

**FIXED POINT RESULT FOR A CLASS OF EXTENDED
INTERPOLATIVE CIRIC-REICH-RUS $(\alpha, \beta, \gamma F)$ -TYPE
CONTRACTIONS ON UNIFORM SPACES**

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Abstract: In this paper we have generalized the extended interpolative Ciric-Reich-Rus type ψF -contraction given in [22] by introducing extended interpolative Ciric-Reich-Rus type $(\alpha, \beta, \gamma F)$ of type-I and type-II contractions. Using these type contractions we have established some unique fixed point results in S -complete Hausdorff uniform spaces. We have discussed about some basic definitions, properties, lemmas and theorems on uniform spaces in the introduction and preliminary sections. Some corollaries and examples are also given on the basis of the results.

Keywords and Phrases: (α, β) -admissible function, S -complete Hausdorff uniform space, E -distance, p -Cauchy sequence, extended interpolative Ciric-Reich-Rus type $(\alpha, \beta, \gamma F)$ -contractions of type-I and type-II.

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1. Introduction

Fixed point theorem is one of the most important and powerful tool in modern Mathematics specially in Functional Analysis. In 1922, Stefan Banach established a contraction, which is well-known as Banach Contraction Principle (BCP) after

the name of S. Banach. Since then many authors have been generalizing this contraction in many direction on different type spaces and establishing fixed point results.

Later in 1968, R. Kannan [12] gave a generalization of Banach contraction principle and made a contraction, which is popularly known as Kannan fixed point theorem. Later in 2012, Wardowski [30] introduced F -contraction on a special type of functions $F : (0, \infty) \rightarrow \mathbb{R}$, and then established and proved a fixed point theorems on a complete metric space. One of the most powerful and interesting generalization of Banach's contraction is F -contraction. Since then F -contraction has been generalizing and extending in various metric spaces. In 2014, Consentino and Vetro [8] introduced F -contraction of Hardy-Roger's type and proved a fixed point theorem in complete metric space.

In 2016, Nicolae-Adrian Secelean et al. [27] introduced a larger type of F -contraction called ψF -contraction.

After then in 2018, Karapinar et al. [16] introduced a new type of contractive mapping via interpolation, called Interpolative Kannan Contraction on a metric space and then established a fixed point theorem, which is given below:

A self-mapping J on a metric space (Y, d) is called Interpolative Kannan contraction if the following hold:

$$d(J\xi, J\eta) \leq L[d(\xi, J\xi)]^a [d(\eta, J\eta)]^{1-a}, \quad \forall \xi, \eta \in Y;$$

where, $L \in [0, 1)$ and $a \in (0, 1)$, with $\xi \neq \eta$.

If J is an Interpolative Kannan contraction on a complete metric space (Y, d) , then it has a fixed point in Y .

Later Karapinar et al. [14] initiated the Ciric-Reich-Rus type contraction, which is stated below:

A self-mapping J on a metric space (Y, d) is called Interpolative Ciric-Reich-Rus type contraction if the following hold:

$$d(J\xi, J\eta) \leq L[d(\xi, \eta)]^a \cdot d(\xi, J\xi)^b \cdot [d(\eta, J\eta)]^{1-a-b}, \quad \forall \xi, \eta \in Y;$$

where, $L \in [0, 1)$ and $a, b \in (0, 1)$, with $\xi \neq \eta$.

In 2019, Mohammadi et al. [20] have extended the result of Karapinar [16] by using F -contraction, namely extended interpolative Ciric-Reich-Rus type F -contraction and established a fixed point result, which is stated in the preliminary section.

In 2012, Samet et al. [24] introduced the notions of α -admissible and α - ψ contractive mappings and established fixed point result on complete metric space.

Also in 2018, Ali et al. [3] established fixed point results on S -complete Hausdorff uniform space in (α, F) -contractive mappings. In 2021, Karapinar et al. [15] established $(\alpha, \beta, \psi, \phi)$ contractions and established fixed point result on this contractions in complete metric space. Also in 2021, Panja et al. [22] have introduced extended interpolative Ciric-Reich-Rus type ψF -contraction and established fixed point result on b -metric space.

Our result is the extension of the results of Panja et al. [22] on S -complete Hausdorff uniform space.

2. Preliminaries

From the references {[3], [21], [2]}, we define the following space as:

Let Y be a non-empty set. A non-empty family ζ of subsets of $Y \times Y$ is called the uniform structure on Y if it satisfies the following properties:

- (y_1) if $U \in \zeta$, then U contains the diagonal $\{(\xi, \xi) | \xi \in Y\}$;
- (y_2) if $U \in \zeta$ and V is a subset of $Y \times Y$ that contains U , then $V \in \zeta$;
- (y_3) if $U, V \in \zeta$, then $U \cap V \in \zeta$;
- (y_4) if $U \in \zeta$, then there exists $V \in \zeta$ such that whenever (ξ, η) and (η, σ) are in V , then (ξ, σ) is in U ;
- (y_5) if $U \in \zeta$, then $\{(\eta, \xi) | (\xi, \eta) \in U\}$ is also in ζ .

The pair (Y, ζ) is called a Uniform Space and the element of ζ is called entourage or neighbourhood or surrounding. The pair (Y, ζ) is called a quasi-uniform space if the condition (y_5) does not hold.

Let $\Delta = \{(\eta, \eta) | \eta \in Y\}$ be the diagonal of a non-empty set Y . For $G, H \in Y \times Y$, we use the following: $GoH = \{(\xi, \eta) | \text{there exists } \sigma \in Y : (\xi, \sigma) \in H \text{ and } (\sigma, \eta) \in G\}$ and $G^{-1} = \{(\xi, \eta) : (\eta, \xi) \in G\}$. For a subset $G \in \zeta$, a pair of points ξ, η are said to be G -closed if $(\xi, \eta) \in G$ and $(\eta, \xi) \in G$.

Definition 2.1. [3] *A sequence $\{\mu_n\} \subset Y$ is called a Cauchy sequence for ζ if for any $G \in \zeta$, there exists $N \in \mathbb{N}$ such that for all $n, m > N$, μ_n and μ_m are G -closed.*

Note. For a Uniform space (Y, ζ) , there is a unique topology $\tau(\zeta)$ on Y generated by $G(\xi) = \{\sigma \in Y | (\xi, \sigma) \in G\}$, where $G \in \zeta$.

Definition 2.2. [22] *A uniform space (Y, ζ) is called Hausdorff if*

$\bigcap G = \Delta$, for all $G \in \zeta$ i.e., if $(\xi, \eta) \in G$ for all $G \in \zeta$, implies $\xi = \eta$.

Definition 2.3. {[3], [21]} *Let (Y, ζ) be a uniform space. A function $p : Y \times Y \rightarrow$*

\mathbb{R}^+ is said to be a A -distance if for any $G \in \zeta$, there exists $\delta > 0$ such that if $p(\sigma, \xi) \leq \delta$ and $p(\sigma, \eta) \leq \delta$ hold for some $\sigma \in Y$, then $(\xi, \eta) \in G$.

Definition 2.4. {[3], [2]} Let (Y, ζ) be a uniform space. A function $p : Y \times Y \rightarrow \mathbb{R}^+$ is said to be a E -distance if following conditions are satisfied:

(e_1) p is an A -distance; (e_2) $p(\xi, \sigma) \leq p(\xi, \eta) + p(\eta, \sigma)$, $\forall \xi, \eta, \sigma \in Y$.

Definition 2.5. [6] Let Y be a non-empty set and $J : Y \rightarrow Y$ and $\alpha, \beta : Y \times Y \rightarrow [0, \infty)$ be functions. Then J is called (α, β) -admissible mapping if $\alpha(\xi, \eta) \geq 1$ and $\beta(\xi, \eta) \geq 1$ implies $\alpha(J\xi, J\eta) \geq 1$ and $\beta(J\xi, J\eta) \geq 1$; for all $\xi, \eta \in Y$.

Definition 2.6. {[3], [27]} A class of functions Γ is given by

$\Gamma = \{\gamma : \mathbb{R} \rightarrow \mathbb{R} \text{ such that (i) } \gamma \text{ is monotone increasing, (ii) } \gamma^n(t) \rightarrow -\infty \text{ as } n \rightarrow \infty,$

$\forall t \in \mathbb{R}; \text{ where } \gamma^n \text{ denotes the } n\text{-th composition of } \gamma.\}$

Lemma 2.1. [22] If $\gamma \in \Gamma$, then $\gamma(t) < t$, for all $t \in \mathbb{R}$.

Lemma 2.2. {[3], [2]} Let (Y, ζ) be a Hausdorff uniform space and p is an A -distance on Y . Let $\{\xi_n\}, \{\eta_n\}$ be two sequences in Y and $\{x_n\}, \{y_n\}$ be two sequence in $[0, \infty)$ converging to 0. Then for $\xi, \eta, \sigma \in Y$ the following results hold:

(p_1) If $p(\xi_n, \eta) \leq x_n$ and $p(\xi_n, \sigma) \leq y_n$, for all $n \in \mathbb{N}$, then $\eta = \sigma$. In particular, if $p(\xi, \eta) = 0$ and $p(\xi, \sigma) = 0$, then $\eta = \sigma$.

(p_2) $p(\xi_n, \eta_n) \leq x_n$ and $p(\xi_n, \sigma) \leq y_n$, for all $n \in \mathbb{N}$, then $\{\eta_n\}$ converges to σ .

(p_3) If $p(\xi_n, \xi_m) \leq x_n$ for all $n, m \in \mathbb{N}$ with $m > n$, then $\{\xi_n\}$ is a Cauchy sequence in (Y, ζ) .

Definition 2.7. [3] Let (Y, ζ) be a uniform space and p -be an A -distance. A sequence $\{\mu_n\}$ in the uniform space (Y, ζ) with an A -distance is said to be p -Cauchy if for every $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $n, m \geq N$, implies $p(\mu_n, \mu_m) < \epsilon$.

Definition 2.8. {[3], [2]} Let (Y, ζ) be a uniform space and p is an A -distance on Y . Then

(s_1) Y is called S -complete if for each p -Cauchy sequence $\{\mu_n\} \subset Y$, there exists $\mu \in Y$ such that $\lim_{n \rightarrow \infty} p(\mu_n, \mu) = 0$.

(s_2) Y is called p -Cauchy complete if for each p -Cauchy sequence $\{\mu_n\} \subset Y$, there is $\mu \in Y$ such that $\lim_{n \rightarrow \infty} \mu_n = \mu$ with respect to $\tau(\zeta)$, where $\tau(\zeta)$ is the unique topology generated by $G(\xi) = \{\sigma \in Y | (\xi, \sigma) \in G\}$, $G \in \zeta$.

Definition 2.9. {[3], [30]} A function $F : (0, \infty) \rightarrow \mathbb{R}$ is said to be F -contraction if it satisfies the following conditions:

(f_1) F is strictly increasing;

(f_2) For every sequence $\{t_n\} \subset (0, \infty)$, $\lim_{n \rightarrow \infty} t_n = 0 \Leftrightarrow \lim_{n \rightarrow \infty} F(t_n) = -\infty$.

(f_3) There exists a constant $l \in (0, 1)$ such that $t^l F(t) \rightarrow 0$, when $t \rightarrow 0^+$.

Note. Υ denotes the class of all such F -contraction functions defined as above.

Example 2.1. [22] Let $F : (0, \infty \rightarrow \mathbb{R})$ be defined by

$$\begin{aligned} \text{(i)} \quad & F(x) = -\frac{1}{x^n}, 0 < n < 1; \quad \text{(ii)} \quad F(x) = x + \ln x; \\ \text{(iii)} \quad & F(x) = \ln x; \quad \text{(iv)} \quad F(x) = \ln(x + x^2). \end{aligned}$$

These are some examples of F -contractions functions.

Definition 2.10. [4] Let (Y, d) be a metric space. A mapping $J : Y \rightarrow Y$ is called an F -contraction of Hardy-Rogers-type if there exists $F \in \Upsilon$ and $\tau > 0$ such that $\tau + F(d(J\xi, J\eta)) \leq F(L_1 d(\xi, \eta) + L_2 d(\xi, J\xi) + L_3 d(\eta, J\eta) + L_4 d(\xi, J\eta) + L_5 d(\eta, J\xi))$, for all $\xi, \eta \in Y$ with $d(J\xi, J\eta) > 0$, where $L_1, L_2, L_3, L_4, L_5 \geq 0, L_1 + L_2 + L_3 + 2L_4 = 1$ and $L_3 \neq 1$.

Definition 2.11. Let (Y, d) be a metric space. We say that a mapping $J : Y \rightarrow Y$ is an extended interpolative Ciric-Reich-Rus type F -contraction if there exist $L_1, L_2 \in (0, 1)$ with $L_1 + L_2 < 1$ and $F \in \Upsilon$ such that

$$\begin{aligned} \tau + F(d(J\xi, J\eta)) &\leq L_1 F(d(\xi, \eta)) + L_2 F(d(\xi, J\xi)) + (1 - L_1 - L_2) F(d(\eta, J\eta)), \\ \forall \xi, \eta \in Y \setminus \text{Fix}(J) \quad &\text{with } d(J\xi, J\eta) > 0. \end{aligned}$$

Theorem 2.1. [20] An extended interpolative Ciric-Reich-Rus type F -contraction self mapping on a complete metric space Y admits a fixed point in Y .

Definition 2.12. [22] In a b -metric space (Y, d, s) with $s \geq 1$, a mapping $J : Y \rightarrow Y$ is an extended interpolative Ciric-Reich-Rus type ψF -contraction if there exist $F \in \Upsilon$ and $\psi \in \Gamma$ such that for all $\xi, \eta \in Y \setminus \text{Fix}(J)$ with $J\xi \neq J\eta$,

$$\begin{aligned} F(sd(J\xi, J\eta)) &\leq L_1 \psi(F(sd(\xi, \eta))) + L_2 \psi(F(sd(\xi, J\eta))) + L_3 \psi(F(sd(\eta, J\xi))); \\ \text{for some constants } L_1, L_2, L_3 &\in [0, 1] \text{ with } 0 < L_1 + L_2 + L_3 \leq 1. \end{aligned}$$

Theorem 2.2. [22] Let $J : Y \rightarrow Y$ an extended interpolative Ciric-Reich-Rus type ψF -contraction on a complete b -metric space (Y, d, s) . Let $w_0 \in Y \setminus \text{Fix}(J)$ be

such that the series $\sum_n \left| \psi^n(F(sa_0)) \right|^{\frac{-1}{k}}$ is convergent, where the constant $k \in (0, 1)$ comes from (f_3) and $a_0 = d(w_0, Tw_0)$. Then J admits a fixed point in Y .

Definition 2.13. [3] Let (Y, ζ) be a uniform space such that p is an E -distance on Y . A mapping $J : Y \rightarrow Y$ is said to be an (α, F) -contractive mapping if there exists a function $\alpha : Y \times Y \rightarrow [0, \infty)$, $F \in \Upsilon$ and constant $\tau > 0$ such that

$$\begin{aligned} \tau + F(\alpha(\xi, \eta)p(J\xi, J\eta)) &\leq F(p(\xi, \eta)), \quad \forall \xi, \eta \in Y; \\ \text{whenever, } \min\{\alpha(\xi, \eta)p(J\xi, J\eta), p(\xi, \eta)\} &> 0. \end{aligned}$$

Theorem 2.3. [3] Let (Y, ζ) be a S -complete Hausdorff uniform space such that p is an E -distance on Y . Let $J : Y \rightarrow Y$ be an (α, F) -contractive mapping which satisfies the following conditions:

(i) J is α -admissible; (ii) there exists $\xi_0 \in Y$ such that $\alpha(\xi_0, J\xi_0) \geq 1$ and $\alpha(J\xi_0, \xi_0) \geq 1$; (iii) J is p -continuous. Then J has a fixed point in Y .

Definition 2.14. Let (Y, ζ) be a uniform space and p is an E -distance on Y . A mapping $J : Y \rightarrow Y$ is said to be an extended interpolative Ciric- Reich-Rus type $(\alpha, \beta, \gamma F)$ - contraction of type-I if J is (α, β) -admissible and if there exist $F \in \Upsilon$ and $\gamma \in \Gamma$ such that the following hold:

$$\begin{aligned} &\alpha(\xi, J\xi)\beta(\eta, J\eta)F\{p(J\xi, J\eta)\} \\ &\leq L_1(t)\gamma\{F(p(\xi, \eta))\} + L_2(t)\gamma\{F(p(\xi, J\xi))\} + L_3(t)\gamma\{F(p(\eta, J\eta))\} + \\ &L_4(t)\gamma\{F([(p(\xi, \eta))]^a[p(\xi, J\xi)]^b[(p(\eta, J\eta))]^c]); \quad \forall \xi, \eta \in Y; \quad \gamma \in \Gamma; F \in \Upsilon; \end{aligned} \tag{2.1}$$

where, $L_i : [0, \infty) \rightarrow [0, 1]$ are continuous, for all $i = 1, 2, 3, 4$;

with $L_1(t) + L_2(t) + L_3(t) + L_4(t) \leq 1, \quad \forall t \in [0, \infty)$;

and $0 \leq a + b + c \leq 1$.

Definition 2.15. Let (Y, ζ) be a uniform space such that p is an E -distance on Y . A mapping $J : Y \rightarrow Y$ is said to be an extended interpolative Ciric-Reich-Rus type $(\alpha, \beta, \gamma F)$ - contraction of type-II if J is (α, β) -admissible and if there exist $F \in \Upsilon$ and $\gamma \in \Gamma$ such that the following hold:

$$\begin{aligned} &\{F\{p(J\xi, J\eta)\} + L\}^{\alpha(\xi, J\xi)\beta(\eta, J\eta)} \\ &\leq L_1(t)\gamma\{F(p(\xi, \eta))\} + L_2(t)\gamma\{F(p(\xi, J\xi))\} + L_3(t)\gamma\{F(p(\eta, J\eta))\} + \\ &L_4(t)\gamma\{F([(p(\xi, \eta))]^a[p(\xi, J\xi)]^b[(p(\eta, J\eta))]^c]\} + L; \quad \forall \xi, \eta \in Y; \quad \gamma \in \Gamma; F \in \Upsilon; \end{aligned} \tag{2.2}$$

where, $L_i : [0, \infty) \rightarrow [0, 1]$ are continuous, for all $i = 1, 2, 3, 4$;
 with $L_1(t) + L_2(t) + L_3(t) + L_4(t) \leq 1, \forall t \in [0, \infty)$;
 and $0 \leq a + b + c \leq 1; L \geq 1$.

3. Main Results

Theorem 3.1. Let (Y, ζ) be an S -complete Hausdorff uniform space with p is an E -distance on Y . Let $J : Y \rightarrow Y$ be an extended interpolative Ciric-Reich-Rus type $(\alpha, \beta, \gamma F)$ -contraction of type-I, given in **definition 2.14**. Let $v_0 \in Y \setminus \text{Fix}(J)$ be such that the series $\sum_{n=1}^{\infty} |\gamma^n(F(a_0))|^{\frac{1}{\gamma}}$ is convergent, where the constant $l \in (0, 1)$ comes from (f_3) and $a_0 = p(v_0, Jv_0)$. Then J has a fixed point in Y .

Further, if all $u, v \in \text{Fix}(J)$ with $u \neq v$ such that $\alpha(u, Ju) \geq 1, \alpha(v, Jv) \geq 1$ and $\beta(u, Ju) \geq 1, \beta(v, Jv) \geq 1$, then fixed point of J is unique in Y .

Proof. Let $v_0 \in Y$ be such that $\alpha(v_0, Jv_0) \geq 1$ and $\beta(v_0, Jv_0) \geq 1$.

Now we can construct a sequence $\{v_n\} \subset Y$ by $v_n = J_n v_0 = Jv_{n-1}, \forall n \in \mathbb{N}$.

We consider two cases as follows:

Case-I: If $v_{n+1} = v_n$, for some $n \in \mathbb{N}$, then v_n is a fixed point of J in Y .

Case-II: Assume $v_{n+1} \neq v_n$, for all $n \in \mathbb{N}$.

Since J is (α, β) -admissible mapping, so

$$\alpha(v_0, Jv_0) = \alpha(v_0, v_1) \geq 1, \alpha(Jv_0, Jv_1) = \alpha(v_1, v_2) \geq 1, \alpha(Jv_1, Jv_2) = \alpha(v_2, v_3) \geq 1.$$

Hence by Induction we have, $\alpha(v_n, v_{n+1}) \geq 1, \forall n \geq 0$.

Similarly, $\beta(v_n, v_{n+1}) \geq 1, \forall n \geq 0$.

For the shake of convenience we assume that

$$L_i(t) \equiv L_i, \forall t \in [0, \infty); \text{ for all } i = 1, 2, 3, 4.$$

Now from (2.1) we have

$$\begin{aligned} & F\{p(v_n, v_{n+1})\} \\ &= F\{p(Jv_{n-1}, Jv_n)\} \\ &\leq \alpha(v_{n-1}, Jv_{n-1})\beta(v_n, Jv_n)F\{p(Jv_{n-1}, Jv_n)\} \\ &\leq L_1\gamma\{F(p(v_{n-1}, v_n))\} + L_2\gamma\{F(p(v_{n-1}, Jv_{n-1}))\} + L_3\gamma\{F(p(v_n, Jv_n))\} + \\ &\quad L_4\gamma\{F([(p(v_{n-1}, v_n))]^a [p(v_{n-1}, Jv_{n-1})]^b [(p(v_n, Jv_n))]^c)\} \\ &= L_1\gamma\{F(p(v_{n-1}, v_n))\} + L_2\gamma\{F(p(v_{n-1}, v_n))\} + L_3\gamma\{F(p(v_n, v_{n+1}))\} + \\ &\quad L_4\gamma\{F([(p(v_{n-1}, v_n))]^a [p(v_{n-1}, v_n)]^b [(p(v_n, v_{n+1}))]^c)\} \end{aligned} \tag{3.1}$$

If $p(v_{n-1}, v_n) < p(v_n, v_{n+1})$, then from (3.1) we have

$$\begin{aligned} F\{p(v_n, v_{n+1})\} &\leq (L_1 + L_2 + L_3)\gamma\{F(p(v_n, v_{n+1}))\} + L_4\gamma\{F[p(v_n, v_{n+1})^{a+b+c}]\} \\ &\leq (L_1 + L_2 + L_3 + L_4)\gamma\{F(p(v_n, v_{n+1}))\} \\ &\leq \gamma\{F(p(v_n, v_{n+1}))\} \\ &< F(p(v_n, v_{n+1})), \quad \text{which is a contradiction as } \gamma(t) < t. \end{aligned}$$

Hence

$$p(v_n, v_{n+1}) \leq p(v_{n-1}, v_n) \tag{3.2}$$

Now from (3.1) and using (3.2) we have

$$\begin{aligned} F\{p(v_n, v_{n+1})\} &\leq (L_1 + L_2 + L_3 + L_4)\gamma\{F(p(v_{n-1}, v_n))\} \\ &\leq \gamma\{F(p(v_{n-1}, v_n))\} \\ &\leq \gamma^2\{F(p(v_{n-2}, v_{n-1}))\} \\ &\leq \gamma^3\{F(p(v_{n-3}, v_{n-2}))\} \\ &\cdot \\ &\cdot \\ &\cdot \\ &\leq \gamma^n\{F(p(v_0, v_1))\} \\ &\rightarrow -\infty \quad \text{as } n \rightarrow \infty. \end{aligned} \tag{3.3}$$

Hence

$$\lim_{n \rightarrow \infty} p(v_n, v_{n+1}) = 0. \tag{3.4}$$

We define $a_n := p(v_n, v_{n+1})$, then $a_n \rightarrow 0$ as $n \rightarrow \infty$. Then there exists $l \in (0, 1)$ such that

$$a_n^l F(a_n) \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

Again using (3.3) we have

$$a_n^l F(a_n) \leq a_n^l \gamma^n\{F(a_0)\} \leq a_n^l F(a_0)$$

Therefore, by Sandwich Theorem, $\lim_{n \rightarrow \infty} a_n^l \gamma^n\{F(a_0)\} = 0$.

So, for $\epsilon = 1$, there exists $k_0 \in \mathbb{N}$ such that

$$a_n < \left| \gamma^n\{F(a_0)\} \right|^{-\frac{1}{l}}, \quad \text{whenever } n \geq k_0. \tag{3.5}$$

Now,

$$\begin{aligned}
p(v_n, v_{n+s}) &\leq p(v_n, v_{n+1}) + p(v_{n+1}, v_{n+2}) + \dots + p(v_{n+s-1}, v_{n+s}) \\
&= a_n + a_{n+1} + \dots + a_{n+s-1} \\
&= \sum_{k=n}^{n+s-1} a_k \\
&< \sum_{k=n}^{n+s-1} \left| \gamma^n \{F(a_0)\} \right|^{-\frac{1}{i}}.
\end{aligned} \tag{3.6}$$

Since the series $\sum_{k=n}^{n+s-1} \left| \gamma^n \{F(a_0)\} \right|^{-\frac{1}{i}}$ is convergent, so for every $s = 1, 2, 3, \dots$ we

have $p(v_n, v_{n+s}) \rightarrow 0$ as $n \rightarrow \infty$ and consequently $\{v_n\}$ is a p -Cauchy in Y .

By the completeness of Y , $\{v_n\}$ is convergent so, there exists $v \in Y$ such that

$$\lim_{n \rightarrow \infty} v_n = v.$$

If $Jv = v$, then v is a fixed point of J in Y .

Assume $Jv \neq v$.

Then from (2.1) we have

$$\begin{aligned}
&F\{p(v_{n+1}, Jv)\} \\
&= F\{p(Jv_n, Jv)\} \\
&\leq \alpha(v_n, Jv_n)\beta(v, Jv)F\{p(Jv_n, Jv)\} \\
&\leq L_1\gamma\{F(p(v_n, v))\} + L_2\gamma\{F(p(v_n, v_{n+1}))\} + L_3\gamma\{F(p(v, Jv))\} + \\
&\quad L_4\gamma\{F([(p(v_n, v))]^a[p(v_n, v_{n+1}))]^b[(p(v, Jv))]^c)\} \\
&\rightarrow -\infty \text{ as } n \rightarrow \infty. \quad [\text{By condition } f_2]
\end{aligned} \tag{3.7}$$

Hence $\lim_{n \rightarrow \infty} p(v_n, v) = 0$ i.e., $p(v, Jv) = 0$, which gives $Jv = v$.

Hence, v is a fixed point of J in Y .

Uniqueness. Suppose w and z be two fixed points of J in Y such that $w \neq z$ and $\alpha(w, Jw) \geq 1$, $\alpha(z, Jz) \geq 1$ and $\beta(w, Jw) \geq 1$, $\beta(z, Jz) \geq 1$.

Now from (2.1)

$$\begin{aligned}
&F\{p(w, z)\} \\
&= F\{p(Jw, Jz)\} \\
&\leq \alpha(w, Jw)\beta(z, Jz)F\{p(Jw, Jz)\} \\
&\leq L_1\gamma\{F(p(w, z))\} + L_2\gamma\{F(p(w, Jw))\} + L_3\gamma\{F(p(z, Jz))\} + \\
&\quad L_4\gamma\{F([(p(w, z))]^a[p(w, Jw))]^b[(p(z, Jz))]^c)\}
\end{aligned}$$

$$\begin{aligned}
&\leq L_1\gamma\{F(p(w, z))\} \\
&\leq \gamma\{F(p(w, z))\} \\
&< F\{p(w, z)\}, \quad \text{which is a contradiction.}
\end{aligned} \tag{3.8}$$

Hence $w = z$. This completes the proof.

Example 3.1. We consider $Y = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$;

Also

$$\alpha(\xi, \eta) = \begin{cases} 2; & \text{if } \xi, \eta \in Y \setminus \{1\}; \\ 1; & \text{otherwise.} \end{cases}$$

$$\beta(\xi, \eta) = \begin{cases} 3; & \text{if } \xi, \eta \in Y \setminus \{1\}; \\ 1; & \text{otherwise.} \end{cases}$$

$$F(x) = -\frac{1}{\sqrt{x}} - 1, \quad \forall x \in (0, \infty); \quad \gamma(t) = t - \frac{1}{4}, \quad \forall t \in \mathbb{R};$$

$$p(\xi, \eta) = \begin{cases} |\xi - \eta|; & \text{if } \xi \neq \eta; \\ 0; & \text{otherwise.} \end{cases} \quad \text{and } J(\xi) = \frac{\xi}{4}, \quad \forall \xi \in Y.$$

We consider $L_1(t) = \frac{1}{16}$, $L_2(t) = 0$, $L_3(t) = 0$, $L_4(t) = \frac{1}{32}$ and $a = 1$, $b = 0$, $c = 0$, then condition (1) holds.

Here, $\xi = 0$ is the unique fixed point of J in Y .

Corollary 3.1. Let (Y, ζ) be an S -complete Hausdorff uniform space with p is an E -distance on Y . Let $J : Y \rightarrow Y$ be a function satisfying the following:

$$\begin{aligned}
&\alpha(\xi, J\xi)\beta(\eta, J\eta)F\{p(J\xi, J\eta)\} \\
&\leq L_1\gamma\{F(p(\xi, \eta))\} + L_2\gamma\{F(p(\xi, J\xi))\} + L_3\gamma\{F(p(\eta, J\eta))\} + \\
&L_3\gamma\{F([p(\xi, \eta)]^a[p(\xi, J\xi)]^b[p(\eta, J\eta)]^c)\}; \quad \forall \xi, \eta \in Y; \quad \gamma \in \Gamma; \quad F \in \Upsilon; \\
&\text{with } 0 \leq L_1 + L_2 + L_3 + L_4 \leq 1; \\
&\text{and } 0 \leq a + b + c \leq 1;
\end{aligned} \tag{3.9}$$

such that

(i) J is (α, β) -admissible;

(ii) Let $v_0 \in Y \setminus \text{Fix}(J)$ be such that the series $\sum_{n=1}^{\infty} |\gamma^n(F(a_0))|^{\frac{1}{T}}$ is convergent, where the constant $l \in (0, 1)$ comes from (f_3) and $a_0 = p(v_0, Jv_0)$. Then J has a fixed point in Y .

Further, if all $u, v \in \text{Fix}(J)$ with $u \neq v$ such that $\alpha(u, Ju) \geq 1$, $\alpha(v, Jv) \geq 1$ and $\beta(u, Ju) \geq 1$, $\beta(v, Jv) \geq 1$, then fixed point of J is unique in Y .

Corollary 3.2. Let (Y, ζ) be an S -complete Hausdorff uniform space with p is an

E -distance on Y . Let $J : Y \rightarrow Y$ be a function satisfying the following:

$$\begin{aligned} & F\{p(J\xi, J\eta)\} \\ & \leq L_1\gamma\{F(p(\xi, \eta))\} + L_2\gamma\{F(p(\xi, J\xi))\} + L_3\gamma\{F(p(\eta, J\eta))\} + \\ & L_4\gamma\{F([(p(\xi, \eta))]^a[p(\xi, J\xi)]^b[(p(\eta, J\eta))]^c]\}; \quad \forall \xi, \eta \in Y; \quad \gamma \in \Gamma; \quad F \in \Upsilon; \quad (3.10) \end{aligned}$$

$$\text{with } 0 \leq L_1 + L_2 + L_3 + L_4 \leq 1,$$

$$\text{and } 0 \leq a + b + c \leq 1.$$

Let $v_0 \in Y \setminus \text{Fix}(J)$ be such that the series $\sum_{n=1}^{\infty} |\gamma^n(F(a_0))|^{\frac{1}{\tau}}$ is convergent, where the constant $l \in (0, 1)$ comes from (f_3) and $a_0 = p(v_0, Jv_0)$. Then J has a unique fixed point in Y .

Theorem 3.2. Let (Y, ζ) be an S -complete Hausdorff uniform space and p is an E -distance on Y . Let $J : Y \rightarrow Y$ be an extended interpolative Ciric-Reich-Rus type $(\alpha, \beta, \gamma F)$ -contraction of type-II, given in **definition 2.15**. Let $v_0 \in \text{Fix}(J)$ be such that the series $\sum_{n=1}^{\infty} |\gamma^n(F(a_0))|^{\frac{1}{\tau}}$ is convergent, where the constant $l \in (0, 1)$ comes from (f_3) and $a_0 = p(v_0, Jv_0)$. Then J has a fixed point in Y .

Further, if all $u, v \in \text{Fix}(J)$ with $u \neq v$ such that $\alpha(u, Ju) \geq 1$, $\alpha(v, Jv) \geq 1$ and $\beta(u, Ju) \geq 1$, $\beta(v, Jv) \geq 1$, then fixed point of J is unique in Y .

Proof. Let $v_0 \in Y$ such that $\alpha(v_0, Jv_0) \geq 1$ and $\beta(v_0, Jv_0) \geq 1$.

Now we can construct a sequence $\{v_n\} \subset Y$ by $v_n = J_n v_0 = Jv_{n-1}$, $\forall n \in \mathbb{N}$.

We consider two cases as follows:

Case-I: If $v_{n+1} = v_n$, for some $n \in \mathbb{N}$, then v_n is a fixed point of J in Y .

Case-II: Assume $v_{n+1} \neq v_n$, for all $n \in \mathbb{N}$.

Since J is (α, β) -admissible mapping, so

$$\alpha(v_0, Jv_0) = \alpha(v_0, v_1) \geq 1, \alpha(Jv_0, Jv_1) = \alpha(v_1, v_2) \geq 1, \alpha(Jv_1, Jv_2) = \alpha(v_2, v_3) \geq 1.$$

Hence by Induction we have, $\alpha(v_n, v_{n+1}) \geq 1$, $\forall n \geq 0$.

Similarly, $\beta(v_n, v_{n+1}) \geq 1$, $\forall n \geq 0$.

For the shake of convenience we assume that

$$L_i(t) \equiv L_i, \quad \forall t \in [0, \infty); \quad \forall i = 1, 2, 3, 4.$$

Now from (2.2) we have

$$\begin{aligned} & F\{p(v_n, v_{n+1})\} + L \\ & = F\{p(Jv_{n-1}, Jv_n)\} + L \\ & \leq \{F\{p(Jv_{n-1}, Jv_n)\} + L\}^{\alpha(v_{n-1}, Jv_{n-1})\beta(v_n, Jv_n)} \\ & \leq L_1\gamma\{F(p(v_{n-1}, v_n))\} + L_2\gamma\{F(p(v_{n-1}, Jv_{n-1}))\} + L_3\gamma\{F(p(v_n, Jv_n))\} + \end{aligned}$$

$$\begin{aligned}
& L_4\gamma\{F([(p(v_{n-1}, v_n))]^a[p(v_{n-1}, Jv_{n-1})]^b[(p(v_n, Jv_n))]^c)\} + L \\
= & L_1\gamma\{F(p(v_{n-1}, v_n))\} + L_2\gamma\{F(p(v_{n-1}, v_n))\} + L_3\gamma\{F(p(v_n, v_{n+1}))\} + \\
& L_4\gamma\{F([(p(v_{n-1}, v_n))]^a[p(v_{n-1}, v_n)]^b[(p(v_n, v_{n+1}))]^c)\} + L
\end{aligned} \tag{3.11}$$

If $p(v_{n-1}, v_n) < p(v_n, v_{n+1})$, then from (3.11) we have

$$\begin{aligned}
F\{p(v_n, v_{n+1})\} + L & \leq (L_1 + L_2 + L_3)\gamma\{F(p(v_n, v_{n+1}))\} + L_4\gamma\{F[p(v_n, v_{n+1})^{a+b+c}]\} + L \\
& \leq (L_1 + L_2 + L_3 + L_4)\gamma\{F(p(v_n, v_{n+1}))\} + L \\
& \leq \gamma\{F(p(v_n, v_{n+1}))\} + L \\
& < F\{p(v_n, v_{n+1})\} + L, \quad \text{which is a contradiction as } \gamma(t) < t.
\end{aligned}$$

Hence,

$$p(v_n, v_{n+1}) \leq p(v_{n-1}, v_n) \tag{3.12}$$

Now from (3.11) and using (3.12) we have

$$\begin{aligned}
F\{p(v_n, v_{n+1})\} + L & \leq (L_1 + L_2 + L_3 + L_4)\gamma\{F(p(v_{n-1}, v_n))\} + L \\
& \leq \gamma\{F(p(v_{n-1}, v_n))\} + L \\
& \leq \gamma^2\{F(p(v_{n-2}, v_{n-1}))\} + L \\
& \leq \gamma^3\{F(p(v_{n-3}, v_{n-2}))\} + L \\
& \cdot \\
& \cdot \\
& \cdot \\
& \leq \gamma^n\{F(p(v_0, v_1))\} + L
\end{aligned} \tag{3.13}$$

Hence from (3.13) we have

$$F\{p(v_n, v_{n+1})\} \leq \gamma^n\{F(p(v_0, v_1))\}, \quad \text{which } \rightarrow -\infty \text{ as } n \rightarrow \infty.$$

Therefore,

$$\lim_{n \rightarrow \infty} p(v_n, v_{n+1}) = 0. \tag{3.14}$$

We define $a_n := p(v_n, v_{n+1})$, then $a_n \rightarrow 0$ as $n \rightarrow \infty$. Then there exists $l \in (0, 1)$ such that

$$a_n^l F(a_n) \rightarrow 0 \text{ as } n \rightarrow \infty.$$

Again using (3.12) we have

$$a_n^l F(a_n) \leq a_n^l \gamma^n\{F(a_0)\} \leq a_n^l F(a_0)$$

Therefore, by Sandwich Theorem, $\lim_{n \rightarrow \infty} a_n^l \gamma^n \{F(a_0)\} = 0$.

So, for $\epsilon = 1$, there exists $k_0 \in \mathbb{N}$ such that

$$a_n < \left| \gamma^n \{F(a_0)\} \right|^{-\frac{1}{l}}, \text{ whenever } n \geq k_0. \quad (3.15)$$

Now

$$\begin{aligned} p(v_n, v_{n+s}) &\leq p(v_n, v_{n+1}) + p(v_{n+1}, v_{n+2}) + \dots + p(v_{n+s-1}, v_{n+s}) \\ &= a_n + a_{n+1} + \dots + a_{n+s-1} \\ &= \sum_{k=n}^{n+s-1} a_k \\ &< \sum_{k=n}^{n+s-1} \left| \gamma^n \{F(a_0)\} \right|^{-\frac{1}{l}}. \end{aligned} \quad (3.16)$$

Since the series $\sum_{k=n}^{n+s-1} \left| \gamma^n \{F(a_0)\} \right|^{-\frac{1}{l}}$ is convergent, so for every $s = 1, 2, 3, \dots$ we have $p(v_n, v_{n+s}) \rightarrow 0$ as $n \rightarrow \infty$ and consequently $\{v_n\}$ is a p -Cauchy in Y .

By the completeness of Y , $\{v_n\}$ is convergent so, there exists $v \in Y$ such that $\lim_{n \rightarrow \infty} v_n = v$. If $Jv = v$, then v is a fixed point of J in Y .

Assume $Jv \neq v$.

Then from (2.2) we have

$$\begin{aligned} &F\{p(v_{n+1}, Jv)\} + L \\ &= F\{p(Jv_n, Jv)\} + L \\ &\leq \{F\{p(Jv_n, Jv)\} + L\}^{\alpha(v_n, Jv_n)\beta(v, Jv)} \\ &\leq L_1 \gamma \{F(p(v_n, v))\} + L_2 \gamma \{F(p(v_n, v_{n+1}))\} + L_3 \gamma \{F(p(v, Jv))\} + \\ &\quad L_4 \gamma \{F([(p(v_n, v))]^a [p(v_n, v_{n+1})]^b [(p(v, Jv))]^c)\} + L \\ &\rightarrow -\infty \text{ as } n \rightarrow \infty. \end{aligned} \quad (3.17)$$

Hence from (3.17) we have

$$\begin{aligned} &F\{p(v_{n+1}, Jv)\} \\ &\leq L_1 \gamma \{F(p(v_n, v))\} + L_2 \gamma \{F(p(v_n, v_{n+1}))\} + L_3 \gamma \{F(p(v, Jv))\} + \\ &\quad L_4 \gamma \{F([(p(v_n, v))]^a [p(v_n, v_{n+1})]^b [(p(v, Jv))]^c)\} \\ &\rightarrow -\infty \text{ as } n \rightarrow \infty. \end{aligned} \quad (3.18)$$

Hence $\lim_{n \rightarrow \infty} p(v_n, v) = 0$ i.e., $p(v, Jv) = 0$. Thus $Jv = v$.

Hence, v is a fixed point of J in Y .

Uniqueness. Suppose w and z be two fixed points of J in Y such that $w \neq z$ and $\alpha(w, Jw) \geq 1, \alpha(z, Jz) \geq 1$ and $\beta(w, Jw) \geq 1, \beta(z, Jz) \geq 1$.

Now from (2.2)

$$\begin{aligned}
 & F\{p(w, z)\} + L \\
 &= F\{p(Jw, Jz)\} + L \\
 &\leq \{F\{p(Jw, Jz)\} + L\}^{\alpha(w, Jw)\beta(z, Jz)} \\
 &\leq L_1\gamma\{F(p(w, z))\} + L_2\gamma\{F(p(w, Jw))\} + L_3\gamma\{F(p(z, Jz))\} + \\
 &\quad L_4\gamma\{F([(p(w, z))]^a[p(w, Jw)]^b[(p(z, Jz))]^c]\} + L \\
 &\leq L_1\gamma\{F(p(w, z))\} + L \\
 &= \gamma\{F(p(w, z))\} + L \\
 &< F\{p(w, z)\} + L, \quad \text{which is a contradiction.}
 \end{aligned} \tag{3.19}$$

Hence $w = z$. This completes the proof.

Example 3.2. We consider $Y = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$;

$$\alpha(\xi, \eta) = \begin{cases} 3; & \text{if } \xi, \eta \in Y \setminus \{1\}; \\ 1; & \text{otherwise.} \end{cases}$$

$$\beta(\xi, \eta) = \begin{cases} 1; & \text{if } \xi, \eta \in Y \setminus \{1\}; \\ 3; & \text{otherwise.} \end{cases}$$

$$F(x) = -\frac{1}{\sqrt{x}} - 2, \quad \forall x \in (0, \infty);$$

$$\gamma(t) = t - \frac{1}{2}, \quad \forall t \in \mathbb{R};$$

$$p(\xi, \eta) = \begin{cases} |\xi - \eta|; & \text{if } \xi \neq \eta; \\ 0; & \text{otherwise.} \end{cases}$$

$$J(\xi) = \frac{\xi}{16}, \quad \forall \xi \in Y.$$

Also we consider $L_1(t) = \frac{4}{5}, L_2(t) = 0, L_3(t) = 0, L_4(t) = 0, L = 2$ and $a = 1, b = 0, c = 0$, then condition (2) holds. Here, $\xi = 0$ is the unique fixed point of J in Y .

Corollary 3.3. Let (Y, ζ) be an S -complete Hausdorff uniform space such that p is an E -distance on Y . Let $J : Y \rightarrow Y$ satisfying the following:

$$\begin{aligned}
 & \{F\{p(J\xi, J\eta)\} + L\}^{\alpha(\xi, J\xi)\beta(\eta, J\eta)} \\
 &\leq L_1\gamma\{F(p(\xi, \eta))\} + L_2\gamma\{F(p(\xi, J\xi))\} + L_3\gamma\{F(p(\eta, J\eta))\} + \\
 &\quad L_4\gamma\{F([(p(\xi, \eta))]^a[p(\xi, J\xi)]^b[(p(\eta, J\eta))]^c]\} + L; \quad \forall \xi, \eta \in Y; \gamma \in \Gamma; F \in \Upsilon;
 \end{aligned}$$

with $0 \leq L_1 + L_2 + L_3 + L_4 \leq 1$;

$$\text{and } 0 \leq a + b + c \leq 1 \text{ and } L \geq 1; \quad (3.20)$$

such that

(i) J is (α, β) -admissible;

(ii) Let $v_0 \in Y \setminus \text{Fix}(J)$ be such that the series $\sum_{n=1}^{\infty} |\gamma^n(F(a_0))|^{\frac{-1}{l}}$ is convergent, where the constant $l \in (0, 1)$ comes from (f_3) and $a_0 = p(v_0, Jv_0)$. Then J has a fixed point in Y .

Further, if all $u, v \in \text{Fix}(J)$ with $u \neq v$ such that $\alpha(u, Ju) \geq 1$, $\alpha(v, Jv) \geq 1$ and $\beta(u, Ju) \geq 1$, $\beta(v, Jv) \geq 1$, then fixed point of J is unique in Y .

4. Conclusion

We have introduced the extended interpolative Ciric-Reich-Rus type $(\alpha, \beta, \gamma F)$ -contractions of type-I and type-II, which are the extensions of extended interpolative Ciric-Reich-Rus type ψF -contractions. we have also established and proved unique fixed point results on S -complete Hausdorff uniform space under these contractions and these are the new ideas in this literature.

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