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ACCELERATING COSMOLOGICAL MODELS WITH VARIABLE G AND Λ-TERM IN GENERAL RELATIVITY

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Abstract: In this paper, we have presented a new class of accelerating universe models with variable cosmological term $\Lambda(t)$ and gravitational constant $G(t)$ in the framework of general relativity. To get exact solution of Einstein's field equations for homogeneous and anisotropic Bianchi type-V space-time, a time varying deceleration parameters is considered as $q = -1 + \frac{n\alpha}{(\alpha+t)^2}$, where n, α are constants. The present model shows a point type singularity at origin. The results establish the quintessence like behavior of model initially, and approaches to ΛCDM model ultimately. Some geometrical and physical properties of the models have been evidenced, and conferred to derive the validity of models with respect to recent astrophysical observations. Stability of the model has been discussed through the means of $Om(z)$ diagnostic and state-finder analysis.

Keywords and Phrases: Bianchi-V universe, ΛCDM Model, Statefinders, Variable DP.

2020 Mathematics Subject Classification: 83C05, 83D05, 83F05.

1. Introduction

Experimental observations like Ia supernovae (SN Ia) observations [41, 46] have confirmed the accelerated expansion of the universe. The dark energy is assumed as a candidate responsible for the accelerated expansion of universe. Observational data [1, 6, 40] specifies that the universe is almost spatially containing 95% energy/matter in the form of dark energy/matter. In the presence of the interaction with matter or radiation, a solution with a variable Λ is obtained while in the absence of interaction Λ remains constant. This demands a energy conservation by the decrease in the energy density of vacuum and an increase in the energy density of matter or radiation [16, 35, 36, 39, 50, 54, 60].

Among many possible alternatives, the simplest and theoretically interesting possibility of dark energy is the energy density stored on the vacuum state of all existing fields in the universe i.e., $\rho_\nu = \frac{\Lambda}{8\pi G}$. The Variable cosmological constant $\Lambda(t)$ becomes the main argument in theories of modern cosmology as it solves the problems of dark energy constant in natural way [24, 30, 31, 35, 36, 51, 54]. The ΛCDM model is a type of standard big-bang hot model that evolves as the best fit to experimental observations. According to these models GR assumptions are valid for large scale cosmological matter and assumes homogeneous distribution. The major issue in this approach is to determine the appropriate dependence of Λ on a scale factor. Motivated with the dimensional ground of quantum cosmology, Chen and Wu [20] consider a variable cosmological constant as $\Lambda \propto a^{-2}$. Several anataz have been proposed in literature showing Λ as decreasing function of time [24, 54]. Various authors have studied the variable cosmological constant in different contexts [7, 8, 20, 24, 30, 31, 35, 36, 51, 54].

The dimensionless parameters r, s termed as state-finder diagnostics are proposed to validate the stability of the model [2, 7, 8, 20, 51]. The parameters are derived by using scale factor and its time derivatives. Previously, various authors have discussed parameters of state-finders in their work [9, 17, 27, 38, 64]. ΛCDM has remarkably verify various experimental data. In general relativity, the Bianchi identities for the Einsteins tensor G_{ij} and the vanishing covariant divergence of the energy momentum tensor T_{ij} together with imply that the cosmological term is constant.

The recent experimental observations do not confirms the Equivalence Principle of general relativity with fundamental constant. Dirac [22, 23] suggests the variable gravitational constant G, and used in modifications of GR. Various tests are developed to get a clear picture of cosmologies with variable G as discussed by Canuto et al. [14, 15, 45]. Few of fundamental work in this direction of cosmological models with variable G in literature may be found in [4, 5, 25]. Later Hubble diagram of Type I_a Supernovae also testify the dynamic gravitational constant [25]. Other models are also developed in the sequences to understand the gravity theory by considering time dependent G, c, and Λ [33, 35, 42, 53, 59]. Many authors also have discussed time varying Λ and G in different frameworks [3, 19, 47, 55, 56, 61]. Recently, many authors have discussed Bianchi type-V universe with variable G and Λ [12, 62, 63]. Bianchi type-I universe with varying G and Λ-term in GR has also been discussed by Pradhan et al. [43]. The present work consist, the study of anisotropic cosmological models with variable Λ and G in Bianchi-V universe.

The manuscript is organized as follow: The present cosmological scenario is studied discussed in section I. In section 2 we present the Bianchi type-V universe. In section three we proposed the solution of field equations. In section-4, physical and geometrical properties are discussed. The stability of model is validate through statefinders in section-5. Om diagnostics analysis is given in section-6. In scetion-7, concluding summary of present manuscript has been given.

2. The Basic Field Equations

The spatially homogenous and anisotropic Bianchi type-V space-time is consider as

$$ds^2 = dt^2 - A^2dx^2 - e^{2kx}B^2dy^2 - e^{2kx}C^2dz^2. \tag{1}$$

Here, k is constant.
Applying time-dependent G and Λ, Einstein equations are defined as

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}. \tag{2}$$

Here, $R = g^{ij}R_{ij}$ and R_{ij} are the Ricci scalar and Ricci tensor respectively, while G and Λ present the Gravitational and Cosmological constants respectively.
The stress-energy-momentum tensor T_{ij} for a perfect fluid given as

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij}, \tag{3}$$

here, ρ denotes the density of matter and p represents the pressure. u_i denotes the four-velocity vector of the fluid which satisfy the relation $u^iu_i = 1$. In the field eq (2), Λ stands for vacuum energy with energy density ρ_ν and pressure p_ν satisfying the equation of state

$$\Lambda = 8\pi G\rho_\nu = -8\pi Gp_\nu \tag{4}$$

The critical density for the case is given by

$$3H^2 = 8\pi G\rho_c \tag{5}$$

The density parameters for matter and cosmological constant are defined as

$$\Omega_m = \frac{8\pi G}{3}H^{-2}\rho \tag{6}$$

$$\Omega_\Lambda = \frac{1}{3}H^{-2}\Lambda \tag{7}$$

The above relations indicate that at $H = 0$, the parameters Ω_m and Ω_Λ have a point kind singularity [32, 44].
For co-moving coordinates system, the field equations for the metric (1), in case of (4) are

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{k^2}{A^2} = \xi, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{k^2}{A^2} = \xi, \tag{9}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{k^2}{A^2} = \xi, \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3k^2}{A^2} = \tau. \tag{11}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{12}$$

Here, $\xi = -8\pi G(t)p + \Lambda(t)$, $\tau = 8\pi G\rho + \Lambda(t)$
The covariant divergence of the (2) affords

$$\dot{\rho} + 3H\rho + 3Hp + \frac{1}{G}\dot{G}\rho + \frac{1}{8\pi G}\dot{\Lambda} = 0. \tag{13}$$

The standard energy conservation equation $T^{ij}_{;j} = 0$ leads to

$$\dot{\rho} + 3H\rho + 3Hp = 0. \tag{14}$$

Now, the Eq. (13) reduces to

$$\rho\frac{\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G} = 0. \tag{15}$$

Hubble parameter can be determined by

$$H = \frac{1}{3}(H_x + H_y + H_z), \tag{16}$$

where $H_x = \frac{\dot{A}}{A}, H_y = \frac{\dot{B}}{B}, H_z = \frac{\dot{C}}{C}$.
The anisotropy parameter and the shear scalar for the model given by

$$A_m = \frac{1}{3H^2}\left[(H_x - H)^2 + (H_y - H)^2 + (H_z - H)^2\right], \tag{17}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[(H_x)^2 + (H_y)^2 + (H_z)^2\right] - \frac{\theta^2}{6}. \tag{18}$$

3. Solution of Field Equations

From equations (8) − (12), we have five equations involving A, B, C, G, Λ, p and ρ seven unknowns, therefore two more relations required to get solution of field equations. So, we consider following two relations:

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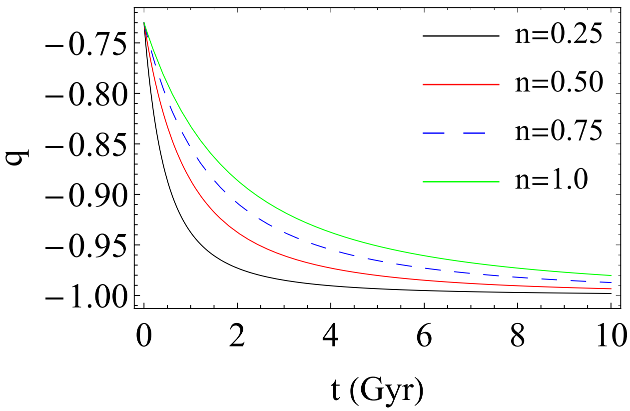

Figure 1: Variation of DP q versus t.

- We consider a power law form of gravitational parameter with scale factor as considered in [19]:
$$G = G_0 a^m \tag{19}$$
Here $G_0 > 0$ is constant and $0 < m < 2$.

- The perfect-gas equation is
$$p = \gamma\rho \tag{20}$$
Here, $0 \leq \gamma \leq 1$; ($\gamma = cons.$)

The DP (deceleration parameter) q for the model can be obtained as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) \tag{21}$$

The metric functions can be determined as the functions of cosmic time t if average scale factor is known. The choice of scale factor attracts a time dependent deceleration parameter, as shown in Eq. (40), which brings that dark energy era, the solution gives inflation and radiation/matter dominance era with subsequent transition from deceleration to acceleration. Now for a Universe, which has been decelerating in past and accelerating at the present time, the deceleration parameter must show signature flipping [37]. This theme motivates to choose such scale factor (22) that yields a time dependent deceleration parameter given by Eq. (40). Thus, to proposed a solution to derived model, we assumed a time dependent scale factor as [10, 11, 26, 48, 57].

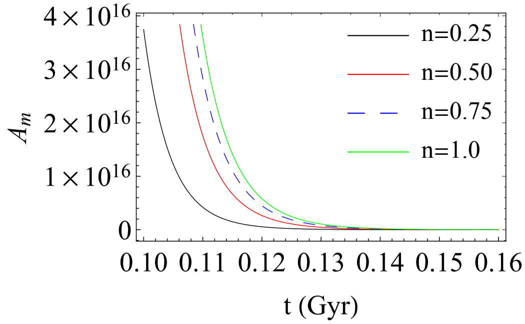

Figure 2: Variation of A_m versus t.

$$a = \left(t^{\alpha} e^{t}\right)^{1/n} \tag{22}$$

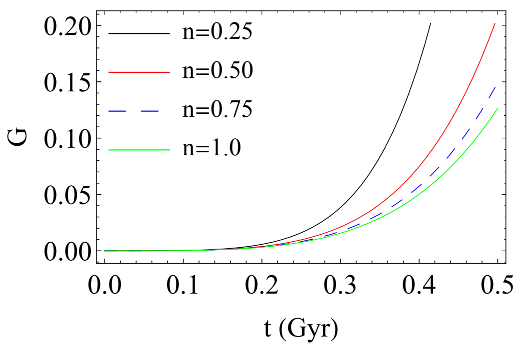

Figure 3: Variation of G versus t.

Integrating (12) and neglecting the integration constant, we get

$$A^2 = BC \tag{23}$$

Solving equations (8)-(11), using (22), we obtained

$$A = \left(t^\alpha e^t\right)^{1/n} \tag{24}$$

$$B = l\left(t^\alpha e^t\right)^{1/n} exp\left[b\int\left(t^\alpha e^t\right)^{-3/n} dt\right] \tag{25}$$

$$C = l^{-1}\left(t^\alpha e^t\right)^{1/n} exp\left[-b\int\left(t^\alpha e^t\right)^{-3/n} dt\right] \tag{26}$$

and the Gravitational constant is

$$G = G_0\left(t^\alpha e^t\right)^{m/n} \tag{27}$$

Using (22) and (27) and solving the field equations (8)-(11), we get

$$\rho = \frac{1}{8\pi G_0(1+\gamma)}\left[\frac{2\alpha}{nt^2}\left(t^\alpha e^t\right)^{-m/n} - 2b^2\left(t^\alpha e^t\right)^{-(m+6)/n}\right] - \frac{2k^2}{8\pi G_0(1+\gamma)}\left[\left(t^\alpha e^t\right)^{-(m+2)/n}\right] \tag{28}$$

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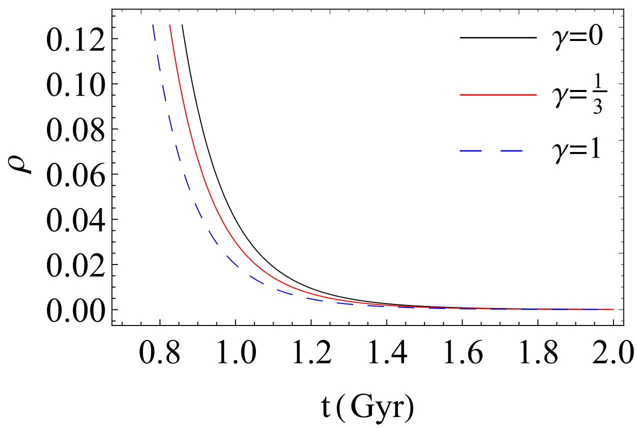

Figure 4: Plot of ρ versus t.

$$p = \frac{\gamma}{8\pi G_0(1+\gamma)}\left[\frac{2\alpha}{nt^2}\left(t^\alpha e^t\right)^{-m/n} - 2b^2\left(t^\alpha e^t\right)^{-(m+6)/n}\right] - \frac{2k^2\gamma}{8\pi G_0(1+\gamma)}\left[\left(t^\alpha e^t\right)^{-(m+2)/n}\right] \tag{29}$$

$$\Lambda = \frac{3}{n^2}\left(\frac{\alpha}{t}+1\right)^2 - \frac{2\alpha}{n(1+\gamma)t^2} - \frac{(\gamma-1)b^2}{(\gamma+1)}\left(t^\alpha e^t\right)^{-6/n} - \frac{(3\gamma+1)k^2}{(\gamma+1)}\left(t^\alpha e^t\right)^{-2/n} \tag{30}$$

The energy and critical densities for vaccum and the density parameters are read as

$$\rho_\nu = \frac{1}{8\pi G_0}\left[\frac{3}{n^2}\left(\frac{\alpha}{t}+1\right)^2\left(t^\alpha e^t\right)^{-m/n} + \frac{2\alpha}{n(1+\gamma)t^2}\left(t^\alpha e^t\right)^{-m/n}\right] - \frac{1}{8\pi G_0}\left[\frac{(\gamma-1)b^2}{(\gamma+1)}\left(t^\alpha e^t\right)^{-(m+6)/n} + \frac{(3\gamma+1)k^2}{(\gamma+1)}\left(t^\alpha e^t\right)^{-(m+2)/n}\right] \tag{31}$$

$$\rho_c = \frac{1}{8\pi G_0}\left[\frac{3}{n^2}\left(\frac{\alpha}{t}+1\right)^2\left(t^\alpha e^t\right)^{-m/n}\right] \tag{32}$$

$$\Omega_m = \frac{1}{3(1+\gamma)}\left[\frac{2n\alpha}{t^2}\left(\frac{\alpha}{t}+1\right)^{-2} - 2n^2b^2\left(\frac{\alpha}{t}+1\right)^{-2}\left(t^\alpha e^t\right)^{-6/n}\right] - \frac{2n^2k^2}{3(1+\gamma)}\left[\left(\frac{\alpha}{t}+1\right)^{-2}\left(t^\alpha e^t\right)^{-2/n}\right] \tag{33}$$

$$\Omega_\Lambda = 1 - \frac{2n\alpha}{3(1+\gamma)t^2}\left(\frac{\alpha}{t}+1\right)^{-2} - \frac{(\gamma-1)n^2b^2}{3(\gamma+1)}\left(\frac{\alpha}{t}+1\right)^{-2}\left(t^\alpha e^t\right)^{-6/n} - \frac{(3\gamma+1)n^2k^2}{3(\gamma+1)}\left(\frac{\alpha}{t}+1\right)^{-2}\left(t^\alpha e^t\right)^{-2/n} \tag{34}$$

Adding (33) and (34), we get

$$\Omega_{total} = 1 - \frac{n^2b^2}{3}\left(\frac{\alpha}{t}+1\right)^{-2}\left(t^\alpha e^t\right)^{-6/n} - n^2k^2\left(\frac{\alpha}{t}+1\right)^{-2}\left(t^\alpha e^t\right)^{-2/n} \tag{35}$$

4. Some Physical and Geometric Properties

The spatial volume and average Hubble parameter for model (1) are given by

$$V = \left(t^\alpha e^t\right)^{3/n} \tag{36}$$

$$H = \frac{1}{n}\left(\frac{\alpha}{t}+1\right) \tag{37}$$

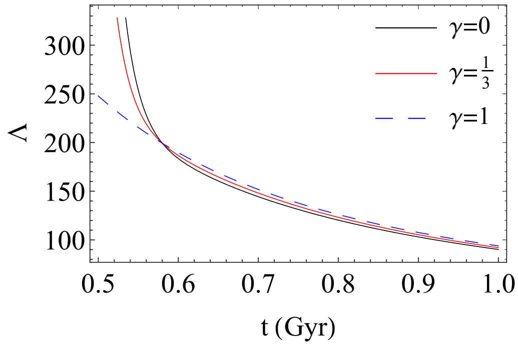

Figure 5: Plot of Λ versus t.

The physical parameters defined in Eqs. (17)-(18) are obtained as

$$A_m = \frac{2n^2b^2}{3}\left(\frac{\alpha}{t}+1\right)^{-2}\left(t^{\alpha}e^{t}\right)^{-6/n} \tag{38}$$

$$\sigma^2 = b^2\left(t^{\alpha}e^{t}\right)^{-6/n} \tag{39}$$

The deceleration parameter is given by

$$q = -1 + \frac{n\alpha}{(\alpha+t)^2} \tag{40}$$

From Eq. (40), we observe that $q > 0$ for $t < \sqrt{n\alpha} - \alpha$ and $q < 0$ for $t > \sqrt{n\alpha} - \alpha$.

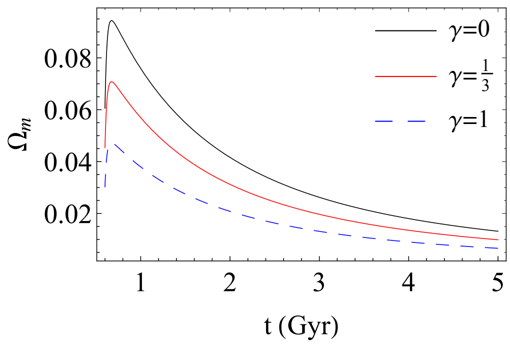

Figure 6: Plot of Ω_m versus t.

From Eq. (40), the value of DP at present can be estimated as

$$q_0 = -1 + \frac{\alpha}{nH_0^2t_0^2} \tag{41}$$

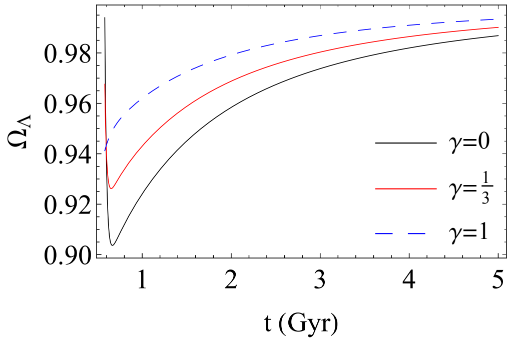

Figure 7: Variation of Ω_Λ versus t.

The DP of the universe vary in the range $-1 \leq q \leq 0$ as affirmed by experimental observation. For $n = 0.27\alpha$, we obtain $q_0 = -0.73$ [21]. The values from table can be used for plotting and numerical validation.

n	α
0.25	0.9259259260
0.50	1.851851852
0.75	2.77777778
3.0	11.11111111

Table 1: Table of Values of n and β

Eq. (38), where $Am \to 0$ as $t \to \infty$, shows a transitioning of the model from early anisotropy to isotropy at late time. The variation of A_m has been plotted in fig.2. It can be confirmed from figure that the universe is isotropic at present epoch.

From above Eqs. (28)-(30), It has been found that the present model is radiating dominated, False vaccum, empty model and stiff fluid model respectively for $\gamma = 1/3$, $\gamma = -1$, $\gamma = 0$ and $\gamma = 1$.

Eq. (28), states that energy density ρ is a time decreasing function. Figure-4, shows the variation of energy density. The cosmological term Λ has a similar effect as a uniform mass density of universe i.e., $\rho_{eff} = \frac{-\Lambda}{4\pi G}$. Figure-5, show variation of cosmological term Λ with time and express that the universe is in acceleration phase, for all three models i.e., radiating dominated, empty and stiff fluid universe models.

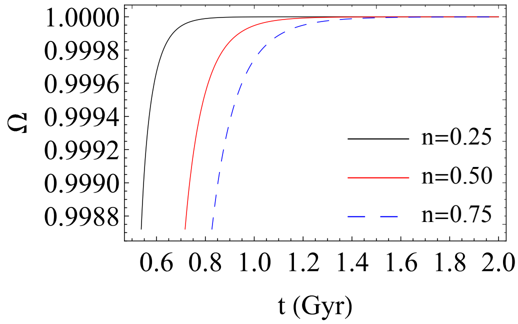

Figure 8: Variation of Ω versus t.

Plots 6 and 7 represent respectively, the variation of matter density parameter, and cosmological term density parameter. These figures explain that evaluation of universe in early phase is matter dominated and later phase is dark energy dominated at present epoch. Figure-8 of total density parameter explains that $\Omega \rightarrow 1$ as $t \rightarrow \infty$ and match with experimental observational results [58].

5. Statefinder Diagnostic

To discuss the geometrical diagnosis of DE models, the state-finders are given as [51]

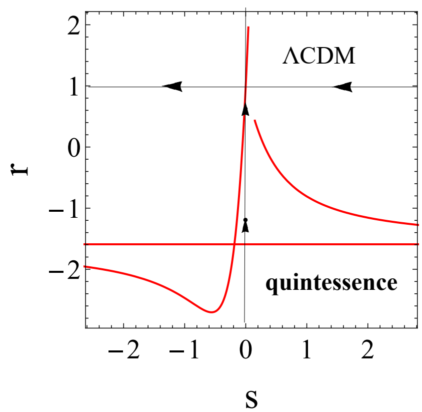

Figure 9: Plots of Variation r versus s.

$$r = \frac{\dddot{a}}{a} H^{-}3 \tag{42}$$

$$s = \frac{r-1}{3(q-0.5)} \tag{43}$$

Here, one can not choose $q = 0.5$. The trajectories of $s - r$ plane is shown in figure 9. From the figure, we observed that the trajectory lying in quintessence region $r < 1$, $s > 0$ and $(s, r) = (0, 1)$, the model is evolving into ΛCDM model. Plot of the trajectory in $q - r$ plane is shown in fig.10. For $r < 1$, $q > 0$ the region belongs to quintessence models, for $r = 1, q > 0$ models evolve as ΛCDM models and for $r > 1$, $q > 0$ the region belongs to Chaplygin gas models [2, 29].

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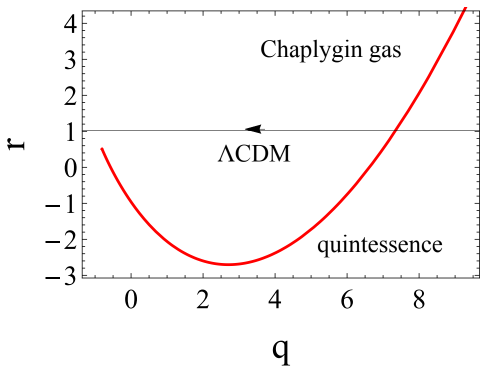

Figure 10: Plots of Variation r versus q.

6. Om Diagnostic Analysis

The $Om(z)$ parameter is given by

$$Om(z) = \frac{\left[\frac{H(z)}{H_0}\right]^2 - 1}{(z+1)^3 - 1} \tag{44}$$

The $Om(z)$ parameter of derived model is given by

$$Om(z) = \frac{\left[\frac{\frac{1}{W\left(\frac{\left(\left(\frac{a_0}{z+1}\right)^n\right)^{1/\alpha}}{\alpha}\right)}+1}{n}\right]^2 - H_0^2}{(z+1)^3 H_0^2 - H_0^2} \tag{45}$$

$Om(z)$ diagnostic analysis is more suitable than the state-finders to validate dynamics of dark energy models [52]. For $Om(z) > 0$, $Om(z) = 0$ and $Om(z) < 0$, the behavior of the models are phantom, quintessence and ΛCDM respectively [28, 49, 52, 65]. The behavior of the derived model is quintessence like as indicated by the negative slope of $Om(z) - z$ trajectories.

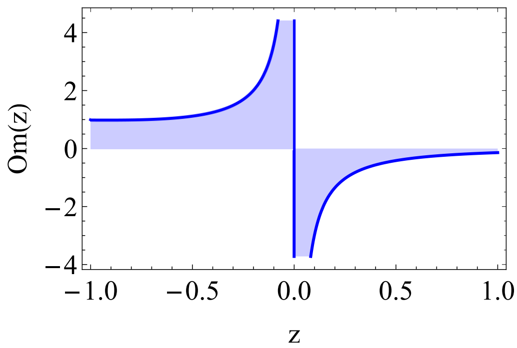

Figure 11: Plot of $Om(z)$ versus z with $H_0 = 71.12 kms^{-1} Mpc^{-1}$.

7. Conclusion

In this work a new class of models of accelerating universe with variable $G(t)$ and $\Lambda(t)$ has been presented. Following are the main aspects of investigation:

- The model suffers point type singularity initially as also described by other authors [15, 32, 34, 44]. The present model shows a transitioning from anisotropy to isotropy, shows good agreement with the observations. The parameters like $p \to 0$, $\rho \to 0$, $\Lambda \to 0$, and $G \to \infty$, $V \to \infty$ as $t \to \infty$.

- The present model is radiating dominated for $\gamma = 1/3$, empty model for $\gamma = 0$ and stiff fluid model for $\gamma = 1$.

- The cosmological constant has a similar consequence as mass density of universe i.e., $\rho_{eff} = \frac{-\Lambda}{4\pi G}$. Figure-5, show variation of Λ and express that the universe is in acceleration phase for possible cases i.e., radiating dominated, empty and stiff fluid universe models.

- The plots 6 and 7, explain that evaluation of universe in early phase is matter dominated and later phase is dark energy dominated phase. Plot of total density parameter explains that at $t \to \infty$, $\Omega \to 1$, and shows a good agreement with observational result [51].

- Plot of the trajectory in $q - r$ plane is shown in fig.10. For $r < 1$, $q > 0$ the region belongs to quintessence models, for $r = 1, q > 0$ models evolve as ΛCDM models and for $r > 1$, $q > 0$ the region belongs to Chaplygin gas models [2].

- The negative slope of parameter $Om(z)$, shows the consistency of the present model.

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