*J. of Ramanujan Society of Mathematics and Mathematical Sciences*
*Vol. 10, No. 1 (2022), pp. 167-172*

DOI: 10.56827/JRSMMS.2022.1001.14 **ISSN (Online): 2582-5461**

**ISSN (Print): 2319-1023**

# DIGITAL TIME: A FINITE FIELD, $T_{\mathbb{F}}$

**G. Manikandan and K. Srinivasa Rao***

Department of Mathematics,
St. Joseph's Institute of Technology,
OMR, Chennai - 600119, Tamil Nadu, INDIA

E-mail : manispark18@gmail.com

*Institute of Mathematical Sciences,
C.I.T Campus, Taramani, Chennai - 600113, INDIA
& Director (Hon.), Srinivasa Ramanujan Academy of Maths Talent,
90/1, Second Main Road, Gandhi Nagar, Adyar, Chennai - 600020, INDIA

E-mail : ksrao18@gmail.com

(**Received:** *Aug. 19, 2022* **Accepted:** *Sep. 15, 2022* **Published:** *Dec. 30, 2022)*

**Abstract:** Digital time was defined KSR-PP [2] with three two-digit positions as $h_2h_1 : m_2m_1 : s_2s_1$. It was identified with appropriate restricted place values on the hours ($H$), minutes ($M$) and seconds ($S$) shown to be 86400-element cyclic Time Group, $T_G$. Here it is shown to be a finite time field, $T_{\mathbb{F}}$. A palindromic sequence of 119-elements and its sub-sequences are shown to be consequences of $T_F$.

**Keywords and Phrases:** Digital Time, Finite Field, Order of Elements, Palindromic Sequence.

**2020 Mathematics Subject Classification:** 05C25, 20F65.

## 1. Introduction and Definitions

Time flows smoothly as it is a continuous real variable. Precision in digital time measurement has been crucial in sophisticated space research and in sports, to proclaim olympic world records. Measurement of time using watches has been a part of a way of life for ages now. The digital Time Group, $T_G$, is indeed shown here to be a finite field, $T_{\mathbb{F}}$. A palindromic sequences are derived, from the first

differences of a subset of elements, of the finite field.
Wrist watches and wall clocks with digital dials are more common than their analogue counterparts with Arabic/Roman numerals. In vogue are watches with 12-hour dials. Digital time became a necessity to specify, unambiguously, the precise time for the arrival and departure of the trains and planes, without the common *A.M.* and *P.M.* suffixes, invented to suit the 12-hour dials in wrist watches and clocks. Let a given instant of time, specified by its hour, minute, second positions be denoted as $H : M : S$ or more precisely as $h_2h_1 : m_2m_1 : s_2s_1$, with the restrictions on the digital values being:

$$0 \le h_2 \le 2,\ 0 \le h_1 \le 3,\ 0 \le m_2 \le 5,\ 0 \le m_1 \le 9,\ 0 \le s_2 \le 5,\ 0 \le s_1 \le 9$$

for the sequence of six digits (from $L$ to $R$): $h_2, h_1, m_2, m_1, s_2, s_1$.
By definition, the first and last elements of $T_{\mathbb{F}}$, correspond to the times $00 : 00 : 00$ and $23 : 59 : 59$. Let these be denoted by $\mathbb{F}_0$ and $\mathbb{F}_N$ where $N = 86399$. Any element $\mathbb{F}_k$ which corresponds to a specific time, say, $h_2h_1 : m_2m_1 : s_2s_1$, is the $k^{th}$ element of the finite field $\mathbb{F}$, where

$$k = 3600 \times H + 60 \times M + S \equiv 3600 \times h_1h_2 + 60 \times m_1m_2 + s_1s_2.$$

For example, 12 Noon is $12 : 00 : 00$ and it is the element with index

$$k = 3600 \times 12 + 60 \times 00 + 60 \times 00 = 43200.$$

A day starts with time specified by $00 : 00 : 00$ and the end of the 24-hour day is specified by the time $23 : 59 : 59$. The $H,\ M,\ S$ places take 24, 60, 60 values, respectively, and therefore the number of elements of the finite field $T_{\mathbb{F}}$ is their product: $24 \times 60 \times 60 = 86400$.

**Finite Field $\mathbb{F}$:** A non-empty set $\mathbb{F}$ with two compositions, addition (+) and multiplication (.), is said to be a finite field, if

1. $(\mathbb{F}, +)$ is an additive commutative group,
   (a) for all $x, y \in \mathbb{F}$, $x + y \in \mathbb{F}$
   (b) for all $x, y, z \in \mathbb{F}$, $x + (y + z) = (x + y) + z$
   (c) for all $x \in \mathbb{F}$, $\exists\ 0 \in \mathbb{F}$ such that $x + 0 = x = 0 + x$, where 0 is the additive identity
   (d) for each $x \in \mathbb{F}$, $\exists\ (-x) \in \mathbb{F}$ such that $x + (-x) = 0 = (-x) + x$
   (e) for all $x, y \in \mathbb{F}$ $x + y \ = \ y + x$

2. $(\mathbb{F}^*, .)$ is a multiplicative commutative group, $(\mathbb{F}^* = \mathbb{F} - \{0\})$

   (a) for all $x, y \in \mathbb{F}$, $x.y \in \mathbb{F}$

   (b) for all $x, y, z \in \mathbb{F}$, $x.(y.z) = (x.y).z$

   (c) there exists an element $I \in \mathbb{F}$, called identity element in $\mathbb{F}$, such that $x.1 = x = 1.x \;\; \forall\, x \in \mathbb{F}$

   (d) for each non zero $x \in \mathbb{F}$, there exist unique $x^{-1}$ in $\mathbb{F}$ such that $x.x^{-1} = I = x^{-1}.x$

   (e) for any $x, y \in \mathbb{F}$, $x.y = y.x$

3. The two distributive laws which hold are:

   (a) $x.(y + z) = (x.y) + (x.z) \;\forall\; x, y, z \in \mathbb{F}$

   (b) $(x + y).z = (x.z) + (y.z) \;\forall\; x, y, z \in \mathbb{F}$

**2. $T_{\mathbb{F}}$ is a Finite Field**

(i) The sum of any two time elements of this finite field is also an element belonging to $T_{\mathbb{F}}$. The addition of any two elements is governed by the modular nature of the $H$, $M$, $S$ positions, which are *mod* 24, *mod* 60 and *mod* 60, respectively. (Since $H \equiv h_2h_1 \; mod \; 24$, $M \equiv m_2m_1 \; mod \; 60$, $S \equiv s_2s_1 \; mod \; 60$, with restrictions on the domains as pointed out earlier.)

(ii) The associativity and distributivity properties are also satisfied under the special condition for modular addition in the hours, minutes and seconds positions of $H : M : S$, as can be trivially verified. For example, to prove associativity,

$$\text{let } t_1 = 12:37:56,\; t_2 = 14:56:29,\; \text{and } t_3 = 09:38:41$$

$$\text{Then, by modular addition: } t_1 + t_2 = 12:37:56 + 14:56:29 = 03:34:25$$

$$\text{and } (t_1 + t_2) + t_3 = 03:34:25 + 09:38:41 = 13:13:06$$

$$\text{Similarly } t_1 + (t_2 + t_3) = 12:37:56 + 00:35:10 = 13:13:06.$$

(iii) $00:00:00$ is the identity element.

(iv) The inverse of a given element is defined as follows: Since $\mathbb{F}_0$ is the first element corresponding to 00 : 00 : 00 and $\mathbb{F}_N$ the last element corresponding to the 23 : 59 : 59. If $t = H : M : S$, then the inverse of the element is $t^{-1} = (23 - H) : (59 - M) : (60 - S)$. For example, if $t = 12 : 34 : 56$ is the $45296^{th}$ element, by definition, and its inverse is: $t^{-1} = 11 : 25 : 04$, the $41104^{th}$ element of $T_{\mathbb{F}}$ and their sum is 24 : 00 : 00, which by modular addition is the identity 00 : 00 : 00, and the index of the inverse of the $k^{th}$ element is given by:

$$(86400 - k) \ mod \ 86400, \ \text{for } 0 \leq k \leq 86399.$$

This is true for all $H : M : S$. The two elements 00 : 00 : 00 and 12 : 00 : 00 are self-inverses[1].

(v) Commutative property is satisfied under addition modulo operation.

(vi) The multiplication of any two time elements of this finite field is also an element belonging to $T_{\mathbb{F}}$. The multiplication of any two elements is governed by the modular nature of the $H, \ M, \ S$ positions, which are *mod* 24, *mod* 60 and *mod* 60, respectively.

(vii) The associativity properties are also satisfied under the special condition for modular multiplication in the hours, minutes and seconds positions of $H : M : S$, as can be trivially verified. For example, to prove associativity,

$$\text{let } t_1 = 12 : 37 : 56, \ t_2 = 14 : 56 : 29, \ \text{and } t_3 = 09 : 38 : 41, \ \text{then}$$

by modular multiplication:

$$t_1 \times t_2 = 12 : 37 : 56 \ \times \ 14 : 56 : 29 = 10 : 59 : 04 \text{ and}$$

$$(t_1 \times t_2) \times t_3 = 10 : 59 : 04 \ \times \ 09 : 38 : 41 = 07 : 24 : 44$$

$$\text{Similarly } t_1 \times (t_2 \times t_3) = 12 : 37 : 56 \ \times \ 11 : 47 : 49 = 07 : 24 : 44.$$

(viii) It is necessary to state that 00 : 00 : 00 is the additive identity element (for the digital time group $T_G$) and 00 : 00 : 01 is the multiplicative identity element for the finite field $T_{\mathbb{F}}$. For example, if $t = 12 : 34 : 56$ is the $45296^{th}$ element, by definition, and its identity element is: 00 : 00 : 01, the $2^{nd}$ element of $T_{\mathbb{F}}$

[1]The 24-hour day is represented on a 12-hour dial on ordinary analogue watches. This results in a specific time belonging to the forenoon and afternoon being designated as *A.M.* and *P.M.*

and their multiplication is 12 : 34 : 56, which by modular multiplication is the identity 00 : 00 : 01 of $H : M : S$. Hence 1 hour, 1 minute and 1 second are the identity elements of $H$, $M$ and $S$ respectively. That is 00 : 00 : 01 is the identity element.

(ix) The inverse of a given element is defined as follows: Since $\mathbb{F}_0$ is the first element corresponding to 00 : 00 : 00 and $\mathbb{F}_N$ the last element corresponding to the 23 : 59 : 59. If $t = H : M : S$, then the inverse of the element $H : M : S$ is $t^{-1} = H(mod\ 24) : M(mod\ 60) : S(mod\ 60)$. For example, if $t = 12 : 34 : 53$ is the $45293^{th}$ element, by definition, and its inverse is: $t^{-1} = 16 : 59 : 17$, the $61157^{th}$ element of $T_{\mathbb{F}}$ and their multiplicative modulo is $61157 \times 45293 \equiv 1\ (mod\ 86400)$. inverse of the $k^{th}$ element is given by:

$$t \times t^{-1} \equiv 1 \mod n.$$

(x) It is important to note that in modular arithmetic, $a^{-1}$ does not mean $1/a$. Not all numbers have a multiplicative inverse *modulo* $n$. In general, a number will only have an inverse if it does not share any common factors with the *modulus* $n$ (apart from the common factor 1, $\gcd(a, b) = 1$). Since 26 has the factors 2 and 13, this means that even numbers, and the number 13, do not have an *inverse modulo* 26.

(xi) Commutative property is satisfied under multiplication modulo operation.

(xii) The distributivity property is also satisfied under the special condition for modular multiplication in the hours, minutes and seconds positions of $H : M : S$, as can be trivially verified. To prove left distributivity,

$$t_1 \times (t_2 + t_3) = (t_1 \times t_2) + (t_1 \times t_3),$$

$$\text{let } t_1 = 12 : 34 : 56, t_2 = 01 : 02 : 03, t_3 = 04 : 05 : 06, \text{ then}$$

$$t_1 \times (t_2 + t_3) = (12 : 34 : 56) \times (05 : 07 : 09) = (16 : 06 : 24) \text{ and}$$

$$t_1 \times t_2 + t_1 \times t_3 = (12 : 34 : 56) \times (01 : 02 : 03) + (12 : 34 : 56) \times (04 : 05 : 06)$$

$$t_1 \times t_2 + t_1 \times t_3 = (13 : 14 : 48) + (02 + 55 : 36) = (16 : 06 : 24),$$

by the definition of modular addition. Similar proof for right distributive law.

**Note:** If $G$ is a finite subgroup of the multiplicative group $\mathbb{F}^*$ of a field $\mathbb{F}$, then $G$ is cyclic (In Grove's book [1], Proposition 3.7 at page 94).

**Acknowledgement**

We wish to thank Mr. S. Thiagarajan, former Registrar of SETS-Chennai and former General Manager of ECIL, for bringing us into contact with each other.

## References

[1] Larry C. Grove., Algerba, Courier Corporation, (2012).

[2] Srinivasa Rao K. and Pankaj Pundir., The ubiquitous digital time group, South East Asian J.Math. Sci., 13 (2017), 01-10.