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SOMBOR INDEX OF EDGE CORONA PRODUCT OF SOME CLASSES OF GRAPHS

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Abstract: The operations of graphs spread their wings in designing complex network structures in various engineering domains. Graph indices, popularly termed topological indices are computed on the basis of distance or degree. The boundless part of graph indices has its foot print in network centrality and the robustness of complex networks. The goal of this paper is to provide a complete expression for the Sombor index of edge corona product of few classes of graphs.

Keywords and Phrases: Paths, Cycles, Edge corona product, Sombor Index.

2020 Mathematics Subject Classification: 05C09, 05C76.

1. Introduction

Graphs considered in this paper are simple graphs. Graphs give a nice structural representations of many physical phenomenon. Very recently, graphs are used to represent the social networks such as Facebook and e-mail networks. The degree of the vertex v_i is denoted by $d(v_i)$ represents the number of edges incident on it. The lower and upper bounds on the Sombor index of graphs in [2] motivate us to work on Sombor index of graph products.

Topological indices characterize the topology of a graph. It is a numerical parameter and is usually graph invariant. Wiener index, Hyper-Wiener index, Hosoya index, Szeged index, Zagreb index, Padmakar–Ivan (PI) index, weighted PI index, etc., are some of the topological indices. The topological indices correlate certain physicochemical properties such as boiling point and stability of chemical components. Molecular descriptors (topological indices) play a significant role in mathematical chemistry, especially in the quantitative structure-activity relationship (QSAR) and quantitative structure-property relationship (QSPR) investigations. In theoretical chemistry, they are used for modeling pharmacological, physicochemical, toxicological, biological and other properties of chemical compounds. Extensive applications of some of these indices are given in [4] and [13]. Among all the indices, Wiener index, PI, Weighted PI and Sombor indices are important in many applications. Wiener showed that the Wiener index number is closely correlated with the boiling points of alkane molecules. Wiener index was well studied in the research. Gopika et al. [3] obtained the weighted PI index for some classes of direct and strong product graphs. PI index for some classes of perfect graphs were obtained in [11].

Sombor index is one of the important indices in the chemical graph theory. Sombor index was introduced by Gutman in [6] and defined it as

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d(u)^2 + d(v)^2}.$$

Mathematical properties and many applications of Sombor index were studied in [5, 7, 8, 9, 10]. Chinglensana et al. [1] explored several properties of Sombor coindex of a finite simple graph and we derive a bound for the total Sombor index.

2. Preliminaries

A path P_n is a connected graph with two vertices of degree one and all other vertices are having degree two. The cycle graph with n vertices is denoted by C_n. A graph in which each pair of graph vertices is connected by an edge is termed as a complete graph. The complete graph with n vertices is denoted by K_n. A graph is said to be regular if degree of each vertex is equal. A graph is called r- regular if degree of each vertex in the graph is r.

The vertex corona product of G and H is the graph $G \circ H$ is obtained by taking one copy of G, called the center graph, $|V(G)|$ copies of H, called the outer graph, and making the i^{th} vertex of G adjacent to every vertex of the i^{th} copy of H, where $1 \leq i \leq |V(G)|$. Vertex corona product is one of the well known graph products. It has many real time applications in chemistry, computer and social networks.

The edge corona of two graphs G and H, denoted by $G \diamond H$, is obtained by taking one copy of G and $|E(G)|$ copies of H and joining each end vertices of i^{th} edge of G to every vertex in the i^{th} copy of H. Figure 1 shows an example for the

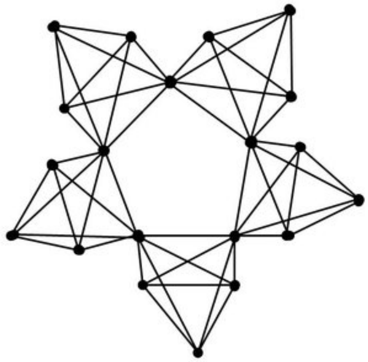

Figure 1: Edge corona product of C_5 and C_3

edge corona product of C_5 and C_3. Chitra et al. [1] and [12] obtained the Sparing number of the edge corona of some classes of graphs. Vignesh et al. [14] proved the total coloring conjecture for edge corona product of graphs.

3. Results

Theorem 3.1. *The Sombor index of edge corona product of a path P_n with a r-regular graph G is* $SO(P_n \diamond G) = 2\sqrt{5} - 6\sqrt{2} - 6\sqrt{2}m + 2\sqrt{5}m + 2\sqrt{2}n + 2\sqrt{2}mn - \sqrt{2}mr$
$+ \sqrt{2}mnr - mr^2/\sqrt{2} + mnr^2/\sqrt{2} + 2m\sqrt{5 + 2m + m^2 + 4r + r^2}$
$- 4m\sqrt{8 + 8m + 4m^2 + 4r + r^2} + 2mn\sqrt{8 + 8m + 4m^2 + 4r + r^2}$.

Proof. By labelling the vertices of the path P_n by $00, 01, 02, ..., 0(n-1)$ and the vertices of the i^{th} copy of G by $i0, i1, i2, ..., i(n-1)$, the edge set of $P_n \diamond G$ is split up into three sets E_1, E_2 and E_3 such that the set of all edges in G, P_n and the edges corresponding to corona product. That is
$E_1 = \{(ij, i(j+1)) : i = 1, 2, ..., m-1, j = 0, 1, 2, ..., n-1\}$
$E_2 = \{(0j, 0(j+1)) : j = 0, 1, 2, ..., m-2\}$
$E_3 = \{(0j, i(j+1)) : i = 1, 2, ..., m-1, j = 0, 1, 2, ..., n-1\}$.
The number of edges of E_1, E_2, E_3 are $mr(n-1)/2, n-1$, corona edges connected with pendent vertices are $2m$, non-pendent vertices are $2m(n-2)$ Sombor index of edge corona product of a path P_n with a r-regular graph G is

$$\begin{aligned} SO(P_n \diamond G) &= \sum_{uv \in P_n \diamond G} \sqrt{d(u)^2 + d(v)^2} \\ &= \sum_{uv \in E_1} \sqrt{d(u)^2 + d(v)^2} \end{aligned}$$

$$+\sum_{uv\in E_2}\sqrt{d(u)^2+d(v)^2} + \sum_{uv\in E_3}\sqrt{d(u)^2+d(v)^2}$$

$$\begin{aligned}
SO(P_n\diamond G) &= \frac{mr(n-1)}{2}\sqrt{2(r+2)^2}+2\sqrt{5(1+m)^2}\\
&\quad +2(n-3)\sqrt{2(1+m)^2}\\
&\quad +2m\sqrt{(1+m)^2+(r+2)^2}\\
&\quad +2m(n-2)\sqrt{4(1+m)^2+(r+2)^2}\\
&= \frac{\sqrt{2}mr(n-1)(r+2)}{2}\\
&\quad +2\sqrt{5}(1+m)+2\sqrt{2}(1+m)(n-3)\\
&\quad +2m\sqrt{(1+m)^2+(r+2)^2}\\
&\quad +2m(n-2)\sqrt{4(1+m)^2+(r+2)^2}
\end{aligned}$$

$$\begin{aligned}
&= 2\sqrt{5}-6\sqrt{2}-6\sqrt{2}m+2\sqrt{5}m+2\sqrt{2}n+2\sqrt{2}mn-\sqrt{2}mr\\
&+\sqrt{2}mnr-mr^2/\sqrt{2}+mnr^2/\sqrt{2}+2m\sqrt{5+2m+m^2+4r+r^2}\\
&-4m\sqrt{8+8m+4m^2+4r+r^2}+2mn\sqrt{8+8m+4m^2+4r+r^2}.
\end{aligned}$$

For example, consider the two graphs P_3 and C_3.
$SO(P_3\diamond C_3)=48\sqrt{2}+32\sqrt{5}$; where $m=3$, $n=3$ and $r=2$ in Theorem 3.1.

Theorem 3.2. *The Sombor index of edge corona product of a cycle C_n with a r-regular graph G is*
$SO(C_n\diamond G)=2^{\frac{3}{2}}n+2^{\frac{3}{2}}mn+\sqrt{2}mnr+mnr^2/\sqrt{2}+2mn\sqrt{8+8m+4m^2+4r+r^2}.$

Proof. Sombor index of edge corona product of a path C_n with a r-regular graph G is

$$\begin{aligned}
SO(C_n\diamond G) &= \sum_{uv\in C_n\diamond G}\sqrt{d(u)^2+d(v)^2}\\
&= \sum_{uv\in E_1}\sqrt{d(u)^2+d(v)^2}\\
&\quad +\sum_{uv\in E_2}\sqrt{d(u)^2+d(v)^2}\\
&\quad +\sum_{uv\in E_3}\sqrt{d(u)^2+d(v)^2}.
\end{aligned}$$

$$\begin{aligned}SO(C_n \diamond G) &= \frac{mrn}{2}\sqrt{2(r+2)^2} + 2n\sqrt{2(1+m)^2} \\ &\quad +2mn\sqrt{(r+2)^2+4(1+m)^2} \\ &= mrn(r+2)/\sqrt{2} + 2\sqrt{2}n(1+m) \\ &\quad +2mn\sqrt{(r+2)^2+4(1+m)^2}.\end{aligned}$$

$$\begin{aligned}SO(C_n \diamond G) &= 2^{\frac{3}{2}}n + 2^{\frac{3}{2}}mn + \sqrt{2}mnr + mnr^2/\sqrt{2} \\ &\quad +2mn\sqrt{8+8m+4m^2+4r+r^2}.\end{aligned}$$

For example, consider the cycle C_3, $SO(C_3 \diamond C_3) = 60\sqrt{2} + 72\sqrt{5}$.

Theorem 3.3. *The Sombor index of edge corona product of a complete graph K_n with a r-regular graph G is*

$$\begin{aligned}SO(K_n \diamond G) = {} & n/\sqrt{2} + mn/\sqrt{2} - \sqrt{2}n^2 - \sqrt{2}mn^2 + n^3/\sqrt{2} \\ & + mn^3/\sqrt{2} - mnr/\sqrt{2} + mn^2r/\sqrt{2} - mnr^2/2\sqrt{2} \\ & + 2^{\frac{3}{2}}mn^2r^2 - mnf(m,n,r) + mn^2f(m,n,r).\end{aligned}$$

where $f(m,n,r)$ *is*

$$\sqrt{5+2m+m^2-2n-4mn-2m^2n+n^2+2mn^2+m^2n^2+4r+r^2}.$$

Proof. Sombor index of edge corona product of a complete graph K_n with a r-regular graph G is

$$\begin{aligned}SO(K_n \diamond G) &= \sum_{uv\in K_n\diamond G}\sqrt{d(u)^2+d(v)^2} \\ &= \sum_{uv\in E_1}\sqrt{d(u)^2+d(v)^2} + \sum_{uv\in E_2}\sqrt{d(u)^2+d(v)^2} \\ &\quad + \sum_{uv\in E_3}\sqrt{d(u)^2+d(v)^2}.\end{aligned}$$

$$\begin{aligned}SO(K_n \diamond G) &= \frac{mrn(n-1)}{4}\sqrt{2(r+2)^2} \\ &\quad +\frac{n(n-1)}{2}\sqrt{2(1+m)^2(n-1)^2} \\ &\quad +mn(n-1)\sqrt{(r+2)^2+(1+m)^2(n-1)^2}.\end{aligned}$$

$$\begin{aligned}SO(K_n \diamond G) &= \frac{mrn(n-1)(r+2)}{2\sqrt{2}} + \frac{n(n-1)^2(1+m)}{\sqrt{2}} \\ &\quad +mn(n-1)\sqrt{(r+2)^2+(1+m)^2(n-1)^2}.\end{aligned}$$

$$\begin{aligned} SO(K_n \diamond G) &= n/\sqrt{2} + mn/\sqrt{2} - \sqrt{2}n^2 - \sqrt{2}mn^2 + n^3/\sqrt{2} \\ &\quad + mn^3/\sqrt{2} - 2^{\frac{5}{2}}mnr + 2^{\frac{5}{2}}mn^2r - 2^{\frac{3}{2}}mnr^2 \\ &\quad + 2^{\frac{3}{2}}mn^2r^2 - mnf(m,n,r) + mn^2f(m,n,r). \end{aligned}$$

where $f(m,n,r)$ *is*
$\sqrt{5+2m+m^2-2n-4mn-2m^2n+n^2+2mn^2+m^2n^2+4r+r^2}$.

Note: *If we take $n=3$ in the theorem 3.3, then the results coincide with the result of theorem 3.2.*

Corollary 3.1. *The Sombor index of edge corona product of a path P_m and a cycle C_n is*

$$SO(P_m \diamond C_n) = \begin{cases} 2\sqrt{5} + 34n - 2^{\frac{5}{2}}n + 2\sqrt{5}n + 2^{\frac{5}{2}}mn \\ \qquad + 4n^2 + 2n^3. \quad for \quad j = 0, n-1. \\ -32^{\frac{3}{2}} + 2^{\frac{3}{2}}m - 40n - 32^{\frac{3}{2}}n - 2^{\frac{5}{2}}n + 20mn \\ \qquad + 2^{\frac{3}{2}}mn + 2^{\frac{5}{2}}mn - 16n^2 + 8mn^2 - 8n^3 \\ \qquad + 4mn^3 \quad for \quad j \neq 0, n-1. \end{cases}$$

Proof. Sombor index of edge corona product of a path P_m with a cycle is C_n is computed as follows.

$$\begin{aligned} SO(P_m \diamond C_n) &= \sum_{uv \in P_m \diamond C_n} \sqrt{d(u)^2 + d(v)^2} \\ &= \sum_{uv \in E_1} \sqrt{d(u)^2 + d(v)^2} + \sum_{uv \in E_2} \sqrt{d(u)^2 + d(v)^2} \\ &\quad + \sum_{uv \in E_3} \sqrt{d(u)^2 + d(v)^2} \\ SO(P_m \diamond C_n) &= (m-1)n\sqrt{32} \\ &\quad + \begin{cases} 2\sqrt{5(1+n)^2}; j = 0, n-1 \\ (m-3)\sqrt{8(1+n)^2}; j \neq 0, n-1. \end{cases} \\ &\quad + \begin{cases} 2n\sqrt{(1+n)^2+16}; j = 0, n-1 \\ 4(mn-2n)\sqrt{(1+n)^2+4}; j \neq 0, n-1. \end{cases} \\ SO(P_m \diamond C_n) &= 4\sqrt{2}(m-1)n \\ &\quad + \begin{cases} 2\sqrt{5}(1+n); j = 0, n-1 \\ 2\sqrt{2}(m-3)(1+n); j \neq 0, n-1. \end{cases} \\ &\quad + \begin{cases} 2n\sqrt{(1+n)^2+16}; j = 0, n-1 \\ 4(mn-2n)\sqrt{(1+n)^2+4}; j \neq 0, n-1. \end{cases} \end{aligned}$$

$$SO(P_m \diamond C_n) = \begin{cases} 2\sqrt{5} + 34n - 2^{\frac{5}{2}}n + 2\sqrt{5}n + 2^{\frac{5}{2}}mn \\ \qquad + 4n^2 + 2n^3. \ \ for \quad j = 0, n-1. \\ -32^{\frac{3}{2}} + 2^{\frac{3}{2}}m - 40n - 32^{\frac{3}{2}}n - 2^{\frac{5}{2}}n + 20mn \\ \qquad + 2^{\frac{3}{2}}mn + 2^{\frac{5}{2}}mn - 16n^2 + 8mn^2 - 8n^3 \\ \qquad + 4mn^3 \quad for \quad j \neq 0, n-1. \end{cases}$$

Corollary 3.2. *The Sombor index of edge corona product of a path P_m and a complete graph K_n is*

$$SO(P_m \diamond K_n) = \begin{cases} 2\sqrt{5} + n/\sqrt{2} + 2^{\frac{3}{2}}n + 2\sqrt{5}n - mn/\sqrt{2} + 2^{\frac{3}{2}}n^2 \\ \qquad - n^3/\sqrt{2} + mn^3/\sqrt{2}; j = 0, n-1. \\ -32^{\frac{3}{2}} + 2\frac{3}{2}m + n/\sqrt{2} - 32\frac{3}{2}n - 4\sqrt{5}n - mn/\sqrt{2} \\ \qquad + 2^{\frac{3}{2}}mn + 2\sqrt{5}mn - 4\sqrt{5}n^2 + 2\sqrt{5}mn^2 \\ \qquad - n^3/\sqrt{2} + mn^3/\sqrt{2}; j \neq 0, n-1. \end{cases}$$

Proof. Sombor index of edge corona product of a path P_m with a complete graph K_n is

$$\begin{aligned} SO(P_m \diamond K_n) &= \sum_{uv \in P_m \diamond K_n} \sqrt{d(u)^2 + d(v)^2} \\ &= \sum_{uv \in E_1} \sqrt{d(u)^2 + d(v)^2} + \sum_{uv \in E_2} \sqrt{d(u)^2 + d(v)^2} \\ &\quad + \sum_{uv \in E_3} \sqrt{d(u)^2 + d(v)^2} \\ SO(P_m \diamond K_n) &= \frac{(m-1)(n^2-1)n}{\sqrt{2}} \\ &\quad + \begin{cases} 2\sqrt{5}(1+n); j = 0, n-1. \\ 2\sqrt{2}(m-3)(1+n); j \neq 0, n-1. \end{cases} \\ &\quad + \begin{cases} 2\sqrt{2}n(1+n)^2; j = 0, n-1. \\ 2\sqrt{5}n(m-2)(1+n); j \neq 0, n-1. \end{cases} \end{aligned}$$

$$SO(P_m \diamond K_n) = \begin{cases} 2\sqrt{5} + n/\sqrt{2} + 2^{\frac{3}{2}}n + 2\sqrt{5}n - mn/\sqrt{2} + 2^{\frac{3}{2}}n^2 \\ \qquad - n^3/\sqrt{2} + mn^3/\sqrt{2}; j = 0, n-1. \\ -32^{\frac{3}{2}} + 2\frac{3}{2}m + n/\sqrt{2} - 32\frac{3}{2}n - 4\sqrt{5}n - mn/\sqrt{2} \\ \qquad + 2^{\frac{3}{2}}mn + 2\sqrt{5}mn - 4\sqrt{5}n^2 + 2\sqrt{5}mn^2 \\ \qquad - n^3/\sqrt{2} + mn^3/\sqrt{2}; j \neq 0, n-1. \end{cases}$$

4. Conclusions

Sombor index is one of the important graph indices with many applications in molecular chemistry and networks. Edge corona product is a variation of famous corona product in graph theory. In this paper, we obtained the Sombor index of edge corona product of some classes of graphs. Sombor index for other product of graphs are still open.

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References

[1] Chithra K. P., Germina K. A. and Sudev N. K., On the Sparing Number of the Edge-Corona of graphs, International Journal of Computer Applications, 118, (1) (2015), 1-5.

[2] Das K. C., Çevik A. S., Cangul, I. N., Shang, Y., On Sombor Index, Symmetry, 13, (2021), 140.

[3] Gopika C., Geetha J. and Somasundaram K., Weighted PI index of tensor product and strong product of graphs, Discrete Mathematics, Algorithms and Applications, 13, (03) (2021), 2150019.

[4] Gutman I. and Trinajstic N., Graph theory and molecular orbitals.Total π-electron energy of alternant hydrocarbons, Chem. Phys. Let., 17, (1972), 535-538.

[5] Gutman I., Geometric approach to degree based topological indices: Sombor indices, MATCH Common. Math, Comput. Chem., 86, (2021), 11-16.

[6] Gutman I., Kulli V. R. and Redzepovic I., Sombor index of Kragujevac trees, Ser. A: Appl. Math. Inform. And Mech., 13, (2) (2021), 61-70.

[7] Kulli V. R., Neighborhood Sombor index of some nanostructures, International Journal of Mathematics Trends and Technology, 67, (5) (2021), 101-108.

[8] Kulli V. R., Sombor indices of two families of dendrimer nanostars, Annals of Pure and Applied Mathematics, 24, (1) (2021), 21-26.

[9] Kulli V. R., Different versions of Sombor index of some chemical structures, International Journal of Engineering Sciences and Research Technology, 10, (7) (2021), 23-32.

[10] Kulli V. R., New irregularity Sombor indices and new Adriatic (a, b)-KA indices of certain chemical drugs, International Journal of Mathematics Trends and Technology, 67, (9) (2021), 105-113.

[11] Manju S. C. and Somasundaram K., Padmakar-Ivan index of some types of perfect graphs, Discrete Math. Lett., 9 (2022), 92-99.

[12] Phanjoubam, Chinglensana, Sainkupar Marwein Mawiong, and Ardeline M. Buhphang, On Sombor coindex of graphs, Communications in Combinatorics and Optimization, 7 (2022), 1-17.

[13] Todeschini R., Consonni V., Molecular Descriptors for Chemoinformatics, Wiley-VCH, Weinheim, 2009.

[14] Vignesh, R., Geetha J., and Somasundaram K., Total coloring conjecture for vertex, edge and neighborhood corona products of graphs, Discrete Mathematics, Algorithms and Applications, 11, No. 01 (2019), 1950014.