gp-w.e to σ. Then $k : (P, \tau) \to (Q, \sigma^*)$ is sw.gp-c.*
Proof. Let $U \in O(Q, \sigma^*)$ with $k^{-1}(U) \neq \phi$, then $U \neq \phi$. As σ and σ^* are weakly equivalent, there exists $W \in O(Q)$ such that $W \neq \phi$ and $W \subset U$. Since $W \in O(Q)$ with $W \neq \phi$, implies $k^{-1}(W) \neq \phi$. From hypothesis, k is sw.gp-c, so $V \in gp\text{-O(P)}$ with $V \neq \phi$ and $V \subset k^{-1}(W)$. So $W \subset U$, implies that $k^{-1}(W) \subset k^{-1}(U)$. Thus, $V \subset k^{-1}(U)$ and hence $k : (P, \tau) \to (Q, \sigma^*)$ is sw.gp-c.

5. Somewhat gp-open Function

Definition 5.1. *A function $k : P \to Q$ is said to be somewhat gp-open (sw.gp-O) provided that for $U \in \tau$ and $U \neq \phi$, there exists $V \in$ gp-O(Q) such that $V \neq \phi$ and $V \subset k(U)$.*

Example 5.1. Let $P = Q = \{m_1, m_2\}$. Let $\tau = \{P, \phi, \{m_1\}\}$ and $\sigma = \{Q, \phi\}$. Let the function $k : P \to Q$ be $k(m_1) = m_2$ and $k(m_2) = m_1$. Then k is sw.gp-O.

Theorem 5.1. *Every sw-O is sw.gp-O.*
Proof. Proof follows from the fact that every open set is gp-open in X.

Example 5.2. From the example 5.1, k is sw.gp-O but k is not sw-O. Since, there does not exists any open $V \neq \phi$ in Q with $V \subset k(U)$.

Theorem 5.2. *The following properties holds for a bijective function $k : P \to Q$:*
(i) k is sw.gp-O

(ii) If $A \subset C(P)$ with $k(A) \neq Q$, there exists $D \in$ gp-C(Q) such that $D \neq Q$ and $k(A) \subset D$.

Proof. $(i) \rightarrow (ii)$: Let $A \subset C(P)$ with $k(A) \neq Q$. Then $P \setminus A \in O(Q)$ and $P \setminus A \neq \phi$. As k is sw-gp-O, $V \in$ *gp*-O(Q) with $V \neq \phi$ such that $V \subset k(P \setminus A)$. Put $D = Q \setminus V$, then $D \in$ *gp*-C(Q). Now, we have to prove that $D \neq \phi$.
Assume that $D = \phi$, then $V = \phi$ which is a contradiction. Since $V \subset k(P \setminus A)$, we get $D = Q \setminus V$, that is $D \supset Q \setminus (k(P \setminus A)) = k(A)$.
$(ii) \rightarrow (i)$ Let $U \in O(P)$ with $U \neq \phi$. Let $A = P \setminus U$, then $A \in C(P)$. We have $k(P \setminus U) = k(A) = Q \setminus k(U)$, then $k(A) \neq \phi$. From (ii), $D \in$ *gp*-C(Q) such that $D \neq Q$ and $k(A) \subset D$. Let $V = Q \setminus D$. Then $V \in$ *gp*-O(Q) with $V \neq \phi$. Further, $V = Q \setminus D \subset Q \setminus k(A) = Q \setminus [Q \setminus k(U)] = k(U)$.

Theorem 5.3. *Let $k : P \rightarrow Q$ be sw.gp-O and $M \in O(P)$. Then $(k/M) : (M, \tau/M) \rightarrow (Q, \sigma)$ is also sw.gp-O.*

Proof. Let $U \in \tau/M$ with $U \neq \phi$. Since $U \in O(M)$ and $M \in O(P)$, then $U \in O(P)$. As k is sw.gp-O, we have $V \in$ *gp*-O(Q) with $V \neq \phi$ and $V \subset k(U)$. Thus for any $U \in O((M, \tau/M))$, there exists $V \in$ *gp*-O(Q) such that $V \neq \phi$ and $V \subset (k/M)(U)$. Thus k/M is sw.gp-O.

Theorem 5.4. *Let $k : P \rightarrow Q$ be any function with $P = A_1 \cup B_1$, where $A_1, B_1 \in O(P)$ such that k/A_1 and k/B_1 are sw.gp-O. Then k is also sw-gp-O.*

Proof. Let $U \in O(Q)$ with $U \neq \phi$. Since $P = A_1 \cup B_1$, then either $A_1 \cap U \neq \phi$ or $B_1 \cap U \neq \phi$ or both $A_1 \cap U \neq \phi$ and $B_1 \cap U \neq \phi$. As $U \in O(P)$, then $U \cap A_1 \in O(P, \tau/A_1)$ and $U \cap B_1 \in O(P, \tau/B_1)$.
Case I. Suppose $U \cap A_1 \neq \phi$, then $U \cap A_1 \in O(P, \tau/A_1)$. As k/A_1 is sw.gp-O, there exists $V \in$ *gp*-O(Q) such that $V \neq \phi$ and $V \subset k(U \cap A_1) \subset k(U)$ and so k is sw.gp-O.
Case II. Suppose $U \cap B_1 \neq \phi$ and rest of the proof is similar to case I.
Case III. Suppose $U \cap A_1 \neq \phi$ and $U \cap B_1 \neq \phi$. Then k is obviously sw.gp-O follows from case I and case II.

Theorem 5.5. *Let $k_1 : P \rightarrow Q$ be an open and $k_2 : Q \rightarrow R$ be sw.gp-O. Then $k_2 \circ k_1 : P \rightarrow R$ is sw.gp-O.*

Proof. Let $U \in O(P)$ with $U \neq \phi$. As k_1 is open, $k_1(U)$ is open with $k_1(U) \neq \phi$. Thus $k_1(U) \in O(Q)$ with $k_1(U) \neq \phi$. Since k_2 is sw.gp-O, $k_1(U) \in O(Q)$ with $k_1(U) \neq \phi$. Thus there exists $V \in$ *gp*-O(R) such that $V \subset k_2(k_1(U))$. Thus $k_2 \circ k_1$ is sw.gp-O.

Theorem 5.6. *Let k be sw.gp-O and A is gp-dense set in Q. Then $k^{-1}(A)$ is dense set of P.*

Proof. Suppose A is gp-dense set in Q. If $k^{-1}(A)$ is not dense in P, then $B \in C(P)$ with $k^{-1}(A) \subset B \subset P$. As k is sw.gp-O and $P \setminus B \in O(P)$, there exists $C \in$ *gp*-O(Q) with $C \neq \phi$ such that $C \subset k(P \setminus B)$. Thus $C \subset k(P \setminus B) \subset k(k^{-1}(Q \setminus A)) \subset Q \setminus A$, that is $A \subset Q \setminus C \subset Q$. But $Q \setminus C \in$ *gp*-C(Q) and $A \subset Q \setminus C \subset Q$, thus A is not gp-dense in Q, which is a contradiction to assumption. Hence $k^{-1}(A)$ is dense in P.

6. Somewhat gp-irresolute Functions

Definition 6.1. *A function $k : P \to Q$ is said to be somewhat gp-irresolute(sw.gp-I) if for each $U_1 \in$ gp-O(Q) and $k^{-1}(U_1) \neq \phi$, there exists $V_1 \in$ gp-O(P) with $V_1 \neq \phi$ such that $V_1 \subset k^{-1}(U_1)$.*

Example 6.2. Let $P = Q = \{m_1, m_2, m_3\}$, $\tau = \{P, \phi, \{m_1\}\}$ and $\sigma = \{Q, \phi, \{m_1\}, \{m_2, m_3\}\}$. Define $k : P \to Q$ as $k(m_1) = m_1, k(m_2) = m_3, k(m_3) = m_2$. Let $U_1 = \{m_1, m_2\} \in$ *gp*-O(Q) and $V_1 = \{m_1\} \in$ *gp*-O(P) such that $V_1 = \{m_1\} \subset k^{-1}(\{m_1, m_2\}) = \{m_1, m_3\})$. Then, k is sw.gp-I.

Theorem 6.1. *The following properties holds for a surjective function $k : P \to Q$:*
(i) k is sw.gp-I
(ii) If $A \in$ gp-C(Q) with $k^{-1}(A) \neq P$, then there is a proper gp-closed set D in P such that $k^{-1}(A) \subset D$.
(iii) If M is gp-dense subset in P, then k(M) is gp-dense subset in Q.

Proof. $(i) \to (ii)$: Let $A \in$ *gp*-C(Q) with $k^{-1}(A) \neq P$. Then $Q \setminus A \in$ *gp*-O(Q) with $k^{-1}(Q \setminus A) = P \setminus k^{-1}(A) \neq \phi$. From (i), $V_1 \in$ *gp*-O(P) such that $V_1 \neq \phi$ and $V_1 \subset k^{-1}(Q \setminus A) = P \setminus k^{-1}(A)$, that is $k^{-1}(A) \subset P \setminus V_1$ and $P \setminus V_1 = D$ which is a proper gp-closed in P.

$(ii) \to (i)$: Let $U_1 \in$ *gp*-O(Q) with $k^{-1}(U_1) \neq \phi$. Then $Q \setminus U_1 \in$ *gp*-C(Q) and $k^{-1}(Q \setminus U_1) = P \setminus k^{-1}(U_1) \neq P$. From (ii), there is a proper gp-closed set D such that $k^{-1}(Q \setminus U_1) \subset D$, that is $P \setminus D \subset k^{-1}(U_1)$, where $P \setminus D \in$ *gp*-O(P) with $P \setminus D \neq \phi$.

$(ii) \to (iii)$: Let M be gp-dense set in P. On the contrary assume that k(M) is not gp-dense in Q. Then there exists a proper gp-closed set A in Q with $k(M) \subset A \subset Q$, so $k^{-1}(A) \neq P$. From (ii), $D \in$ *gp*-C(P) such that $M \subset k^{-1}(A) \subset D \subset P$, which is contradiction to the fact that M is gp-dense in P.

$(iii) \to (ii)$: Let the property (ii) is not true. Then $\exists\ A \in$ *gp*-C(Q) such that $k^{-1}(A) \neq P$. But, there exists no proper gp-closed set D such that $k^{-1}(A) \subset D$, that is $k^{-1}(A)$ is gp-dense in P. From (iii), $k(k^{-1}(A)) = A$ must be gp-dense in Q, which is a contradiction.

Theorem 6.2. *Let k be any function with $P = A_1 \cup B_1$, where $A_1, B_1 \in O(P)$. If*

$k/A_1 : (A_1, \tau/A_1) \to (Q, \sigma)$ and $k/B_1 : (B_1, \tau/B_1) \to (Q, \sigma)$ are sw.gp-I then k is sw.gp-I.

Proof. Let $U_1 \in gp$-O(Q) with $k^{-1}(U_1) \neq \phi$. Then $(k/A_1)^{-1}(U_1) \neq \phi$ or $(k/B_1)^{-1}(U_1) \neq \phi$ or both $(k/A_1)^{-1}(U_1) \neq \phi$ and $(k/B_1)^{-1}(U_1) \neq \phi$.

Case I. Suppose $(k/A_1)^{-1}(U_1) \neq \phi$. Since k/A_1 is sw.gp-I, $V_1 \in gp\text{-}O(A_1)$ such that $V_1 \neq \phi$ and $V_1 \subset (k/A_1)^{-1}(U_1) \subset k^{-1}(U_1)$. As $V_1 \in gp\text{-}O(A_1)$ and $A_1 \in O(P)$, then $V_1 \in gp$-O(P). Thus k is sw.gp-I.

Case II. Suppose $(k/B_1)^{-1}(U_1) \neq \phi$. The rest of the proof follows as case I.

Case III. Let $(k/A_1)^{-1}(U_1) \neq \phi$ and $(k/B_1)^{-1}(U_1) \neq \phi$. The rest of the proof follows from case I and case II.

Theorem 6.3. *Let $k : P \to Q$ be sw.gp-I surjective function and $\sigma*$ be the topology on Q which is equivalent to τ. Then $k(P, \tau) \to (Q, \sigma^*)$ is sw.gp-I.*

Proof. Similar to Theorem 4.2.

Theorem 6.4. *Let $k : P \to Q$ be sw.gp-I surjective function and τ^* be the topology on P, which is gp-equivalent to τ. Then $k : (P, \tau^*) \to (Q, \sigma)$ is sw.gp-I.*

Proof. Similar to Theorem 4.1.

7. Conclusion

The concept of sets and continuous functions are broadly developed and applicable in many fields like particle physics, computational topology, quantum physics. By generalizing the properties of generalizations of closed sets, new separation axioms and continuous functions which have been founded and they are very much useful in the study of digital topology. Thus, the study of somewhat gp-continuous functions provides many applications in digital topology and computer graphics.

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