South East Asian J. of Mathematics and Mathematical Sciences
Vol. 19, No. 1 (2023), pp. 211-224

DOI: 10.56827/SEAJMMS.2023.1901.17 **ISSN (Online): 2582-0850**

ISSN (Print): 0972-7752

DIVISOR CORDIAL LABELING FOR SOME SNAKES AND DEGREE SPLITTING RELATED GRAPHS

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(**Received:** *Dec. 29, 2021* **Accepted:** *Apr. 14, 2023* **Published:** *Apr. 30, 2023*)

Abstract: For a graph $G = (V(G), E(G))$, the vertex labeling function is defined as a bijection $f : V(G) \rightarrow \{1, 2, \ldots, |V(G)|\}$ such that an edge uv is assigned the label 1 if one $f(u)$ or $f(v)$ divides the other and 0 otherwise. f is called divisor cordial labeling of graph G if the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. In 2011, Varatharajan *et al.* [24] have introduced divisor cordial labeling as a variant of cordial labeling. In this paper, we study divisor cordial labeling for triangular snake and quadrilateral snake. Moreover, we investigate divisor cordial labeling for the degree splitting graph of path, shell, cycle with one chord, crown and comb graph.

Keywords and Phrases: Graph Labeling, Cordial Labeling, Divisor Cordial Labeling, Snake Graph, Degree Splitting graph.

2020 Mathematics Subject Classification: 05C78, 05C76, 05C38.

1. Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$. For all standard terminologies and notations we follow Harary [12]. We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1. *A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex labeling (an edge labeling).*

For an extensive survey on graph labeling and bibliographic references we refer to Gallian [9].
In 1967, Rosa [19] introduced $\beta - valuation$ labeling of a graph. Golamb [10] subsequently called such labeling as a graceful labeling. In 1980, Graham and Sloane [11] introduced harmonious labeling. In 1987, Cahit [7] introduced cordial labeling as a weaker version of graceful labeling and harmonious labeling, which is defined as follows.

Definition 1.2. *For a graph $G = (V(G), E(G))$, the vertex labeling function is defined as $f : V(G) \to \{0, 1\}$ and induced edge labeling function $f^* : E(G) \to \{0, 1\}$ such that for each edge uv, $f^*(uv) = |f(u) - f(v)|$. f is called cordial labeling of graph G if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a cordial labeling is called a cordial graph.*
Many researchers have explored variants of cordial labeling like product cordial labeling [20], k-product cordial labeling [15], k-difference cordial labeling [16], k-prime cordial labeling [17] etc. In 2011, Varatharajan *et al.* [24] have introduced divisor cordial labeling as a variant of cordial labeling, which is defined as follows.

Definition 1.3. *For a graph $G = (V(G), E(G))$, the vertex labeling function is defined as a bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ such that induced edge labeling function $f^* : E(G) \to \{0, 1\}$ is given by*

$$f^*(e = uv) = \begin{cases} 1, & \text{if } f(u)/f(v) \text{ or } f(v)/f(u); \\ 0, & \text{otherwise.} \end{cases}$$

Denote the number of edges labeled with 0 and 1 by $e_f(0)$ and $e_f(1)$ respectively. f is called divisor cordial labeling of graph G if $|e_f(0) - e_f(1)| \leq 1$. The graph that admits a divisor cordial labeling is called a divisor cordial graph.
Varatharajan *et al.* [24, 25] have investigated divisor cordial labeling for many standard graph families. Vaidya and Shah [22, 23] have discussed divisor cordial labeling for some star related graphs. Raj and Valli [18] have obtained divisor cordial labeling for some new graphs. Bosamia and Kanani [5, 6] have investigated divisor cordial labeling in the context of some graph operations.
Barasara and Thakkar [1, 2, 3] have obtained results related to divisor cordial labeling for cycle, wheel and ladder related graphs. Also we have discussed divisor cordial labeling for larger graphs obtained using graph operations. In [4] we have

studied divisor cordial labeling for degree splitting graph of some graphs and graph obtained by duplication of some graph elements. Also, Murugan and Devakiruba [13], Thirusangu and Madhu [21], Devaraj *et al.* [8] and Muthaiyan and Pugalenthi [14] have proved results related to divisor cordial labeling.

Definition 1.4. *The triangular snake T_n is obtained from the path P_n by replacing every edge of a path by a cycle C_3.*

Definition 1.5. *The quadrilateral snake Q_n is obtained from the path P_n by replacing every edge of a path by a cycle C_4.*

Definition 1.6. *Let $G = (V(G), E(G))$ be a graph with $V = S_1 \cup S_2 \cup ...S_t \cup T$ where each S_i is a set of all vertices having same degree (at least two vertices) and $T = V \setminus \cup S_i$. The degree splitting graph of G denoted by $DS(G)$ is obtained from the graph G by adding vertices $w_1, w_2, ..., w_t$ and joining to each vertex of S_i for $1 \leq i \leq t$.*

Definition 1.7. *A chord of a cycle C_n is an edge joining two non-adjacent vertices of C_n.*

Definition 1.8. *The shell graph S_n is a graph obtained by taking $n-3$ concurrent chords in cycle C_n. The vertex at which all the chords are concurrent is called an apex vertex.*

Definition 1.9. *Let G and H be two graphs. The corona product of G and H, denoted by $G \odot H$, is obtained by taking one copy of G and $|V(G)|$ copies of H, and by joining each vertex of the i^{th} copies of H to the i^{th} vertex of G, where $1 \leqslant i \leqslant |V(G)|$.*

Definition 1.10. *The crown graph is defined as $C_n \odot K_1$.*

Definition 1.11. *The comb graph is defined as $P_n \odot K_1$.*

In this paper, we study divisor cordial labeling for triangular snake and quadrilateral snake. Moreover, we investigate divisor cordial labeling for the degree splitting graph of path, shell, cycle with one chord, crown and comb graph.

2. Main Results

Theorem 2.1. *The triangular snake T_n is a divisor cordial graph.*
Proof. Let P_n be the path with vertices $v_1, v_3, v_5, ..., v_{2n-1}$. To construct T_n from P_n, join vertices v_{2i-1} and v_{2i+1} to a new vertex v_{2i} for $i = 1, 2, ..., n-1$. Then $|V(T_n)| = 2n-1$ and $|E(T_n)| = 3n-3$.

We define the divisor cordial labeling $f : V(T_n) \to \{1, 2, \ldots, 2n-1\}$ by following three cases:

Case 1: For $n = 2$.
The graph T_2 is isomorphic to cycle C_3 and Varatharajan *et al.* [24] proved that cycles are divisor cordial graph. Hence, T_2 is divisor cordial graph.

Case 2: For $n = 3$.
$f(v_i) = i + 1$; for $1 \leq i \leq 4$,
$f(v_5) = 1$.
In view of above defined labeling pattern, we have $e_f(0) = 3$ and $e_f(1) = 3$. Thus, $|e_f(0) - e_f(1)| = 0$.

Case 3: For $n \geq 4$.
$f(v_{2n-3}) = 1$,
$f(v_i) = 1 \times 1 \times 2^{p_1+1-i}$; for $1 \leq i \leq a_1$,
such that $1 \times 1 \times 2^{p_1+1-i} \geq 1 \times 1 \times 2$;
where p_1 is largest number such that $2^{p_1} \leq n$,
$f(v_{i+a_1}) = 3 \times 1 \times 2^{p_2+1-i}$; for $1 \leq i \leq a_2$,
such that $3 \times 1 \times 2^{p_2+1-i} \geq 3 \times 1 \times 1$;
where p_2 is largest number such that $3 \times 2^{p_2} \leq n$,
$f(v_{i+a_1+a_2}) = 3 \times 3 \times 2^{p_3+1-i}$; for $1 \leq i \leq a_3$,
such that $3 \times 3 \times 2^{p_3+1-i} \geq 3 \times 3 \times 1$;
where p_3 is largest number such that $3 \times 3 \times 2^{p_3} \leq n$,
$f(v_{i+a_1+a_2+a_3}) = 3 \times 3^2 \times 2^{p_4+1-i}$; for $1 \leq i \leq a_4$,
such that $3 \times 3^2 \times 2^{p_4+1-i} \geq 3 \times 3^2 \times 1$;
where p_4 is largest number such that $3 \times 3^2 \times 2^{p_4} \leq n$,
$f(v_{i+a_1+a_2+a_3+a_4}) = 3 \times 3^3 \times 2^{p_5+1-i}$; for $1 \leq i \leq a_5$,
such that $3 \times 3^3 \times 2^{p_5+1-i} \geq 3 \times 3^3 \times 1$;
where p_5 is largest number such that $3 \times 3^3 \times 2^{p_5} \leq n$,
Continuing in this way upto $3^{m_1} \leq n$,
$f(v_{i+a_1+a_2+...+a_{m_1}}) = 5 \times 1 \times 2^{q_1+1-i}$; for $1 \leq i \leq b_1$,
such that $5 \times 1 \times 2^{q_1+1-i} \geq 5 \times 1 \times 1$;
where q_1 is largest number such that $5 \times 1 \times 2^{q_1} \leq n$,
$f(v_{i+a_1+a_2+...+a_{m_1}+b_1}) = 5 \times 3 \times 2^{q_2+1-i}$; for $1 \leq i \leq b_2$,
such that $5 \times 3 \times 2^{q_2+1-i} \geq 5 \times 3 \times 1$;
where q_2 is largest number such that $5 \times 3 \times 2^{q_2} \leq n$,
$f(v_{i+a_1+a_2+...+a_{m_1}+b_1+b_2}) = 5 \times 3^2 \times 2^{q_3+1-i}$; for $1 \leq i \leq b_3$,
such that $5 \times 3^2 \times 2^{q_3+1-i} \geq 5 \times 3^2 \times 1$;
where q_3 is largest number such that $5 \times 3^2 \times 2^{q_3} \leq n$,
$f(v_{i+a_1+a_2+...+a_{m_1}+b_1+b_2+b_3}) = 5 \times 3^3 \times 2^{q_4+1-i}$; for $1 \leq i \leq b_4$,
such that $5 \times 3^3 \times 2^{q_4+1-i} \geq 5 \times 3^3 \times 1$;
where q_4 is largest number such that $5 \times 3^3 \times 2^{q_4} \leq n$.

Continuing in this way till we get at least $\left\lfloor \dfrac{3n-3}{2} \right\rfloor - 1$ edges with label 1.

If we get $\left\lfloor \dfrac{3n-3}{2} \right\rfloor - 1$ edges with label 1, take $f(v_{2n-1}) = 2p'$ and $f(v_{2n-2}) = p'$, where p' is largest prime number such that $2p' < 2n-1$.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices.

In view of above defined labeling pattern, we have $e_f(0) = \left\lceil \dfrac{3n-3}{2} \right\rceil$ and $e_f(1) = \left\lfloor \dfrac{3n-3}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the triangular snake T_n is a divisor cordial graph.

Example 2.2. The triangular snake T_6 and its divisor cordial labeling is shown in Fig 1.

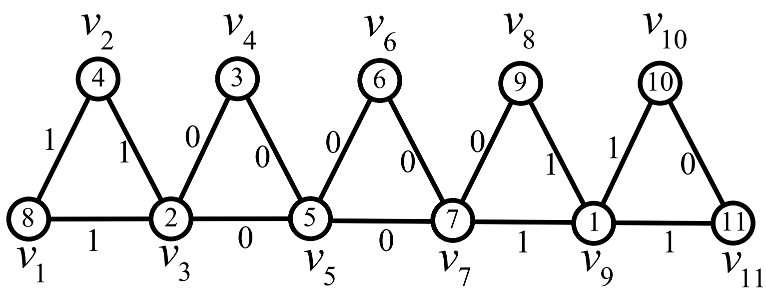

Fig 1: The triangular snake T_6 and its divisor cordial labeling.

Theorem 2.3. *The quadrilateral snake Q_n is a divisor cordial graph.*
Proof. Let P_n be the path with vertices $v_1, v_4, v_7, ..., v_{3n-2}$. To construct Q_n from P_n, join v_{3i-2} to a new vertex v_{3i-1}, v_{3i+1} to a new vertex v_{3i} and v_{3i-1} to v_{3i} for $i = 1, 2, ..., n-1$. Then $|V(Q_n)| = 3n-2$ and $|E(T_n)| = 4n-4$.

We define the divisor cordial labeling $f : V(Q_n) \to \{1, 2, \ldots, 3n-2\}$ by following two cases:

Case 1: For $n = 2$.
The graph Q_2 is isomorphic to cycle C_4 and Varatharajan *et al.* [24] proved that cycles are divisor cordial graph. Hence, Q_2 is divisor cordial graph.

Case 2: For $n \geq 3$.

$f(v_{3n-5}) = 1,$
$f(v_i) = 1 \times 1 \times 2^{p_1+1-i};$ for $1 \leq i \leq a_1,$
such that $1 \times 1 \times 2^{p_1+1-i} \geq 1 \times 1 \times 2;$
where p_1 is largest number such that $2^{p_1} \leq n,$
$f(v_{i+a_1}) = 3 \times 1 \times 2^{p_2+1-i};$ for $1 \leq i \leq a_2,$
such that $3 \times 1 \times 2^{p_2+1-i} \geq 3 \times 1 \times 1;$
where p_2 is largest number such that $3 \times 2^{p_2} \leq n,$
$f(v_{i+a_1+a_2}) = 3 \times 3 \times 2^{p_3+1-i};$ for $1 \leq i \leq a_3,$
such that $3 \times 3 \times 2^{p_3+1-i} \geq 3 \times 3 \times 1;$
where p_3 is largest number such that $3 \times 3 \times 2^{p_3} \leq n,$
$f(v_{i+a_1+a_2+a_3}) = 3 \times 3^2 \times 2^{p_4+1-i};$ for $1 \leq i \leq a_4,$
such that $3 \times 3^2 \times 2^{p_4+1-i} \geq 3 \times 3^2 \times 1;$
where p_4 is largest number such that $3 \times 3^2 \times 2^{p_4} \leq n,$
$f(v_{i+a_1+a_2+a_3+a_4}) = 3 \times 3^3 \times 2^{p_5+1-i};$ for $1 \leq i \leq a_5,$
such that $3 \times 3^3 \times 2^{p_5+1-i} \geq 3 \times 3^3 \times 1;$
where p_5 is largest number such that $3 \times 3^3 \times 2^{p_5} \leq n,$
Continuing in this way upto $3^{m_1} \leq n,$
$f(v_{i+a_1+a_2+...+a_{m_1}}) = 5 \times 1 \times 2^{q_1+1-i};$ for $1 \leq i \leq b_1,$
such that $5 \times 1 \times 2^{q_1+1-i} \geq 5 \times 1 \times 1;$
where q_1 is largest number such that $5 \times 1 \times 2^{q_1} \leq n,$
$f(v_{i+a_1+a_2+...+a_{m_1}+b_1}) = 5 \times 3 \times 2^{q_2+1-i};$ for $1 \leq i \leq b_2,$
such that $5 \times 3 \times 2^{q_2+1-i} \geq 5 \times 3 \times 1;$
where q_2 is largest number such that $5 \times 3 \times 2^{q_2} \leq n,$
$f(v_{i+a_1+a_2+...+a_{m_1}+b_1+b_2}) = 5 \times 3^2 \times 2^{q_3+1-i};$ for $1 \leq i \leq b_3,$
such that $5 \times 3^2 \times 2^{q_3+1-i} \geq 5 \times 3^2 \times 1;$
where q_3 is largest number such that $5 \times 3^2 \times 2^{q_3} \leq n,$
$f(v_{i+a_1+a_2+...+a_{m_1}+b_1+b_2+b_3}) = 5 \times 3^3 \times 2^{q_4+1-i};$ for $1 \leq i \leq b_4,$
such that $5 \times 3^3 \times 2^{q_4+1-i} \geq 5 \times 3^3 \times 1;$
where q_4 is largest number such that $5 \times 3^3 \times 2^{q_4} \leq n.$

Continuing in this way till we get at least $2n - 3$ edges with label 1.
If we get $2n - 3$ edges with label 1, take $f(v_{3n-2}) = 2p'$ and $f(v_{3n-3}) = p'$, where p' is largest prime number such that $2p' \leq 3n - 2$.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices.

In view of above defined labeling pattern, we have $e_f(0) = e_f(1) = 2n - 2$. Thus, $|e_f(0) - e_f(1)| = 0$.

Hence, the quadrilateral snake Q_n is a divisor cordial graph.

Example 2.4. The quadrilateral snake Q_5 and its divisor cordial labeling is shown in Fig 2.

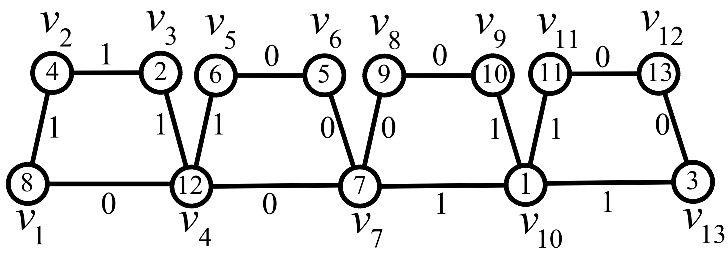

Fig 2: The quadrilateral snake Q_5 and its divisor cordial labeling.

Theorem 2.5. *The graph $DS(P_n)$ is a divisor cordial graph.*
Proof. Let P_n be the path with vertices $v_1, v_2, v_3, ..., v_n$. To obtain $DS(P_n)$, join vertices v_1 and v_n to a new vertex v' and join vertices $v_2, v_3, ..., v_{n-1}$ to a new vertex v''. Then $|V(DS(P_n))| = n + 2$ and $|E(DS(P_n))| = 2n - 1$.
We define the divisor cordial labeling $f : V(DS(P_n)) \to \{1, 2, \ldots, n+2\}$ by following three cases:

Case 1: For $n = 2$.
The graph $DS(P_2)$ is isomorphic to cycle C_3 and Varatharajan *et al.* [24] proved that cycles are divisor cordial graph. Hence, $DS(P_2)$ is divisor cordial graph.

Case 2: For $n = 3$.
The graph $DS(P_3)$ is isomorphic to cycle C_4 and Varatharajan *et al.* [24] proved that cycles are divisor cordial graph. Hence, $DS(P_3)$ is divisor cordial graph.

Case 3: For $n \geq 4$.
$f(v'') = 1,$
$f(v_1) = 2,$
$f(v_2) = 4.$
Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices.

In view of above defined labeling pattern, we have $e_f(0) = \left\lceil \frac{2n-1}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{2n-1}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, the graph $DS(P_n)$ is a divisor cordial graph.

Example 2.6. The graph $DS(P_8)$ and its divisor cordial labeling is shown in Fig 3.

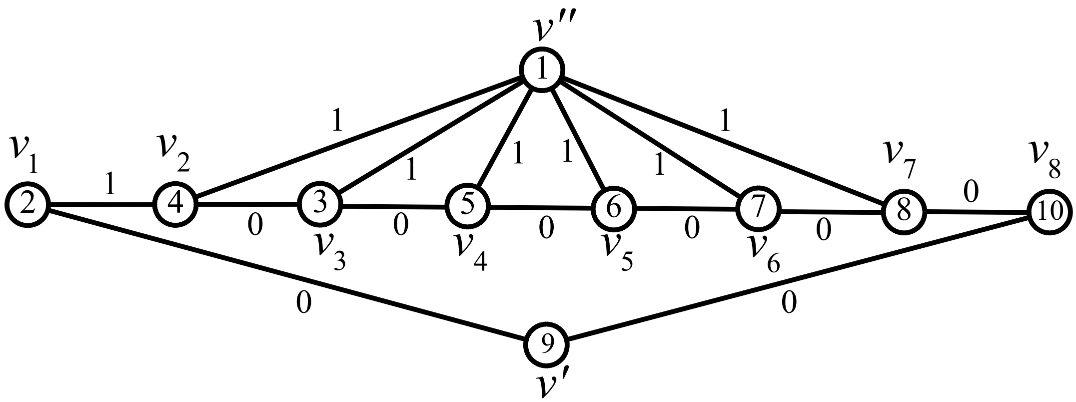

Fig 3: The graph $DS(P_8)$ and its divisor cordial labeling.

Theorem 2.7. *The graph $DS(S_n)$ is a divisor cordial graph for $n \geq 5$.*

Proof. Let S_n be the shell with vertices $v_1, v_2, v_3, ..., v_n$. To obtain $DS(S_n)$, let the added vertices be v' and v''. Here $E(DS(S_n)) = \{v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{v_1v_i/3 \leq i \leq n-1\} \cup \{v_iv''/3 \leq i \leq n-1\} \cup \{v_nv_1, v_2v', v_nv'\}$. Then $|V(DS(S_n))| = n+2$ and $|E(DS(S_n))| = 3n-4$.

We define the divisor cordial labeling $f : V(DS(S_n)) \to \{1, 2, \ldots, n+2\}$ as follows.

$f(v_1) = 1,$
$f(v'') = 2,$
$f(v_{1+i}) = 2i;$ for $i \geq 2,$ such that $2i \leq n+2$.

Continuing in this way till we get $\left\lfloor \frac{3n-4}{2} \right\rfloor$ edges with label 1.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices.

In view of above defined labeling pattern, we have $e_f(0) = \left\lceil \frac{3n-4}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{3n-4}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the graph $DS(S_n)$ is a divisor cordial graph for $n \geq 5$.

Example 2.8. The graph $DS(S_8)$ and its divisor cordial labeling is shown in Fig 4.

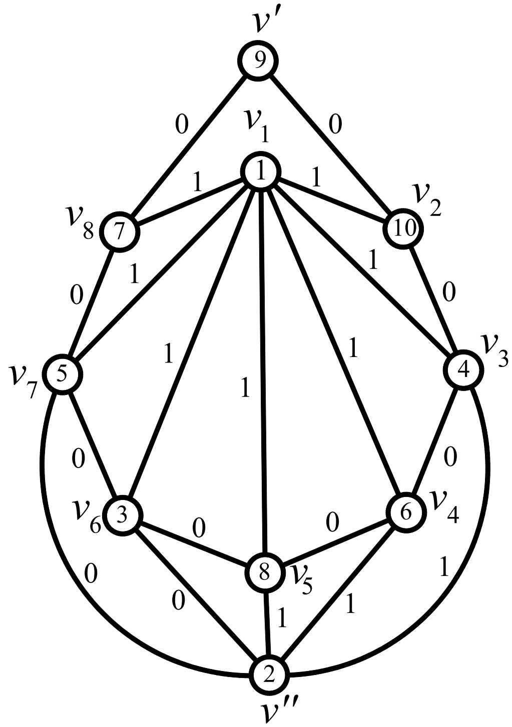

Fig 4: The graph $DS(S_8)$ and its divisor cordial labeling.

Theorem 2.9. *The degree splitting graph of cycle C_n with one chord is a divisor cordial graph.*

Proof. Let C_n be the cycle with vertices $v_1, v_2, v_3, ..., v_n$ and graph G be cycle with one chord. Let v_a and v_b be the end vertices of chord in cycle. To obtain $DS(G)$, join the vertices v_a and v_b to new vertex w_1 and join all other vertices to w_2. Then $|V(DS(G))| = n+2$ and $|E(DS(G))| = 2n+1$.

We define the divisor cordial labeling $f : V(G) \to \{1, 2, \ldots, n+2\}$ as follows.

$f(w_2) = 1$,

$f(w_1) = 2$,

$f(v_a) = 4$,

$f(v_b) = 6$.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices.

In view of above defined labeling pattern, we have $e_f(0) = \left\lceil \frac{2n+1}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{2n+1}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the degree splitting graph of cycle C_n with one chord is a divisor cordial graph.

Example 2.10. The degree splitting graph of cycle C_6 with one chord and its divisor cordial labeling is shown in Fig 5.

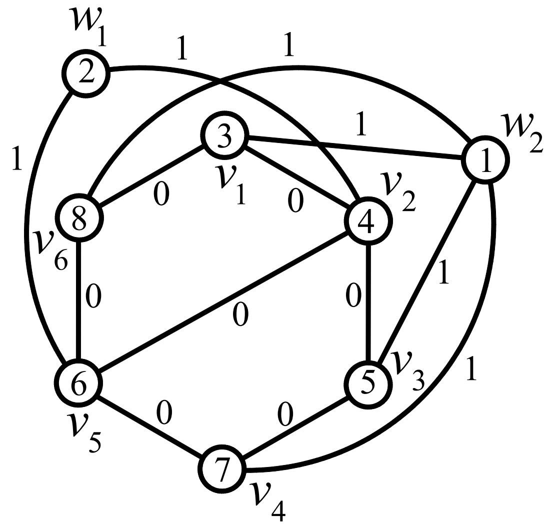

Fig 5: The degree splitting graph of cycle C_6 with one chord and its divisor cordial labeling.

Theorem 2.11. *The graph $DS(C_n \odot K_1)$ is a divisor cordial graph.*
Proof. For crown graph $C_n \odot K_1$, let $v_1, v_2, v_3, ..., v_n$ be the vertices corresponding to cycle C_n and v_i' be the pendent vertices attached to v_i for $i = 1.2, ..., n$. To obtain $DS(C_n \odot K_1)$, join vertices v_i to a new vertex w_1 and vertices v_i' to a new vertex w_2 for $i = 1, 2, ..., n$. Then $|V(DS(C_n \odot K_1))| = 2n + 2$ and $|E(DS(C_n \odot K_1))| = 4n$. We define the divisor cordial labeling $f : V(C_n \odot K_1) \to \{1, 2, \ldots, 2n + 2\}$ as follows.
$f(w_1) = 1$,
$f(v_i) = 2i + 1$; for $1 \le i \le n$,
$f(w_2) = 2$,
$f(v_i') = 2i + 2$; for $1 \le i \le n$.
In view of above defined labeling pattern, we have $e_f(0) = e_f(1) = 2n$. Thus, $|e_f(0) - e_f(1)| = 0$.
Hence, the graph $DS(C_n \odot K_1)$ is a divisor cordial graph.

Example 2.12. The graph $DS(C_7 \odot K_1)$ and its divisor cordial labeling is shown in Fig 6.

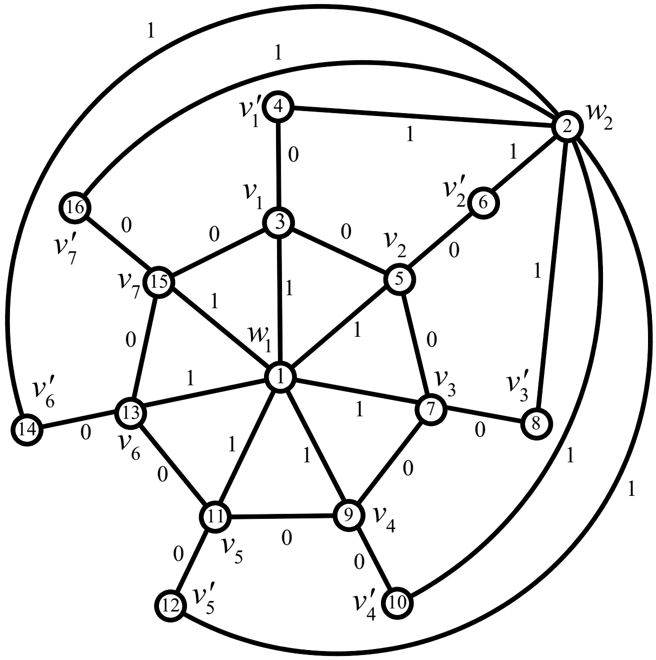

Fig 6: The graph $DS(C_7 \odot K_1)$ and its divisor cordial labeling.

Theorem 2.13. *The graph $DS(P_n \odot K_1)$ is a divisor cordial graph for $n \geq 4$.*
Proof. For comb graph $P_n \odot K_1$, let $v_1, v_2, v_3, ..., v_n$ be the vertices corresponding to path P_n and v_i' be the pendent vertices attached to v_i for $i = 1, 2, ..., n$. To obtain $DS(P_n \odot K_1)$, join vertices v_i to a new vertex w_1 for $i = 2, 3, ..., n-1$, v_i' to a new vertex w_2 for $i = 1, 2, ..., n$ and vertices v_1 and v_n to a new vertex w_3. Then $|V(DS(P_n \odot K_1))| = 2n + 3$ and $|E(DS(P_n \odot K_1))| = 4n - 1$.
We define the divisor cordial labeling $f : V(P_n \odot K_1) \rightarrow \{1, 2, \ldots, 2n + 2\}$ by follows.

$f(v_1) = 9$,
$f(v_i) = 2i$; for $2 \leq i \leq n - 1$,
$f(w_1) = 2$,
$f(w_2) = 1$,
$f(w_3) = 3$.

Now label the remaining vertices in such a way that they neither divide nor divided by the label of adjacent vertices.
In view of above defined labeling pattern, we have $e_f(0) = \left\lceil \frac{4n-1}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{4n-1}{2} \right\rfloor$. Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the graph $DS(P_n \odot K_1)$ is a divisor cordial graph for $n \geq 4$.

Example 2.14. The graph $DS(P_6 \odot K_1)$ and its divisor cordial labeling is shown in Fig 7.

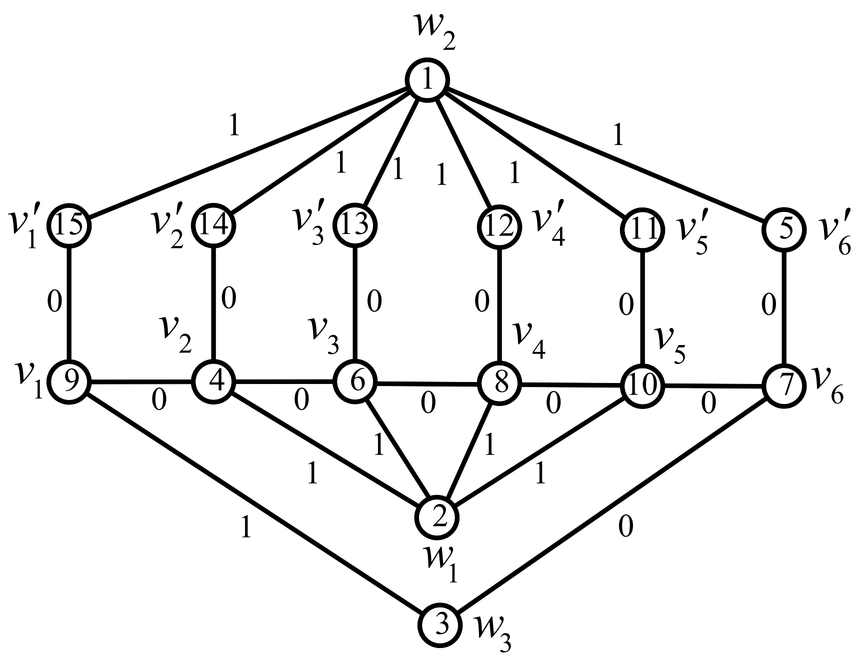

Fig 7: The graph $DS(P_6 \odot K_1)$ and its divisor cordial labeling.

3. Concluding Remarks

In this paper we have investigated divisor cordial labeling for triangular snake and quadrilateral snake. Moreover, we have discussed divisor cordial labeling for the degree splitting graph of path, shell, cycle with one chord, crown and comb graph. To obtain similar results for different graph families or for other labeling schemes are open areas of research.

Acknowledgement

The authors are highly thankful to the anonymous referees for the kind comments and fruitful suggestions on the first draft of this paper. The present work is a part of the research work done under the Minor Research Project No. HNGU/UGC/5658/2023, Dated: 4th January, 2023 of Hemchandracharya North Gujarat University, Patan(Gujarat), INDIA. The second author is supported by Knowledge Consortium of Gujarat, Government of Gujarat, Ahmedabad through SHODH Scholarship-2021-23 with Ref. No.: 202001400029.

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