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BASIC BILATERAL HYPERGEOMETRIC FUNCTION ${}_2\Psi_2$ AND CONTINUED FRACTIONS

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Abstract: In this paper certain continued fractions of the ratios of ${}_2\Psi_2$-series have been established.

Keywords and Phrases: Basic bilateral hypergeometric series, basic hypergeometric series, continued fraction and transformation formula.

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1. Introduction, Notations and Definitions

Since the time of Euler and Gauss, continued fractions have been playing a very important role in number theory and classical analysis. Generalized hypergeometric series, both ordinary and basic have been a very significant tool in the derivation of continued fraction representations. A fresh impetus to this interesting branch of analysis has been given by the works of Ramanujan which are reputed with some beautiful continued fraction representations without any reference to their number theoretic importance or interpretations. Various continued fraction representations for the ratio of ${}_2\Psi_2$'s are known in the literature. Bhagirathi [3], Denis [5], Gupta [7,

8], Pathak and Srivastava [9], Singh S. N., Singh Satya Prakash and Yadav Vijay [10, 12], Singh A. K. and Yadav Vijay [11] and Srivastava [13] have established a good number of continued fractions for the ratios of ${}_2\Psi_2$'s. In this paper, our attempt is also to establish continued fraction results for the ratios of ${}_2\Psi_2$'s which are believed to be interesting and new. In what follows, we shall use the following usual notations and definitions.
The $q-$ rising factorial is defined as,

$$(a;q)_0 = 1, \quad (a;q)_n = (1-a)(1-aq)...(1-aq^{n-1}), \quad n \in (1,2,3,...),$$

where the parameter q is called the base and $|q| < 1$.
The infinite $q-$rising factorial is defined as,

$$(a;q)_\infty = \prod_{r=0}^{\infty}(1-aq^r) = \lim_{n\to\infty}(a;q)_n.$$

When k is complex number, we write

$$(a;q)_k = \frac{(a;q)_\infty}{(aq^k;q)_\infty}.$$

We define a basic hypergeometric series as,

$${}_r\Phi_s\left[\begin{array}{l} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s \end{array}\right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, ..., a_r; q)_n z^n}{(q, b_1, b_2, ..., b_s; q)_n} \left[(-1)^n q^{n(n-1)/2}\right]^{1+s-r}, \quad (1.1)$$

where $(a_1, a_2, ..., a_r; q)_n = (a_1;q)_n(a_2;q)_n...(a_r;q)_n$.
The infinite series in (1.1) converges in $|z| < \infty$ if $r \leq s$ and for $r = 1+s$, it converges in the unit disc $|z| < 1$.
We also define a basic bilateral hypergeometric series as,

$${}_r\Psi_r\left[\begin{array}{l} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_r \end{array}\right] = \sum_{n=-\infty}^{\infty} \frac{(a_1, a_2, ..., a_r; q)_n z^n}{(b_1, b_2, ..., b_r; q)_n}. \quad (1.2)$$

The series in (1.2) converges in the region $\left|\frac{b_1 b_2 ... b_r}{a_1 a_2 ... a_r}\right| < |z| < 1$.
Following Bowman and Schaumburg [4] write a coninued fraction as,

$$b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \ddots}}},$$

with the more compact notation

$$b_0 + \frac{a_1}{b_1+} \frac{a_2}{b_2+} \frac{a_3}{b_3 + \ldots}. \tag{1.3}$$

The k^{th} classical numerator A_k and k^{th} classical denominator B_k of the continued fraction (1.3) are the respective numerator and denominator when the finite continued fraction

$$\frac{A_k}{B_k} = b_0 + \frac{a_1}{b_1+} \frac{a_2}{b_2+} \frac{a_3}{b_3 + \ldots} \quad \frac{a_k}{+b_k}$$

is simplified in the usual way.
There is no loss of generality due to the simple identity,

$$b_0 + \frac{a_1}{b_1+} \frac{a_2}{b_2+} \frac{a_3}{b_3 + \ldots} \quad \frac{a_k}{b_k} = b_0 \left(1 + \frac{a_1/b_0 b_1}{1+} \frac{a_2/b_1 b_2}{1 + \ldots} \quad \frac{a_k/b_{k-1} b_k}{1} \right). \tag{1.4}$$

In this paper, we establish certain continued fractions for the ratios of ${}_2\Psi_2$ by making use of Bailey's transformations of ${}_2\Psi_2$ given below

$${}_2\Psi_2 \left[\begin{matrix} a, b; q; z \\ c, d \end{matrix} \right] = \frac{(az, d/a, c/b, dq/abz; q)_\infty}{(z, d, q/b, cd/abz; q)_\infty} \; {}_2\Psi_2 \left[\begin{matrix} a, abz/d; q; d/a \\ az, c \end{matrix} \right]. \tag{1.5}$$

[Gasper, G. and Rahman, M. (6); (5.20) (i), p. 150]

$${}_2\Psi_2 \left[\begin{matrix} a, b; q; z \\ c, d \end{matrix} \right] = \frac{(az, bz, cq/abz, dq/abz; q)_\infty}{(q/a, q/b, c, d; q)_\infty} \; {}_2\Psi_2 \left[\begin{matrix} abz/c, abz/d; q; cd/abz \\ az, bz \end{matrix} \right]. \tag{1.6}$$

[Gasper, G. and Rahman, M. (6); (5.20) (ii), p. 150]

2. Main Results

In this section we shall establish our main results involving continued fractions. Putting $d = q$ in (1.5) and (1.6) we get following transformations

$${}_2\Phi_1 \left[\begin{matrix} a, b; q; z \\ c \end{matrix} \right] = \frac{(az, q/a, c/b, q^2/abz; q)_\infty}{(q, z, q/b, cq/abz; q)_\infty} \; {}_2\Psi_2 \left[\begin{matrix} a, abz/q; q; q/a \\ az, c \end{matrix} \right]. \tag{2.1}$$

$${}_2\Phi_1 \left[\begin{matrix} a, b; q; z \\ c \end{matrix} \right] = \frac{(az, bz, cq/abz, q^2/abz; q)_\infty}{(q/a, q/b, c, q; q)_\infty} \; {}_2\Psi_2 \left[\begin{matrix} abz/c, abz/q; q; cq/abz \\ az, bz \end{matrix} \right]. \tag{2.2}$$

(A) Replacing b and c by bq and cq respectively in (2.1) and (2.2) we have

$$ {}_2\Phi_1\left[\begin{matrix} a, bq; q; z \\ cq \end{matrix}\right] = \frac{(az, q/a, c/b, q/abz; q)_\infty}{(q, z, 1/b, cq/abz; q)_\infty}\ {}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ az, cq \end{matrix}\right]. \tag{2.3} $$

$$ {}_2\Phi_1\left[\begin{matrix} a, bq; q; z \\ cq \end{matrix}\right] = \frac{(az, bzq, cq/abz, q/abz; q)_\infty}{(q/a, 1/b, cq, q; q)_\infty}\ {}_2\Psi_2\left[\begin{matrix} abz/c, abz; q; cq/abz \\ az, bzq \end{matrix}\right]. \tag{2.4} $$

Taking the ratios of {(2.3), (2.1)}, {(2.3), (2.2)}, {(2.4), (2.1)}, {(2.4), (2.2)} and making use of Agarwal, R. P. [1; (3.1) p. 65] we have,

$$ \frac{\left(1-\frac{q}{abz}\right)}{\left(1-\frac{1}{b}\right)} \frac{{}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ az, cq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} a, abz/q; q; q/a \\ az, c \end{matrix}\right]} = \frac{\left(1-\frac{q}{abz}\right)\left(\frac{q}{a}; q\right)_\infty^2 \left(c, \frac{c}{b}; q\right)_\infty}{\left(1-\frac{1}{b}\right)\left(\frac{cq}{abz}; q\right)_\infty^2 (z, bz; q)_\infty} \times $$

$$ \times \frac{{}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ az, cq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} abz/c, abz/q; q; cq/abz \\ az, bz \end{matrix}\right]} $$

$$ = \frac{(z, bzq; q)_\infty \left(\frac{cq}{abz}; q\right)_\infty^2 \left(1-\frac{q}{abz}\right)}{\left(cq, \frac{c}{b}; q\right)_\infty \left(\frac{q}{a}; q\right)^2 \left(1-\frac{1}{b}\right)} \frac{{}_2\Psi_2\left[\begin{matrix} abz/c, abz; q; cq/abz \\ cz, bzq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} a, abz/q; q; q/a \\ az, c \end{matrix}\right]} $$

$$ = \frac{\left(1-\frac{q}{abz}\right)(1-c)}{(1-bz)\left(1-\frac{1}{b}\right)} \frac{{}_2\Psi_2\left[\begin{matrix} abz/c, abz; q; cq/abz \\ cz, bzq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} abz/c, abz/q; q; cq/abz \\ az, bz \end{matrix}\right]} $$

$$ = \frac{1}{1+}\frac{a_1}{1+}\frac{a_2}{1+}\frac{a_3}{1+\ldots}, \tag{2.5} $$

where

$$ a_{2k} = \frac{-zaq^{k-1}(1-bq^k)\left(1-\frac{cq^k}{a}\right)}{(1-cq^{2k-1})(1-cq^{2k})}, \quad k \geq 1 $$

and

$$a_{2k+1} = -zbq^k \frac{(1-aq^k)\left(1-\dfrac{cq^k}{b}\right)}{(1-cq^{2k})(1-cq^{2k+1})}, \qquad k \geq 0.$$

(B) Replacing z by zq in (2.1) and (2.2) we have

$$ {}_2\Phi_1\left[\begin{matrix} a, b; q; zq \\ c \end{matrix}\right] = \frac{(azq, q/a, c/b, q/abz; q)_\infty}{(q, zq, q/b, c/abz; q)_\infty} \; {}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ azq, c \end{matrix}\right]. \tag{2.6}$$

and

$$ {}_2\Phi_1\left[\begin{matrix} a, b; q; zq \\ c \end{matrix}\right] = \frac{(azq, bzq, c/abz, q/abz; q)_\infty}{(q/a, q/b, c, q; q)_\infty} \; {}_2\Psi_2\left[\begin{matrix} abzq/c, abz; q; c/abz \\ azq, bzq \end{matrix}\right]. \tag{2.7}$$

Taking the ratios of {(2.6), (2.1)}, {(2.6), (2.2)}, {(2.7), (2.1)}, {(2.7), (2.2)} and making use of Agarwal, R. P. [1; (3.13) p. 69] we have,

$$\frac{(1-z)\left(1-\dfrac{q}{abz}\right)}{(1-az)\left(1-\dfrac{c}{abz}\right)} \frac{{}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ azq, c \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} a, abz/q; q; q/a \\ az, c \end{matrix}\right]}$$

$$= \frac{\left(\dfrac{q}{a}; q\right)_\infty^2 \left(c, \dfrac{c}{b}; q\right)_\infty \left(1-\dfrac{q}{abz}\right)}{\left(\dfrac{cq}{abz}; q\right)_\infty^2 (zq, bz; q)_\infty \left(1-\dfrac{c}{abz}\right)} \frac{{}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ azq, c \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} abz/c, abz/q; q; cq/abz \\ az, bz \end{matrix}\right]}$$

$$= \frac{\left(1-\dfrac{c}{abz}\right)\left(1-\dfrac{q}{abz}\right)(z, bzq; q)_\infty \left(\dfrac{cq}{abz}; q\right)_\infty^2}{(1-az)\left(c, \dfrac{c}{b}; q\right)_\infty \left(\dfrac{q}{a}; q\right)_\infty^2} \frac{{}_2\Psi_2\left[\begin{matrix} abzq/c, abz; q; c/abz \\ azq, bzq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} a, abz/q; q; q/a \\ az, c \end{matrix}\right]}$$

$$= \frac{\left(1-\dfrac{c}{abz}\right)\left(1-\dfrac{q}{abz}\right)}{(1-az)(1-bz)} \frac{{}_2\Psi_2\left[\begin{matrix} abzq/c, abz; q; c/abz \\ azq, bzq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} abz/c, abz; q; cq/abz \\ az, bzq \end{matrix}\right]}$$

$$= \frac{1}{1+}\frac{z(1-a)(1-b)}{(1-z)+}\frac{abzq-c}{1+\ldots}\quad\frac{z(1-aq^n)(1-bq^n)}{1+}\frac{abzq^{2n+1}-cq^n}{1+\ldots} \tag{2.8}$$

(C) Putting bq for b in (2.1) and (2.2) we get

$$ {}_2\Phi_1\left[\begin{matrix} a, bq; q; z \\ c \end{matrix}\right] = \frac{(az, q/a, c/bq, q/abz; q)_\infty}{(q, z, 1/b, c/abz; q)_\infty}\ {}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ az, c \end{matrix}\right]. \tag{2.9}$$

and

$$ {}_2\Phi_1\left[\begin{matrix} a, bq; q; z \\ c \end{matrix}\right] = \frac{(az, bzq, c/abz, q/abz; q)_\infty}{(q/a, 1/b, c, q; q)_\infty}\ {}_2\Psi_2\left[\begin{matrix} abzq/c, abz; q; c/abz \\ az, bzq \end{matrix}\right]. \tag{2.10}$$

Putting zq for z in (2.1) and (2.2) we have

$$ {}_2\Phi_1\left[\begin{matrix} a, b; q; zq \\ c \end{matrix}\right] = \frac{(azq, q/a, c/b, q/abz; q)_\infty}{(q, zq, q/b, c/abz; q)_\infty}\ {}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ azq, c \end{matrix}\right]. \tag{2.11}$$

and

$$ {}_2\Phi_1\left[\begin{matrix} a, b; q; zq \\ c \end{matrix}\right] = \frac{(azq, bzq, c/abz, q/abz; q)_\infty}{(q/a, q/b, c, q; q)_\infty}\ {}_2\Psi_2\left[\begin{matrix} abzq/c, abz; q; c/abz \\ azq, bzq \end{matrix}\right]. \tag{2.12}$$

Taking the ratios of {(2.9), (2.11)}, {(2.9), (2.12)}, {(2.10), (2.11)}, {(2.10), (2.12)} and using Agarwal, R. P. [1; (21) p. 70] we have,

$$\frac{(1-az)\left(1-\dfrac{c}{bq}\right)}{(1-z)\left(1-\dfrac{1}{b}\right)}\frac{{}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ az, c \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ azq, c \end{matrix}\right]}$$

$$= \frac{(1-az)\left(c, \dfrac{c}{bq}; q\right)_\infty\left(\dfrac{q}{a}; q\right)_\infty^2}{\left(1-\dfrac{1}{b}\right)(z, bzq; q)_\infty\left(\dfrac{c}{abz}; q\right)_\infty^2}\ \frac{{}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ az, c \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} abzq/c, abz; q; c/abz \\ azq, bzq \end{matrix}\right]}$$

$$= \frac{(1-az)}{\left(1-\dfrac{1}{b}\right)}\frac{(zq, bzq; q)_\infty\left(\dfrac{c}{abz}; q\right)_\infty^2}{\left(c, \dfrac{c}{b}; q\right)_\infty\left(\dfrac{q}{a}; q\right)_\infty^2}\ \frac{{}_2\Psi_2\left[\begin{matrix} abzq/c, abz; q; c/abz \\ az, bzq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} a, abz; q; q/a \\ azq, c \end{matrix}\right]}$$

$$= \frac{(1-az)}{\left(1-\frac{1}{b}\right)} \frac{{}_2\Psi_2\left[\begin{matrix} abzq/c, abz; q; c/abz \\ az, bzq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} abzq/c, abz; q; c/abz \\ azq, bzq \end{matrix}\right]}$$

$$= 1 + \frac{z(1-a)}{(1-z)(1-b)+} \frac{(b-c)(1-azq)}{1+} \frac{z(1-aq)}{(1-z)(1-b)+} \frac{(b-cq)(1-azq^2)}{1+\ldots}. \tag{2.13}$$

(D) Putting cq for c in (2.1) and (2.2) we find

$${}_2\Phi_1\left[\begin{matrix} a, b; q; z \\ cq \end{matrix}\right] = \frac{(az, q/a, cq/b, q^2/abz; q)_\infty}{(q, z, q/b, cq^2/abz; q)_\infty} \; {}_2\Psi_2\left[\begin{matrix} a, abz/q; q; q/a \\ az, cq \end{matrix}\right]. \tag{2.14}$$

and

$${}_2\Phi_1\left[\begin{matrix} a, b; q; z \\ cq \end{matrix}\right] = \frac{(az, bz, cq^2/abz, q^2/abz; q)_\infty}{(q/a, q/b, cq, q; q)_\infty} \; {}_2\Psi_2\left[\begin{matrix} abz/cq, abz/q; q; cq^2/abz \\ az, bz \end{matrix}\right]. \tag{2.15}$$

Now, taking the ratios of {(2.14), (2.1)}, {(2.14), (2.2)}, {(2.15), (2.1)}, {(2.15), (2.2)} and using Agarwal, R. P. [1; (23 (i)) p. 71] we have,

$$\frac{\left(1-\frac{cq}{abz}\right)}{\left(1-\frac{c}{b}\right)} \frac{{}_2\Psi_2\left[\begin{matrix} a, abz/q; q; q/a \\ az, cq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} a, abz/q; q; q/a \\ az, c \end{matrix}\right]}$$

$$= \frac{\left(\frac{q}{a}; q\right)_\infty^2 \left(c, \frac{cq}{b}; q\right)_\infty}{\left(\frac{cq^2}{abz}; q\right)_\infty^2 \left(1-\frac{cq}{abz}\right)(z, bz; q)_\infty} \frac{{}_2\Psi_2\left[\begin{matrix} a, abz/q; q; q/a \\ az, cq \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} abz/c, abz/q; q; cq/abz \\ az, bz \end{matrix}\right]}$$

$$= \frac{\left(1-\frac{cq}{abz}\right)\left(\frac{cq^2}{abz}; q\right)_\infty^2 (z, bz; q)_\infty}{\left(\frac{q}{a}; q\right)_\infty^2 \left(cq, \frac{c}{b}; q\right)_\infty} \frac{{}_2\Psi_2\left[\begin{matrix} abz/cq, abz/q; q; cq^2/abz \\ az, bz \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} a, abz/q; q; q/a \\ az, c \end{matrix}\right]}$$

$$= \frac{(1-c)}{\left(1-\dfrac{cq}{abz}\right)} \frac{{}_2\Psi_2\left[\begin{matrix} abz/cq, abz/q; q; cq^2/abz \\ az, bz \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} abz/c, abz/q; q; cq/abz \\ az, bz \end{matrix}\right]}$$

$$= \frac{1}{1+} \frac{cz(1-a)(1-b)/(1-c)(1-cq)}{(1-abz/cq)+} \frac{z(a-cq)(b-cq)/cq(1-cq)(1-cq^2)}{1+}$$
$$\frac{czq^2(1-aq)(1-bq)/(1-cq^2)(1-cq^3)}{(1-abz/cq)+} \frac{z(a-cq^2)(b-cq^2)/cq(1-cq^3)(1-cq^4)}{1+\ldots}. \tag{2.16}$$

(E) Putting $c = q$ in (1.6) we have

$${}_2\Phi_1\left[\begin{matrix} a, b; q; z \\ d \end{matrix}\right] = \frac{(az, bz, q^2/abz, dq/abz; q)_\infty}{(q/a, q/b, d, q; q)_\infty} \, {}_2\Psi_2\left[\begin{matrix} abz/q, abz/d; q; dq/abz \\ az, bz \end{matrix}\right]. \tag{2.17}$$

Putting z/ab for z in (2.17) we have

$${}_2\Phi_1\left[\begin{matrix} a, b; q; z/ab \\ d \end{matrix}\right] = \frac{(1/a, 1/b, q^2/z, dq/z; q)_\infty}{(q/a, q/b, d, q; q)_\infty} \, {}_2\Psi_2\left[\begin{matrix} z/q, z/d; q; dq/z \\ 1/a, 1/b \end{matrix}\right]. \tag{2.18}$$

As $a, b \to \infty$ we get,

$$\sum_{n=0}^{\infty} \frac{q^{n^2-n} z^n}{(d;q)_n (q;q)_n} = \frac{(q^2/z, dq/z; q)_\infty}{(d, q; q)_\infty} \sum_{n=-\infty}^{\infty} (z/q, z/d; q)_n \frac{d^n q^n}{z^n}. \tag{2.19}$$

Taking $d = -q$, $z = \lambda q$ in (2.19) and comparing with Andrews G. E. and Berndt B. C. [2; (6.2.23) p. 150] we get after simplification,

$$\sum_{n=-\infty}^{\infty} (\lambda^2; q^2)_n \left(-\frac{q}{\lambda}\right)^n = \frac{(q^2;q^2)_\infty (-\lambda q; q^2)_\infty}{\left(\dfrac{q^2}{\lambda^2}; q^2\right)_\infty}. \tag{2.20}$$

Taking $d = q$ in (2.19) we have,

$$\sum_{n=0}^{\infty} \frac{q^{n^2-n} z^n}{(q;q)_n^2} = \frac{\left(\dfrac{q^2}{z}; q\right)_\infty^2}{(q;q)_\infty^2} \sum_{n=-\infty}^{\infty} \left(\frac{z}{q}; q\right)_n^2 \frac{q^{2n}}{z^n}. \tag{2.21}$$

Putting $d = -q$ in (2.19) we find

$$\sum_{n=0}^{\infty} \frac{q^{n^2-n} z^n}{(q^2;q^2)_n} = \frac{\left(\frac{q^4}{z^2};q^2\right)_\infty}{(q^2;q^2)_\infty} \sum_{n=-\infty}^{\infty} \left(\frac{z^2}{q^2};q^2\right)_n (-1)^n \left(\frac{q^2}{z}\right)^n. \tag{2.22}$$

Applying Ramanujan's summation formula Gasper G. and Rahman M. [6; App. II (II.29), p. 357] on the right hand side of (2.22) we get

$$\sum_{n=0}^{\infty} \frac{q^{n^2-n} z^n}{(q^2;q^2)_n} = \frac{(-z;q^2)_\infty (q^2;q^2)_\infty}{\left(\frac{q^4}{z^2};q^2\right)_\infty}. \tag{2.23}$$

Putting $z = q$ in (2.23) we have

$$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q^2;q^2)_n} = (-q;q^2)_\infty, \tag{2.24}$$

where the right hand side of (2.24) is the generating function of the partitions into distinct odd parts.

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