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THE QUINTESSENTIAL NUMBER 24

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Abstract: In this article, the authors refer to the interesting nature and properties of the number 24 and its occurrence and relevance in different contexts and conjecture that it is the raison d'être for its occurrence in the definition of the τ-function of Ramanujan.

Keywords and Phrases: Aryabhatia, Ashoka chakra, cube, fundamental theorem of Algebra, Gayatri mantra, Greek alphabet, Hardy-Ramanujan theorem, Highly Composite Numbers, hypergeometric functions, Kummer solutions of Gauss ODE, prime numbers, Surya Sidhanta, symmetries, tau-function, tetrahedral symmetries.

2020 Mathematics Subject Classification: 11.

1. Introduction

One should ask why the number of hours in a day is 24? The answer to this question requires an understanding of the *Surya Sidhanta* of the late 4th or early 5th

century B.C. and Aryabhatiya, the magnum opus of the 5th Century Indian Mathematician Aryabhatta (476–550 A.D.). These astronomy texts dealt with time at length vis-a-vis the movement of astronomical bodies in the Solar system, thereby laying a foundation for modern Astronomy and Mathematics.

Calendars are based either on the Solar or Lunar astronomical system. The Solar calendar is based on the movement of the Earth in an Elliptic orbit with the Sun at one of its two foci. The Earth completes an orbit in 365.256 days in a full sidereal year, travelling a distance of 940 million kilometers. The lunar calendar gives the monthly cycles of the phases of the Moon. There are 12 lunar cycles in a year.

Ancient Hindus in India reckoned from the observations of the night sky, the celestial Solar system, long before the idea was carried by travellers to Europe and to the Western world. Today, all Arrival and Departure times of Flights, Trains and Buses are given in the 24-hours-a-day mode, as well as in the forenoon A.M. and afternoon P.M. mode.

Sanskrit is the sacred language of Sanatana Dharma, called by the British as an endonym to refer to Hinduism, the language of Hindu Philosophy — referring to the eternal truth and teachings of Hinduism. It is an Indo-Aryan language and Vedic Sanskrit belongs to 1500 – 600 BCE, while classical Sanskrit belongs to 700 — 1350 BCE.

The great Epic of sage Valmiki[1], Ramayana has its 24000 Sanskrit verses in 7 Kaandas or parts – Bala kaandam, Ayodhya kaandam, Aaranya Kaandam, Kishkinda kaandam, Sundara kaandam, Yuddha kaandam and Uththara kaandam. It is remarkable that it has been preserved verbatim from the second of the four yugas, called the Tretaa Yuga, till date, in its entirity. Chronologically, the four Yugas are: Krita Yuga (1,728,000 years), Tretaa Yuga (1,296,000 years), Dvaapara Yuga (864,000 years) and the present Kali Yuga (432,000 years). An 'Itihaasa', means 'thus it happened', the narration of the events that occurred in the life time of the narrator. Of the two Itihaasas, Srimad Ramayana and Mahabharata, Valmiki Ramayana is considered as an Aadhikavya (the first literary creation). The first syllables, respectively, of the first verse, the 1001th, 2001th verse, and so on, begin with the successive syllables of the gAytri Mantra[2].

This ancient gAyatri mantra is mentioned in three other Puranas[3], *viz.,* the Shakthi Purana, the Bhagawatha Purana and the Padma Purana.

The gAyatri meter is made of 8 x 3 =24 letters/syllables. The gAyatri mantra is considered as the most important of the mantras in the Rig Veda and the analysis of the word gAyatri given by most Sanskrit scholars is "gAyantaM trAyate iti

gAyatrI" which means 'one that saves the singer (reciter) of the mantra', anywhere at anytime.

2. A few interesting occurrences of the number 24:

- Around 1600 A.D., Johannes Bayer (1572–1625) used the Greek alphabet to name the brighter stars and the Greek alphabet has the following 24 Greek letters, attributed rightly to him: Alpha (α A), Beta (β, B), Gamma (γ, Γ), Delta (δ, Δ), Epsilon (ε, E), Zeta (ζ, Z), Eta (η, I), Theta (θ, Θ), Iota (i, I), Kappa (κ, K), Lambda (λ, Λ), Mu (μ, M), Nu (ν, N), Xi (ψ, Ψ), Omicron (o, O), Pi (π, Π), Rho (ρ, P), Sigma (σ, Σ), Tau (τ, T), Upsilon (υ, Y), Phi (ϕ, Φ), Chi (χ, X), Psi (ψ, Ψ), Omega (ω, Ω). This Greek alphabet was the model on which other European alphabets were created, including that of English. These symbols are extensively used in mathematical equations.

- The human spine is a vertebral column, a bony tubular structure through which the vegas nerve is threaded. The number of vertebrae which govern movement in the neck are, the cervical (or, the neck, 7), the thoracic (or, middle of the back, 12) and the lumbar (or, the lower back, 5) regions, totaling 24 – out of the 33 stacked bones that are interlocked to form the spine.

- The Musical instrument in the hands of the goddess of learning Saraswati, evolved in Thanjavur, Tamizhnadu, during the reign (1600 - 1634) of the king Raghunatha Naik. It has 24 fixed, brass or bell-metal, frets under four parallel strings, each of length about 1.2m. This musical instrument is aptly referred to as the Saraswati Veena (or Vina) – an ancient ($\sim$1700 A.D.) South Indian stringed instrument, consisting of a large resonator 'kudam' carved out of a jackfruit wooden log. There are many variants of this beautiful vibrating instrument of the lute family, such as the Rudra veena, the Vichitra veena, etc.

- The number of spokes in the wheel at the center of the National Flag of India is 24 corresponding to the 24 cardinal principles of virtue.

- The standardized frame rate for commercial sound film is 24 frames per second. Jean Luc Godard (1930–2022), film maker said, "Photography is truth. And cinema is truth twenty four times a second", referring to the fact that most film and TV shows are shot and viewed at 24 frames per second. The 35mm movie projector, the medium of light and sound, runs at a speed of 24 frames a second.

- The number of symmetries of the ordinary Cube is 24 – ref. Budden[5].

- The number of solutions found by Ernst Eduard Kummer[6] (1810 - 1893) for the second order Ordinary Differential Equation, discovered in 1812 by Carl Friedrich Gauss[7] (1777 - 1855), is 24 and these have been put in one-to-one correspondence, in recent times, with the symmetries of the cube, by one of the present authors (KSR) and his collaborators, Lievens and Vander Jeugt[8].

- Mathematically, the number 24 is, according to the definition given by Srinivasa Ramanujan[4], a Highly Composite Number – a positive integer with more divisors than any preceding positive integer. The divisors of 24 are eight – $1, 2, 3, 4, 6, 8, 12$ **and** 24. Thus, $d(24) = 8$, while the number 23 is a prime number with only two divisors, $d(23) = 2$, and the number 25 is a square number 25 with three divisors $1, 5, 25,\ d(25) = 3$.

- The Ramanujan tau-function, $\tau(n)$, definition[9] involves the 24-th power of an infinite product function: $x[(1-x)(1-x^2)(1-x)^3\cdots]^{24}$.

- The 'astonishing theorem' of Hardy and Ramanujan[10] involves a certain 24-th q-th root of unity.

The reader may explore to find such occurrences in Science and in Nature!

We also have:
– The great Epic of Valmiki, Ramayanam (or, Ramayana) has its 24000 Slokas with the first letter of the first verse, followed by those of the 1001-th, 2001-th, ..., 23001-th being the 24 syllables of the gAyatri mantra and this gAyatri mantra is mentioned in three other Puranas, *viz.* the Shakthi Purana, the Bhagawat Purana and the Padma Purana.

– The gAyatri meter is made of $8 \times 3 = 24$ syllables. The ancient gAyatri mantra is considered as the most important of the mantras in the Vedas and the analysis of the word given by most Sanskrit scholars is "gAyantaM trAyate iti gAyatrI" which means 'one that saves the singer (reciter) of the mantra', anywhere at anytime.

3. gAyatri Mantra:

This is a highly revered mantra from the Rig Veda (Mandala 3.62.10) and is in praise of the Goddess gAyatri, the Goddess of the Vedic meter.

This is a highly revered mantra from the Rig Veda (Mandala 3.62.10) and is in praise of the Goddess gAyatri, the Goddess of the Vedic meter. Traditionally the recitation of the gAyatri Mantra is preceded by OM the sound of the sacred spiritual symbol in Indic religions. It is found in not only ancinet manuscripts, but the icon for Omkara can also be found in spiritual retreats of Hinduism, Budhism, Jainism and Sikhism. The recommended yogic posture is to sit cross legged on the floor, with a straight back, to recite the Gayatri mantra, which has 24 syllables. They are:

1. tat, 2. sa, 3. vi, 4. tur, 5. va, 6. re, 7. ni, 8. yam,
9. bhar, 10. go, 11. de, 12. va, 13. sya, 14. dhī, 15. ma, 16. hi,
17. dhi, 18. yo, 19. jo, 20. nah, 21. pra, 22. Cho, 23. da, 24. yAt.

The Gayatri Mantra is popular in all well-known Hindu texts, dating back to the Bhagavad Gita in Mahabharata, Harivamsa and Manusmrti.
The Royal Sage (Rajarishi / King-Saint) Vishvamitra, an author of 'the Mandala 3 of the Rigveda, including the Gayatri Mantra', is mentioned in the Puranas to be the first of the 24 Rishis (Yajnavalkya, the last) who understood the entire meaning and power of the Gayatri Mantra. King Viswamitra renounced his Kingdom and did penance in the Himalayas for ages to become one of the seven Brahmarishis (Angiras, Atri, Bhrigu, Kashyapa, Shandilya, Vasishta and Vishwamitra – the enlightened rishis or seers / sages), who had the ultimate infinite knowledge, including self-knowledge, Brahmajnana.
A Brahmarishi on his death, frees himself from the eternal birth-death cycle and is considered to be on par with the Devas, in power and piety. The Puranas mention that it is only these 24 Rishis who understood the whole meaning of the Gayatri Mantra and also wielded the full power of the Mantra.

4. National Flag of Bharath (INDIA):

The meanings of the 24 spokes of the Ashok Chakra are:

1. The first: Chastity	2. Second: Health
3. Third: Peace	4. Fourth: Sacrifice
5. Fifth: Morality	6. Sixth: Service
7. Seventh: Forgiveness	8. Eighth: Love
9. Ninth: Friendship	10. Tenth: Fraternity
11. Eleventh: Organization	12. Twelfth: Welfare
13. Thirteenth: Prosperity	14. Fourteenth: Industry
15. Fifteen: Safety	16. Sixteenth: Awareness
17. Seventeenth: Equality	18. Eighteenth: Artha
19. Nineteenth: Policy	20. Twentieth: Justice
21. Twenty-one: Co-operation	22. Twenty-second: Duties
23. Twenty-third: Rights	24. Twenty-Fourth: Wisdom

(Notes for the numbered spokes in the table given:

9. Friendship is in the sense of having cordial relation with all the citizens.

10. Fraternity is to develop a sense of brotherhood among the citizens of country.

11. Organization is to Strengthening the unity and integrity of the Nation.

12. Welfare refers to participation in the welfare activities related to country and society.

13. Prosperity is to actively participate in the development of the country.

14. Industries should assist the country in its industrial progress.

15. Safety is to be always ready for the protection of the frontiers of the country.

16. Awareness is to to be aware of the truth and don't believe in or spread rumours.

17. Equality is in the establishment of a society based on equality.

18. Artha is for the optimum utilization of money.

19. Policy is to have faith in the country's unifying National policies.

20. Justice is to talk about and implement justice for all.)

The following are ten interesting facts about the Ashoka Chakra in the Indian National Flag:

- On August, 1906, the first unofficial flag of India was hoisted in Green Park's Parsee Bagan Square, Kolkata. It was a tricolour flag with three horizontal strips of red, yellow and green.
- The Tricolour flag is a symbol of National pride for every citizen of the Nation.
- The Ashoka chakra features in the middle of the white portion in the tricolour.
- There are 24 spokes in the wheel, which is known as the 'Wheel of Duty'.
- 'Har Ghar Tiranga' is a campaign under the aegis of Azadi Ka Amrit Mahotsav, as India completed its seventyfive (75) years of Independence on August 15, 2022.
- The Ashok Chakra on the National flag is in the Navy Blue colour on a white background.
- A representation of the Dharma Chakra, adopted on July 22, 1947.
- Mahatma Gandhi, the Father of the Nation, commissioned Pingali Venkayya to design a flag on a red and green banner.
- Each spoke on the wheel symbolises one principle of life. The 24 principles are: love, courage, patience, self-sacrifice, truthfulness, righteousness, spiritual knowledge, morality, welfare, industry, faith and prosperity, among others.

5. The Highly Composite Number 24:

The ordinary whole numbers or integers are discrete and they are:

$$1, 2, 3, ..., n, ..., \infty,$$

the last being called as infinity. Adding or subtracting anything to infinity results in infinity only. The integers are normally divided into even and odd integers. The fundamental theorem of Algebra states that every integer can be written in terms of the powers of prime numbers:

$$N = 2^{p_1} 3^{p_2} 5^{p_3} \cdots \qquad \text{and} \qquad d(N) = (p_1 + 1)(p_2 + 1)(p_3 + 1) \cdots ,$$

where $p_1, p_2, p_3, \cdots$ are integers and $d(N)$ defines the number of divisors of N. A prime number is a number which has only two divisors, 1 and itself. The first 25 prime numbers are:

$$2, 3, 5, 7; 11, 13, 17, 19; 23, 29; 31, 37; 41, 43, 47; 53, 59; 61, 67; 71, 73, 79; 83, 89; 97.$$

The semi-colons used above are to enable one to write the formulas for the number of primes below a given number n, as $p(n)$. Clearly, it is straightforward to write down:

$$p(10) = 4; p(20) = 8; p(30) = 10; p(40) = 12; p(50) = 15; p(60) = 17;$$

$$p(70) = 19; p(80) = 22; p(90) = 24; p(100) = 25.$$

The asymptotic formula, as $n \to \infty$, was known to the legendary mathematician Carl Friedrich Gauss (1777–1867) as

$$p(n) \sim n/\log n,$$

$p(n)$ being the number of primes less than or equal to n, the prime counting function.

6. Ramanujan's definition of a highly composite number:

A number N is said to be a highly composite number, if its divisors $d(N)$ satisfy the condition:

$$d(N') < d(N), \text{for all } N' < N.$$

Ramanujan observed in his highly creative and original paper[4] on this topic:
"The number of divisors of N varies with extreme irregularity as $N \to \infty$, tending itself to infinity or remaining small according to the form of N. In this paper I prove a large number of results which add a good deal to our knowledge of the behaviour of $d(N)$."
In a Table Ramanujan classified the first 25 natural numbers into Prime, Composite and/or Highly Composite numbers. The number 24 is:

$$24 = 2^3 \cdot 3^1, \text{with } 1, 2, 3, 4, 6, 8, 12, 24 \text{ as its 8 divisors.}$$

Therefore, $d(24) = 8$.
In his second Notebook, Ramanujan listed the first 102 of the highly composite numbers and the last entry in the table is:

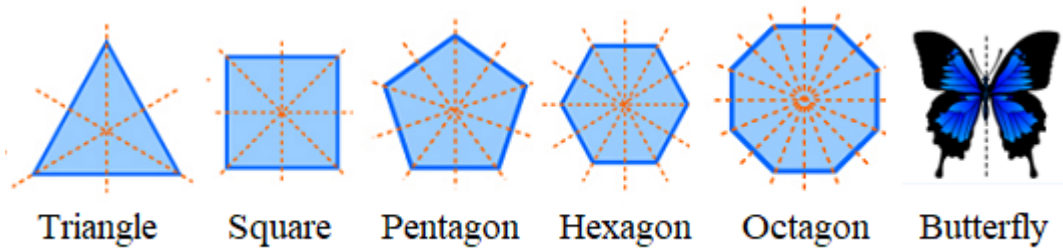
$$N = 6746328388800 = 26.34.52.72.11.13.17.19.23$$

$$d(N) = 10080 = 25.32.5.7 \text{ and } dd(N) = 72.$$

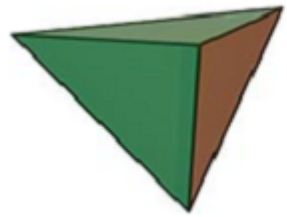
Since $d(N)$ is a large number, Ramanujan defined the divisors of the divisors of N, as $dd(N)$. Hardy considered this 69 printed pages paper as one "of an elementary but highly ingenious character", to enthuse Ramanujan who was diagnosed and treated for the then considered as highly contagious Tuberculosis, he was admitted into Sanatoria for nearly half of his five year sojourn, between 1913-1919. Ramanujan returned to India in March 1919 and despite the best medical attention to him he died at about 10:00 a.m. on April 26, 1920. However, he returned with the B.A. degree of Cambridge University - awarded for a dissertation containing this paper on Highly Composite Numbers (1915) along with half a dozen other published papers of his put together by his mentor G.H. Hardy. The Fellowship of the Royal Society (F.R.S.) of London, was awarded to him, he being the first Indian Mathematician for that coveted honour.

7. Symmetries of the cube:

Symmetry means "agreement in dimensions due proportion, arrangement". Simply stated symmetry refers to a sense of harmonious and beautiful proportion. Poet John Keats said that "a thing of beauty is a joy forever: its loveliness increases, it will never pass into nothingness". From time immemorial, symmetry in Nature has fascinated architects, artists, astronomers, mathematicians, philosophers, physicists and above all it is in the eye of the beholder, the common man! Its absence (asymmetry) is also as fascinating as its presence. For instance, it is said that some of the most famous of the Palaces (Schlosses in Germany) reveal on scrutiny, asymmetry in the number/positioning of the windows!
Mathematical symmetry means that the shape appears exactly like another when the object possessing symmetry is moved in some ways: turn orbrotate, flip or slide. Two objects are said to be symmetrical, if they have the same size and shape, with one object having a different orientation compared to the first. The following are a few examples:



The above are all regular 2-D shapes having the same number of symmetries as their sides. All of shapes above and the butterfly also have an axis of symmetry which divides the object into exactly two halves such that each half is a perfect reflection of the other. Most of the living species including the human beings have bilateral symmetry.



Regular Tetrahedron

Cube

A regular tetrahedron has 12 rotational (or orientation preserving) symmetries, and a symmetry order of 24 including transformations that combine a reflection and a rotation.
A cube has 24 symmetries: Let the Cartesian coordinate system pass through the center of the cube. Then 90^0 rotations about each of the 3 axis, give rise to 12 symmetries. About each one of the 4 block diagonals, 120^0 rotations are symmetries, giving rise to 12 more symmetries for the cube.

8. Gauss differential equation and its Kummer solutions:

The hypergeometric function is defined as the solution of the second order Ordinary Differential Equation (ODE), called as the Gauss differential equation (1812):

$$z(1-z)w'' + [c-(a+b+1)z]w' - abw = 0$$

Normally a second order ODE has two independent solutions. Gauss wrote the solution of this equation as $w = F(a,b,c,z)$ and made the important and significant observation that this is not a function of the variable z, but it has to be treated as a function of all the four variables, a, b, c, z. He also wrote down four solutions and four three-term recurrence relations satisfied by them.
It was E. E. Kummer (1810–1898) who showed that the above Gauss ODE, has one solution which in the later day notation is written as $w = {}_2F_1(a,b;c;z)$, is but one of 24 solutions (including the four written down by Gauss), all in terms of the Gauss functions with variables which are a linear combination of a, b, c, z. Kummer

wrote down a set of six different solutions of the Gauss ODE, each of which has four different forms, so that there are in all 24 solutions which may be written as $w_1, w_2, ..., w_{24}$.
The sides of the ordinary cube are labeled as: $x_1, x_2, x_3, x_4, x_5, x_6$. We note that the sides opposite to each other (in the dice) are: 1,6; 3,4; 2,5 so that the sum of the two labels is 7. The authors (KSR, et.al.) in ref.8, have shown that there exists a unique mapping of the four variables a, b, c, d onto the six variables for the six sides of the cube $x_1, x_2, x_3, x_4, x_5, x_6$. Therefore, the 24 solutions are due to the 24 symmetries of the cube (out of a possible 6! = 720 permutations of the six variables which can be assigned to the six sides of the cube). Thus, Lievens, Srinivasa Rao and Vander Jeugt (2005) showed that the Kummer solutions of the hypergeometric function ${}_2F_1(A, B; C; D)$ are related to the subset of 24 (out of the 6!) permutations of a, b, c, d, e, f. Further more there is one Master Equation from which the 24 solutions can be written down – till this discovery, all the books on Special Functions, in their chapter on Hypergeometric Equation, write down the 24 solutions $w_1, w_2, ..., w_{24}$ in two or three pages!

9. Ramanujan's tau (τ) function:

The τ-function of Ramanujan is defined by

$$g(q) = \sum_{n>0} \tau(n)\ q^n = q\{(1–q)(1–q^2)(1–q^3)...\}^{24} = q\prod_{n>0}(1-q^n)^{24}$$

where $q = \exp(2\pi iz)$ with $Im\ z > 0$. Ramanujan tabulated $\tau(n)$ up to $n = 30$ and showed that $\tau(p) \equiv 0(mod\ p)$, for every n. Ramanujan (1916) observed, but did not prove, the following three properties of $\tau(n)$:

- $\tau(mn) = \tau(m)\tau(n)$ if $gcd(mn) = 1$
  (implying that $\tau(n)$ is a multiplicative function)
- $\tau(p^{r+1}) = \tau(p)\ \tau(p^r)\ -\ p^{11}\ \tau(p^{r-1})$ for p prime and $r > 0$.
- $|\tau(p)| \leq\ 2p^{11/2}$ for all primes p.

The first two properties were proved by Mordell (1917) and the third one, renowned as Ramanujan's τ-conjecture was proved by the Belgian Algebraic Geometer and Algebraic Number Theorist, Pierre Deligne (1944 born) and finds mention in the citation for the Fields Medal awarded to him in 1978.
Ramanujan tabulated $\tau(n)$ up to $n = 30$ and showed that $\tau(p) \equiv 0(\text{mod } p)$, for every n. He was the first to observe interesting congruence properties of the function,

and also made the famous conjecture $|\tau(p)| \leq 2p^{11/2}$ for every prime number p, as stated above. Another Fields Medalist, Alte Selberg, in 1988, at an extemporaneous talk at the Tata Institute of Fundamental Research (TIFR), Mumbai, observed that a "felicitous but unproved conjecture may be of much more consequence for mathematics than the proof of many a respectable theorem." By leaving behind in his Notebooks about 4000 theorems without proofs, Ramanujan gave an opportunity to George Andrews, Bruce Berndt, Robert Rankin, K.G. Ramanathan, and many mathematicians of repute opportunities provide the proofs over decades and some are considered not the simplest possible as they make use of the work discovered long after the demise of Ramanujan! Thus, Ramanujan and his work continue to be a source of inspiration to generations of mathematicians even in the present century!

10. Astonishing theorem of Hardy-Ramanujan:

While only the asymptotic formula for $p(n)$ was known to Gauss, the number of unrestricted partitions of n, Hardy and Ramanujan showed that $p(n)$ can be calculated to any value of n, the integer nearest to:

$$\frac{1}{\sqrt{2}} \sum_{q=1}^{\nu} \sqrt{q} A_q(n) \psi_q(n),$$

where

$$A_q(n) = \sum \omega_{p,q} \exp\{(-2np\pi i)/q\},$$

the sum being over all p prime to q and less than it, $\omega_{p,q}$ is the 24-th q-th root of unity, ν is of order $\sqrt{n}$ and

$$\Psi_q(n) = \frac{d}{dn}\left[\exp\left\{\frac{\pi}{q}\sqrt{\frac{2}{3}\left(n - \frac{1}{24}\right)}\right\}\right].$$

Taking $\nu = 5,$ for $n = 200$, Hardy-Ramanujan predicted the value,

$$p(200) = 3972999029388$$

which was computed by Major MacMahon, without the use of computers, to be correct. Littlewood opined that the collaboration between the two men of "quite unlike gifts, in which each contributed the best, most characteristic, and most fortunate work that was in him."

The author wishes to thank Dr. Christian Krattenthaler who states that the reason for the appearance of 24 around the tau-function and in the Hardy-Ramanujan

formula is the same: "both cases feature the infinite product: $(1-q)(1-q^2)(1-q^3)\cdots$. For the same reason, there is another (famous) appearance of the number 24, *viz.*, in Ramanujan's partition congruences modulo powers of 5, 7, and 11. Ramanujan conjectured them but did not prove them, and his conjecture for the powers of 7 was not quite correct – refer to the paper of Garvan[1]. More generally, when you are in the world of modular forms/functions then frequently the number 24 will appear, again for the same reason".

In fine, in this article we have pointed out that the number 24 occurs, surprisingly, not only in Vedic texts but also in some of the modern mathematical formulas, prompting the observation that this is the *raison d'être* for Ramanujan's choice of the 24-th power in $g(x)$, which defines his τ-function!

Acknowledgement

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Appendix: Gayatri Ramayanam

Unlike what is suggested by the title, this is not the story of Rama. All the twenty four slokas of this stotra have been taken from Valmiki Ramayanam. They are not taken in an order but in such a way the first letters of the 24 slokas form the Gayathri mantra. It signifies that Ramayana is the essence of the entire Vedas and Sri Rama is Narayana. Listening and chanting of Sri Gayathri Ramayanam bestows peace and bliss.

[1]https://www.ams.org/journals/tran/1988-305-01/S0002-9947-1988 -0920146-8/S0002-9947-1988-0920146-8.pdf

1.	***Ta***paswadhyaya niratham, ***Th***apaswee vag vidhamvaram, Narada pariprucha, Valmikir muni pungavam	The great sage Valmiki asked Narada Who does penance and reading of Vedas And who himself was a great sage, As well as an expert over words.
2.	***Sa*** hathwaa Rakshasaan sarvaan, Yagnagnaan Raghu nandana, Rishibhi poojithasthathra, Yadendro vijaye puraa.	That son of the clan of Raghu, By killing all the Rakshasas, And protecting the sacrificial fires Was worshipped by the sages.
3.	***Vi***swamitharasthu dharmathma, Sruthwa janaka bhashitham, Vathsa Rama, dhanu pasya, Ithi Raghabamabraveeth.	Viswamithra, the personification of dharma After hearing the words spoken by Janaka, told: "Child Rama, please see this bow." This he told Raghava.
4.	***Thu***shtyavaasya thadhaa vamsam, Pravisya sa visaampathe, Sathaneeyam Narendrasya, Thadasadhya vyathishtitha.	Then he reached the bedroom of the King, went near and standing outside, praised and narrated the detailed pedigree of the king.
5.	***Va***navasam hi sankhyaya, vasamsyabharanani cha, Bhartharamanugachanthyai, Seethayai swasuro dhadhou.	Based on the years that Sita had to live in the forest, when she would accompany her husband, her father-in-law gave her sufficient dresses and ornaments.
6.	***Ra***ja sathyam cha dharmam cha, Raja kulavatham kulam, Raja Matha pitha chaiva, Raja hithakaro nrunaam.	The king is Truth and Dharma. He is the Lord of the people of good families. He is the Father as well as the Mother. And he is the one who does good to the people.
7.	***Ni***reekshya sanmuhurtham thu, Dadarsa bharatho Gurum, Utaje Ramamaseenam, Jata valkala dharinam.	After waiting for an auspicious time, Bharatha saw his teacher Rama, Who was having matted hair and was, Wearing cloth made of bark.
8.	***Ya***dhi budhi krutha drushtum, Magasthyam tham mahamunim, Adviava gamane budhim, Rochayaswa Mahasaya.	Oh great one! If you are desirous of seeing the great sage Agasthya, take the decision to start to do so.

9. ***Bhar***athasya aaryaputhrasya,
Swasroonam mama cha prabho,
Mruga roopamidham vyaktham,
Vismayam janayishyathi.

Oh Lord, this deer which is very pretty,
would create a great sense of wonder,
to Bharatha, you and my in-laws.

10. ***Ga***cha seegramitho Rama,
Sugreevam tham Mahabalam,
Vayasyam tham kuru kshipra,
Mitho gathwadhya Raghava.

Oh Rama, go immediately from here,
to the very strong Sugreeva. And,
Oh Raghava, make him your intimate
friend.

11. ***De***sa kalou Pratheekshaswa,
Kshama mana priyaa priye,
Sukha dukha saha kale,
Sugreeva vasago bhavam.

Understanding the time and place,
being patient for likes and dislikes,
treating joy and sorrow as equal,
become obedient to Sugreeva.

12. ***Va***ndhyasthe Thapasa sidhaa,
Sthapasaa veethakalmasha,
Prashtavyachapi seethaaya,
Pravruthi vinayanvithou.

The Sidhaas who do penance are fit to be
saluted, for they are devoid of sins due
to their penance. You may enquire humbly
about the whereabouts of Sita from them.

13. ***Sa***nirjithya purim sreshtaam,
Lankaam tham Kamaroopinim,
Vikramena maha thejo,
Hanuman maruthathmaja.

Hanuman, the son of Wind God, who is
valourous and shining, won over Lanka,
the town's goddess, who can assume
any form she wishes.

14. ***Dh***anyodhayaa sa gandharwa,
Sidhascha paramarshya,
Mama pasyanthi ye nadham,
Ramam rajeeva lochanam.

I see Rama who is my Lord, who has eyes
as pretty as lotuses, is seen by the
Gandharvas, Siddhas and Saints, as if it
is the rise of their Luck.

15. ***Ma***ngalabhimukhi thasya,
Saa thadassn maha kape,
Upathasthe Visalakshi.
Prayathaa havya vahanam.

That lady with an auspicious face, deciding
to Bless the great Monkey, saluted and
prayed to the God of Fire, after making
herself pure.

16. ***Hi***tham mahartham mruduhethu
sammitham, Vyatheetha
kalayutha samrathikshamam,
Nisamya thadvakya
mupasthitha jwara, Prasanga
vanuthara methad abhraveeth.

After hearing those words which were
meant
for good, which would give great results,
which were sweet, logical and suitable for
the past, present and the future.

With very great anger that Ravana replied thus.

17. ***Dh***armathma Rakshasa sreshta,
Samprapthoyam Vibheeshana,
Langaiswaryam dhruvam sreema-,
Nayam prapthothya gandakam.

That great Rakshasa, a personification of Dharma, called Vibheeshana went and joined me, undoubtely, with certainty, he would attain the wealth of Lanka.

18. ***Yo***vajrapathaasani
sannipathavajra -N na chukshubhe
napi chachala Raja, ayudha,
Sa Ramabhanabhi hatho
brusatha, Schchala
chapancha mumocha

That King who never bothered by the hit of that thunder is now hit by Rama's arrows, and that hero becomes sad by the shock; became nervous, and trembling, veera and his bow slipped from his hand.

19. ***Ya***sya Vikrama masadhya,
Rakshasa nidhanam gathaa,
Tham manye Raghavam veeram,
Narayanamana matam.

Caught by his fame, many Raakshasaas died. And I consider that the valourous, Rama, is the real God Narayana.

20. ***Na*** ye dhadru sire Rama,
Chindanda mari vahineem,
Mohithaa paramasthrena,
Gandharvena mahathmana.

The Raakshasaas were unable to see Rama who was burning the army of enemies. They lost their senses by the great arrows, called Gandharwa sent by great Rama.

21. ***Pr***anamya devadhabyascha,
Brahmanebhascha Mythili,
Badanjali putaa chedha-

After offering her saluations, to Brahamans and Devas, Mythili with saluting hands went near the fire, said: Muvachagni sameepatha.

22. ***Ch***alanath parvathendrasya,
Ganam devascha kambitha,
Chachala Parvathi chapi,
Thadaslishta Maheswaram.

When the King of the Mountains moved the Ganaas of Shiva, and the Devas trembled. Even the Goddess Parvathi trembled, and Rushed and embraced the great Maheswara.

23. ***Da***raa puthra puram Rashtram,
Bhogachadana bhajanam,
Sarva mevapi bhakthanno,
Bhavishyati Hareaswara.

Oh Lord of the Monkeys, from today, let the fields, children, towns, countries, luxuries, meals and the fields, become our common property and be shared equally among us.

24. ***Ya***meva rathrimn Shathrugna,
Parna salaam samavisad,
Thameva rathrim Seethapi,
Prasoothaa darakadwayam.

On the night when Shatrughna, stayed in the hermitage, and on the same night, Sita, gave birth to two sons.

Minor changes in the text of the translation by Mr. P.R. Ramachander – who deserves the full credit as the TRANSLATOR of the 24 verses – were made! One of us (KSR) is thankful for the permission granted for his translation of the Gayatri Ramayanam to be reproduced here.
Note: The first syllable in each of the 24 verses is underlined and italicised, as these 24 verses constitute the Gayathri Ramayanam!

References

[1] Valmiki Ramayana: https://www.amazon.in/Ramayana-C-Rajagopalachari -ebook /dp/B00EEE499Q

[2] Gāyatrī Mantra is also known as the Sāvitri Mantra, a highly revered mantra from the Rig Veda, Mandala 3.62.10. Gayatri is the name of the Goddess of the Vedic metre in which it is composed. See also, Amit Ray, "Gayatri Mantra Meditation 24 Syllables Meanings", //amitray.com

[3] The Puranas, A compact English-only version of the 18 Puranas in one document, Issue 1, Draft 1, compiled by the Dharmic Scriptures Team, October 3, 2002.

[4] Ramanujan S., Highly Composite Numbers, Proc. London Math. Soc., Vol.2 (1915) 347 – 409. See also, "Srinivasa Ramanujan: a natural mathematical genius, Swayambhu", K. Srinivasa Rao (Springer, 2021).

[5] Budden F. J., The fascination of groups, Cambridge University Press (1972).

[6] E. E., Uber hypergeometrische Reihe $F(\alpha, \beta, x)$, J. fur Math. 15 (1836) 39–83 and 127–172.

[7] Gauss Carl Friedrich: Titan of Science', Waldo G. Dunnington, Hafner Publishing, New York (1955), reprinted by the Mathematical Association of America (2004).

[8] Lievens S., Rao K. Srinivasa and J. Vander Jeugt, Integral Transforms and Special Functions, Vol. 16 (2005), 153-158.

[9] Tau-function defn. given in ref. 4 above, p.88. For the interested reader: see (1.4) – (1.6), in the article of F.G. Garvan, Trans. of the AMS, Vol. 305 (1988) – https: //www.ams.org/journals/tran/1988-305-01/S0002-9947-1988-0920146-8/S0002-9947-1988-0920146-8.pdf.

[10] Hardy-Ramanujan, An astonishing theorem, in ref.4, Srinivasa Rao, 73-74.

[11] The Rig Veda is one of the four Vedas – Rig, Yajur, Saama, Atharva. It has 1028 hymns, organized in ten mandalas (or books), in praise of the Gods. It is perhaps the oldest text in Sanskrit (dating to a period between 1500 and 1200 BCE).

[12] See ref. 4, Highly Composite Numbers, pp. 62 – 65.

[13] Notebooks of Srinivasa Ramanujan, (facsimile edition) 2 volumes, Tata Institute of Fundamental Research, Bombay, 1957; Narosa, New Delhi, 1987. Reprinted in 2012 by the National Board for Higher Mathematics (NBHM).

[14] See ref. 4, The beginning of the end, 30-31.

[15] Garvan P., New Combinatorial Interpretations of Ramanujan's Partition Congruences Mod 5, 7 and 11, in Transactions of the American Mathematical Society, Vol. 305, Number 1 (1988), 47-77.