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SOME ENERGIES OF COCKTAIL PARTY GRAPH

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Abstract: In this paper, we compute the distance energy, degree sum energy, degree exponent energy and degree sum exponent distance energy of Cocktail party graph.

Keywords and Phrases: Distance energy, degree sum energy; degree exponent energy, degree sum exponent distance energy, cocktail party graph.

2020 Mathematics Subject Classification: 05C15, 05C50.

1. Introduction and Preliminaries

The concept of energy of a graphs was introduced by I. Gutman in the year 1978. Let G be a Simple graph with p vertices and q edges. Let $D = [d_{ij}]$ be the distance matrix of the graph. The eigen values $\rho_1, \rho_2, \ldots, \rho_p$ of D, assumed in non increasing order, are the eigen values of the graph G. The distance energy $E_D(G)$ [4] of G is defined to be the sum of the absolute values of the eigen values of the distance matrix of G. Let K_{2p} be a complete graph with $2p$ vertices $p = 1, 2, 3, \ldots, n$. We delete the edge joining the vertices i and $p + i$,$1 \leq i \leq p$, i.e., we delete p independent edges, i.e., we delete a perfect matching fromK_{2p}. The resulting graph, denoted by CP_{2p} has order $2p$ and has $2p(p-1)$ edges and is regular of degree $2p - 2$. Such a graph is referred to as a "cocktail party graph". The degree sum energy [10] $E_{DS}(G)$ is the sum of the absolute values of the eigen values of the degree sum matrix, $DS(G)=\begin{cases} d_i + d_j, & if \ \ i \neq j \\ 0, & \ \ \ \ i = j \end{cases}$ $\gamma_1, \gamma_2, \ldots, \gamma_p$ are the eigenvalues of degree sum matrix of G. A degree exponent matrix $DE(G)$ of a graph G, $DE(G) = [de_{ij}] = \begin{cases} d_i^{d_j}, & if\ i \neq j \\ 0, & i = j \end{cases}$. $\beta_1, \beta_2, \ldots, \beta_p$ are called degree exponent eigenvalues of G. The sum of the absolute values of the degree exponent eigenvalues of G is called the degree exponent energy [15] of G. The degree sum exponent distance matrix $M_{Xdist}(G)$ of a graph G is a square matrix whose $(i,j)^{th}$ entry is $(d_i + d_j)^{d_{ij}}$ whenever $i \neq j$ otherwise it is zero where d_i is the degree of i^{th} vertex of G and $d_{ij} = d(v_i, v_j)$ is distance between v_i and v_j . $\alpha_1, \alpha_2, \ldots, \alpha_p$ are the corresponding eigen values of degree sum exponent distance matrix. The degree sum exponent distance energy [12] is the sum of the absolute values of the eigen values of the degree sum exponent distance matrix of G.

2. Main Results

Lemma 2.1. [5] *Let M,N,P and Q be matrices with M invertible then* $\begin{bmatrix} M & N \\ P & Q \end{bmatrix}$ $= \left|M\right| \left|Q - PM^{-1}N\right|$

Lemma 2.2. [5] *Let M,N,P and Q be matrices.Let S=* $\begin{bmatrix} M & N \\ P & Q \end{bmatrix}$ *if M and P commutes then* $\left|S\right| = \left|MQ - PN\right|$.

Lemma 2.3. [19] *If $A(K_p)$ is the adjacency matrix of K_p and the spectrum of $A(K_p)$ are p-1 and $(-1)^{p-1}$ then $A^2(K_p) = (p-2)A(K_p) + (p-1)I_p$.*

Theorem 2.4. *Let CP_{2p} be the Cocktail party graph then*
$Sp(D(CP_{2p})) = \begin{bmatrix} -2 & 2p & 0 \\ p & 1 & p-1 \end{bmatrix}$ *and* $E_D(CP_{2p}) = 4p$.

Proof. Let CP_{2p} be the cocktail party graph then it has $2p$ vertices $2p(p-1)$ edges and regular of degree $2p-2$.
Let $\rho = \{\rho_1, \rho_2, ..., \rho_{2p}\}$ be the eigen values of distance matrix of CP_{2p}.
Then the distance matrix of CP_{2p} is $D(CP_{2p}) = \begin{bmatrix} A(K_p) & A(K_p) + 2I_p \\ A(K_p) + 2I_p & A(K_p) \end{bmatrix}$
and its Characteristic polynomial is $\begin{vmatrix} \rho I_p - A(K_p) & -(A(K_p) + 2I_p) \\ -(A(K_p) + 2I_p) & \rho I_p - A(K_p) \end{vmatrix}$

$$\begin{aligned} \left|\rho I_{2p} - D(CP_{2p})\right| &= (\rho I_p - A(K_p))^2 - (A(K_p) + 2I_p)^2 \\ &= \rho^2 I_p^2 - 2\rho A(K_p) - 4A(K_p) - 4I_p^2 \\ &= (2\rho + 4)\left[\frac{(\rho^2 - 4)I_p^2}{(2\rho + 4)} - A(K_p)\right] \end{aligned}$$

By lemma 2.3 we get,

$$\left|\rho I_{2p} - D(CP_{2p})\right| = (2\rho+4)\left[\frac{(\rho^2-4)}{(2\rho+4)} - (p-1)\right](2\rho+4)\left[\frac{(\rho^2-4)}{(2\rho+4)}+1\right]^{p-1}$$
$$= (\rho^2-4-2\rho p-4p+2\rho+4)(\rho^2-4+2\rho+4)^{p-1}.$$
$$= (\rho+2)(\rho-2p)(\rho)^{p-1}(\rho+2)^{p-1}$$
$$= (\rho+2)^p(\rho)^{p-1}(\rho-2p).$$

Therefore spectrum of $D(CP_{2p}) = \begin{bmatrix} -2 & 2p & 0 \\ p & 1 & p-1 \end{bmatrix}$

and $E_D(CP_{2p}) = 4p$.

Theorem 2.5. *If CP_{2p} is cocktail party graph then degree sum energy of CP_{2p} is $16p^2-24p+8$.*

Proof. Let $\gamma = \{\gamma_1, \gamma_2, ..., \gamma_{2p}\}$ be the eigen values of degree sum matrix of CP_{2p}.
Then $DS(CP_{2p}) = \left[(4p-4)A(K_{2p})\right]$.
By lemma 2.3 we get the Characteristic polynomial is
$(\gamma-(2p-1)(4p-4))(\gamma+(4p-4))^{2p-1}$.
Hence $Sp(DS(CP_{2p})) = \begin{bmatrix} 8p^2-12p+4 & -(4p-4) \\ 1 & 2p-1 \end{bmatrix}$
and $E_{DS}(CP_{2p}) = 16p^2-24p+8$.

Theorem 2.6. *If CP_{2p} is cocktail party graph then*
$Sp(DE(CP_{2p})) = \begin{bmatrix} (2p-1)r & r \\ 1 & 2p-1 \end{bmatrix}$,
and $DEE(CP_{2p}) = 2r(2p-1)$ where $r = (2p-2)^{2p-2}$

Proof. Consider the cocktail party graph and the eigen value of degree exponent matrix are CP_{2p} and β respectively.
Then the degree exponent matrix of CP_{2p} is $DE(CP_{2p}) = \left[(2p-2)^{2p-2}A(K_{2p})\right]$.
$DE(CP_{2p}) = \left[rA(K_{2p})\right]$,where $r = (2p-2)^{2p-2}$.
By lemma 2.3 we get $\left|\beta I_{2p} - DE(CP_{2p})\right| = (\beta - r(2p-1))(\beta+r)^{2p-1}$
and $DEE(CP_{2p}) = 2r(2p-1)$ where $r = (2p-2)^{2p-2}$.

Theorem 2.7. *Let CP_{2p} be cocktail party graph then*
$Sp(M_{Xdist}(CP_{2p})) = \begin{bmatrix} -(4p-4)^2 & (4p-4)(6p-6) & (4p-4)(4p-6) \\ p & 1 & p-1 \end{bmatrix}$
and $E_{M_{Xdist}}(CP_{2p}) = 32p(p-1)^2$.

Proof. Let CP_{2p} be cocktail party graph then the degree sum exponent distance matrix
$M_{Xdist}(CP_{2p}) = \begin{bmatrix} (4p-4)A(K_p) & (4p-4)A(K_p)+(4p-4)^2I_p \\ (4p-4)A(K_p)+(4p-4)^2I_p & (4p-4)A(K_p) \end{bmatrix}$

$$\left|\alpha I_{2p} - M_{Xdist}(CP_{2p})\right| = (4p-4)\begin{vmatrix} \alpha I_p - A(K_p) & -(A(K_p)+(4p-4)I_p) \\ -(A(K_p)+(4p-4)I_p) & \alpha I_p - A(K_p) \end{vmatrix}$$
$= (4p-4)\left[(\alpha I_p - A(K_p))^2 - ((4p-4)I_p + A(K_p))^2\right]$
$= (4p-4)\left[(\alpha I_p)^2 - 2\alpha A(K_p) + A^2(K_p)) - ((4p-4)I_p)^2 + 2(4p-4)A(K_p) + A^2(K_p))\right]$
$= (4p-4)\left[(\alpha I_p)^2 - 2\alpha A(K_p) - ((4p-4)I_p)^2 - 2(4p-4)A(K_p))\right]$
$= (4p-4)\left[(\alpha-(4p-4))I_p)^2 - (2\alpha+8p-8)A(K_p)\right]$
$= (4p-4)(2\alpha+8p-8)\left[\frac{(\alpha-(4p-4)I_p)^2}{(2\alpha+8p-8)} - A(K_p)\right]$
$= (4p-4)[(\alpha^2 - 16p^2 + 32p - 16 - 2\alpha p + 2\alpha - 8p^2 + 8p + 8p - 8)$
$(\alpha^2 - 16p^2 + 32p - 16 + 2\alpha + 8p - 8)^{p-1}]$
$= (4p-4)\left[(\alpha^2 - 24p^2 + 48p - 2\alpha p + 2\alpha - 24)(\alpha^2 - 16p^2 + 40p + 2\alpha - 24)^{p-1}\right]$
$= (4p-4)\left[(\alpha-(4-4p))(\alpha-(6p-6))(\alpha-(4-4p))^{p-1}(\alpha-(4p-6))^{p-1}\right]$
Therefore characteristic polynomial is
$(\alpha+(4p-4)^2)^p(\alpha-(4p-4)(4p-6))^{p-1}(\alpha-(4p-4)(6p-6))$,
and $Sp(M_{Xdist}(CP_{2p})) = \begin{bmatrix} -(4p-4)^2 & (4p-4)(6p-6) & (4p-4)(4p-6) \\ p & 1 & p-1 \end{bmatrix}$
Hence $E_{M_{Xdist}}(CP_{2p}) = 32p(p-1)^2$.

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