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## SCHULTZ INDICES AND THEIR POLYNOMIALS OF MYCIELSKIAN GRAPHS

Lavanya E., M. A. Sriraj\* and Bhagyashri R. Doddamani\*\*

Department of Mathematics,  
Acharya Institute of Graduate Studies,  
Bengaluru, Karnataka, INDIA

E-mail : lavanya.hup@gmail.com

\*Department of Mathematics,  
Vidyavardhaka College of Engineering, Mysuru, Karnataka, INDIA

E-mail : masriraj@gmail.com

\*\*Department of Mathematics,  
CDEVL, Jain (Deemed-to-be University), Bengaluru, Karnataka, INDIA

E-mail : bhagyadoddamani1@gmail.com

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**Abstract:** Topological indices are studied extensively due to its vibrant applicability in the field of chemical graph theory. These connectivity indices (topological indices) is a numerical value resulting in an unequivocal process based on the structure of graph. Numerous topological indices are classified based on their distance and degree. The Schultz and modified Schultz indices considered in this paper have been expansively studied by various authors on different types of graphs. In this paper, we established the results on Schultz, modified Schultz indices and their polynomials for mycielskian graphs.

**Keywords and Phrases:** Mycielskian graph, Schultz indices, Schultz polynomials.

**2020 Mathematics Subject Classification:** 05C07, 05C09, 05C12, 05C31.

## 1. Introduction

Let  $G$  be a connected graph.  $V(G)$  be the vertex set and  $E(G)$  be the edge set. The distance between two vertices  $v_1$  and  $v_2$  denoted by  $d(v_1, v_2)$  is the length of the shortest path between them. The degree of a vertex  $v$  is the number of edges incident with it and is denoted by  $d_v$ . For undefined terminologies refer [5].

The Schultz index was introduced by H. Schultz [10] in 1989 and is defined as:

$$Sc(G) = \frac{1}{2} \sum_{u,v \in V(G)} (d_u + d_v)d(u, v) \quad (1)$$

The Schultz polynomial is

$$Sc(G, x) = \frac{1}{2} \sum_{u,v \in V(G)} (d_u + d_v)x^{d(u,v)} \quad (2)$$

Modified Schultz index was introduced by S. Klavzar and I. Gutman [7] as:

$$Sc^*(G) = \frac{1}{2} \sum_{u,v \in V(G)} (d_u \times d_v)d(u, v) \quad (3)$$

The modified Schultz polynomial is:

$$Sc^*(G, x) = \frac{1}{2} \sum_{u,v \in V(G)} (d_u \times d_v)x^{d(u,v)} \quad (4)$$

The Schultz, modified Schultz indices and their polynomials are studied and computed for various graphs by many authors [2, 4, 6]. Mycielskian graph of a graph  $G$  is denoted by  $\mu(G)$  has the vertex set  $V(G) \cup U(G) \cup w$  and edge set  $E(G) \cup \{vu' : uv \in E(G)\} \cup \{u'w : u' \in U(G)\}$ . Motivated by [1,3 8, 9], we have computed the Schultz and modified Schultz polynomial and their indices of Mycielskian path, Mycielskian cycle and Mycielskian complete bipartite graph.

## 2. Results on Mycielskian Path

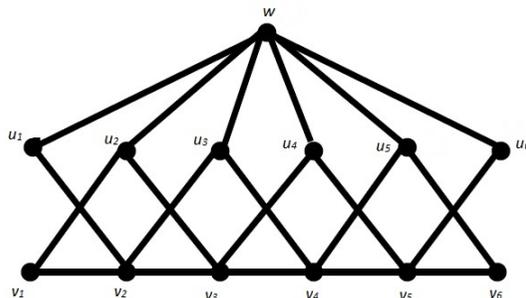


Figure 1. Mycielskian of  $P_6$

**Theorem 2.1.** Let  $\mu(P_m)$  be a Mycielskian of path graph, then for all  $m \geq 5$

$$Sc(\mu(P_m), x) = (m^2 + 25m - 34)x + (4m^2 + 28m - 62)x^2 + (7m^2 - 33m + 32)x^3 + (4m^2 - 32m + 64)x^4.$$

$$Sc(\mu(P_m)) = 46m^2 - 104m + 52.$$

$$Sc^*(\mu(P_m), x) = (3m^2 + 38m - 76)x + \left(\frac{17}{2}m^2 + \frac{75}{2}m - 125\right)x^2 + (12m^2 - 64m + 72)x^3 + (8m^2 - 72m + 164)x^4.$$

$$Sc^*(\mu(P_m)) = 88m^2 - 367m + 546.$$

**Proof.** Consider Mycielskian path graph  $\mu(P_m)$ ,  $\forall m \geq 5$  which contains  $2m + 1$  vertices and  $4m - 3$  edges. Among  $2m + 1$  vertices, four vertices have degree two,  $m - 2$  vertices have degree three,  $m - 2$  vertices have degree four and one vertex has degree  $m$ . Thus we have four partitions of the vertex set  $V(\mu(P_m))$  as follow.

$$V_1 = \{v \in V(\mu(P_m)) / d_v = 2\}$$

$$V_2 = \{v \in V(\mu(P_m)) / d_v = 3\}$$

$$V_3 = \{v \in V(\mu(P_m)) / d_v = 4\}$$

$$W = \{v \in V(\mu(P_m)) / d_v = m\}$$

$$\mu(P_m) = V_1 \cup V_2 \cup V_3 \cup W \text{ and } V_1 \cap V_2 \cap V_3 \cap W = \phi \text{ thus}$$

$$|E(\mu(P_m))| = \frac{1}{2}[2|V_1| + 3|V_2| + 4|V_3| + m|W|] = 4m - 3.$$

We observe from the structure of Mycielskian path graph  $\mu(P_m)$  contains the distances from one to four for every vertices  $u, v \in V(\mu(P_m))$ .

In other words  $\forall u, v \in V(\mu(P_m)) \exists d(u, v) \in \{1, 2, 3, 4\}$  and the diameter  $D$  of Mycielskian path  $\mu(P_m)$  is equal to  $D(\mu(P_m)) = 4$ . Now, we compute the cases of  $d(u, v)$  - edge paths  $d(u, v) = 1, 2, 3, 4$  of  $\mu(P_m)$  in table 1.

For  $d(u, v) = 1$ , term of  $Sc(\mu(P_m), x)$  and  $Sc^*(\mu(P_m), x)$  will be

$$5(2)x + 6(4)x + (m + 2)2x + 7(2m - 6)x + (m + 3)(m - 2)x + 8(m - 3)x = (m^2 + 25m - 34)x. \text{ and}$$

$$6(2)x + 8(4)x + 2m(2)x + 12(2m - 6)x + 3m(m - 2)x + 16(m - 3)x = (3m^2 + 38m -$$

Table 1: All cases of  $d(u, v)$ 

The distance $d(u, v) = i$	Degrees of $d_u, d_v$	Number of i-edge paths	Term of Schultz polynomial	Term of modified Schultz polynomial
1	2, 3	2	5	6
1	2, 4	4	6	8
1	2, $m$	2	$(2 + m)$	$2m$
1	3, 4	$(2m - 6)$	7	12
1	3, $m$	$(m - 2)$	$(3 + m)$	$3m$
1	4, 4	$(m - 3)$	8	16
2	2, 2	3	4	4
2	2, 3	$(2m - 2)$	5	6
2	2, 4	4	6	8
2	2, $m$	2	$(2 + m)$	$2m$
2	3, 3	$\frac{1}{2}(m - 2)(m - 3)$	6	9
2	3, 4	$(3m - 10)$	7	12
2	4, 4	$(m - 4)$	8	16
2	4, $m$	$(m - 2)$	$(4 + m)$	$4m$
3	2, 2	2	4	4
3	2, 3	$2(m - 4)$	5	6
3	2, 4	$2(m - 3)$	6	8
3	3, 4	$(m - 4)(m - 5)$	7	12
3	4, 4	$(m - 5)$	8	16
4	2, 2	1	4	4
4	2, 4	$2(m - 5)$	6	8
4	4, 4	$\frac{1}{2}(m - 5)(m - 6)$	8	16

76) $x$  respectively.

For  $d(u, v) = 2$ , term of  $Sc(\mu(P_m), x)$  and  $Sc^*(\mu(P_m), x)$  is equal to

$$[4(3) + 5(2m - 2) + 6(4) + (2 + m)(2) + 6(\frac{1}{2}(m - 2)(m - 3)) + 7(3m - 10) + 8(m - 4)]x^2 = (4m^2 + 28m - 62)x^2. \text{ and}$$

$$[4(3) + 6(2m - 2) + 8(4) + 2m(2) + 9(\frac{1}{2}(m - 2)(m - 3)) + 12(3m - 10) + 16(m - 4)]x^2 = (\frac{17}{2}m^2 + \frac{75}{2}m - 125)x^2$$

For  $d(u, v) = 3$ , term of  $Sc(\mu(P_m), x)$  and  $Sc^*(\mu(P_m), x)$  is equal to

$$[4(2) + 5(2(m - 4)) + 6(2(m - 3)) + 7(m - 4)(m - 5) + 8(m - 5)]x^3 = (7m^2 - 33m + 32)x^3$$

and

$$[4(2) + 6(2(m - 4)) + 8(2(m - 3)) + 12(m - 4)(m - 5) + 16(m - 5)]x^3 = (12m^2 - 64m + 72)x^3.$$

For  $d(u, v) = 4$ , term of  $Sc(\mu(P_m), x)$  and  $Sc^*(\mu(P_m), x)$  is equal to  $[8(1) + 6(2(m - 5)) + 8(\frac{1}{2}(m - 5)(m - 6))]x^4 = (4m^2 - 32m + 64)x^4$

and

$[16(1) + 8(2(m - 5)) + 16(\frac{1}{2}(m - 5)(m - 6))]x^4 = (8m^2 - 72m + 164)x^4$ , respectively.

Hence from the following definitions the  $Sc(\mu(P_m), x)$  and  $Sc^*(\mu(P_m), x)$  is given by

$$Sc(\mu(P_m), x) = \sum_{u, v \in V(\mu(P_m))} (d_u + d_v)x^{d(u, v)}, \quad Sc^*(\mu(P_m), x) = \sum_{u, v \in V(\mu(P_m))} (d_u \times d_v)x^{d(u, v)}$$

$$Sc(\mu(P_m), x) = (m^2 + 25m - 34)x + (4m^2 + 28m - 62)x^2 + (7m^2 - 33m + 32)x^3 + (4m^2 - 32m + 64)x^4.$$

$$Sc^*(\mu(P_m), x) = (3m^2 + 38m - 76)x + (\frac{17}{2}m^2 + \frac{75}{2}m - 125)x^2 + (12m^2 - 64m + 72)x^3 + (8m^2 - 72m + 164)x^4.$$

From the definition,  $Sc(\mu(P_m))$  and  $Sc^*(\mu(P_m))$  is given by

$$\begin{aligned} Sc(\mu(P_m)) &= \left. \frac{\partial Sc(\mu(P_m), x)}{\partial x} \right|_{x=1} \\ &= \frac{\partial}{\partial x} [(m^2 + 25m - 34)x + (4m^2 + 28m - 62)x^2 + (7m^2 - 33m + 32)x^3 + (4m^2 - 32m + 64)x^4] \\ &= 46m^2 - 104m + 52 \end{aligned}$$

$$\begin{aligned} Sc^*(\mu(P_m)) &= \left. \frac{\partial Sc^*(\mu(P_m), x)}{\partial x} \right|_{x=1} \\ &= \frac{\partial}{\partial x} [(3m^2 + 38m - 76)x + (\frac{17}{2}m^2 + \frac{75}{2}m - 125)x^2 + (12m^2 - 64m + 72)x^3 + (8m^2 - 72m + 164)x^4] \\ &= 88m^2 - 367m + 546 \end{aligned}$$

**Corollary 2.2.** *The  $\mu(P_m)$  be a Mycielskian of path graph, then for  $m = 2, 3, 4$  is.*

$$Sc(\mu(P_2), x) = 20x + 20x^2; \quad Sc(\mu(P_2)) = 60$$

$$Sc^*(\mu(P_2), x) = 20x + 20x^2; \quad Sc^*(\mu(P_2)) = 60$$

$$Sc(\mu(P_3), x) = 50x + 51x^2; \quad Sc(\mu(P_3)) = 152$$

$$Sc^*(\mu(P_3), x) = 65x + 60x^2; \quad Sc^*(\mu(P_3)) = 185$$

$$Sc(\mu(P_4), x) = 82x + 114x^2 + 8x^3; \quad Sc(\mu(P_4)) = 334$$

$$Sc^*(\mu(P_4), x) = 124x + 161x^2 + 8x^3; \quad Sc^*(\mu(P_4)) = 376$$

### 3. Results on Mycielskian Cycle

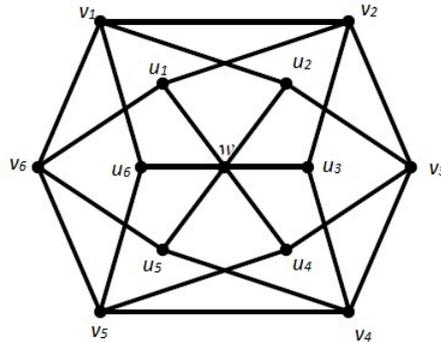


Figure 2. Mycielskian graph of  $C_6$

**Theorem 3.1.** Let mycielskian cycle of cycle graph be denoted by  $\mu(C_m)$  for all  $m \geq 8$

$$Sc(\mu(C_m), x) = m(m + 25)x + 2m(2m + 15)x^2 + m(7m - 27)x^3 + 4m(m - 7)x^4.$$

$$Sc(\mu(C_m)) = m(46m - 92).$$

$$Sc^*(\mu(C_m), x) = m(3m + 40)x + \frac{m}{2}(17m + 95)x^2 + 4m(3m - 11)x^3 + 8m(m - 7)x^4.$$

$$Sc^*(\mu(C_m)) = m(88m - 189).$$

**Proof.** Consider the Mycielskian cycle graph  $\mu(C_m)$ ,  $\forall m \geq 8$  and it contains  $2m + 1$  vertices and  $4m$  edges. From the structure of Mycielskian cycle there are  $m$  vertices that have degree 3 and  $m$  vertices that have degree 4 and one vertex of  $\mu(C_m)$  that has degree  $m$ . Thus we have three partitions of the vertex set  $V(\mu(C_m))$  as follows.

$$V_1 = \{v \in V(\mu(C_m)) / d_v = 3\}$$

$$V_2 = \{v \in V(\mu(C_m)) / d_v = 4\}$$

$$W = \{v \in V(\mu(C_m)) / d_v = m\}$$

$$\mu(C_m) = V_1 \cup V_2 \cup W \text{ and } V_1 \cap V_2 \cap W = \phi$$

$$\text{thus } |E(\mu(C_m))| = \frac{1}{2}[3|V_1| + 4|V_2| + m|W|] = 4m.$$

From the structure of Mycielskian cycle graph  $\mu(C_m)$ , we observe that there are distances from one to four  $\forall u, v \in V(\mu(C_m)) \exists d(u, v) \in \{1, 2, 3, 4\}$  and the diameter  $D$  of Mycielskian path  $\mu(C_m)$  is equal to  $D(\mu(C_m)) = 4$ .

Now we compute the edge paths  $d(u, v) = \{1, 2, 3, 4\}$  of  $\mu(C_m)$  in table 2.

For  $d(u, v) = 1$ , term of  $Sc(\mu(C_m), x)$  and  $Sc^*(\mu(C_m), x)$  will be  $7(2m)x + (3 + m)mx + 8mx = m(m + 25)x$ .

$$12(2m)x + 3m(m)x + 16mx = m(3m + 40)x.$$

For  $d(u, v) = 2$ , term of  $Sc(\mu(C_m), x)$  and  $Sc^*(\mu(C_m), x)$  is

$$6\frac{m}{2}(m - 1)x^2 + 7(3m)x^2 + 8mx^2 + (4 + m)mx^2 = 2m(2m + 15)x^2$$

Table 2: All cases of  $d(u, v)$ 

The distance $d(u, v) = i$	Degrees of $d_u, d_v$	Number of i-edge paths	Term of Schultz polynomial	Term of modified Schultz polynomial
1	3, 4	$2m$	7	12
1	3, $m$	$m$	$3 + m$	$3m$
1	4, 4	$m$	8	16
2	3, 3	$\frac{m}{2}(m - 1)$	6	9
2	3, 4	$3m$	7	12
2	4, 4	$m$	8	16
2	4, $m$	$m$	$4 + m$	$4m$
3	3, 4	$m(m - 5)$	7	12
3	4, 4	$m$	8	16
4	4, 4	$\frac{m}{2}(m - 7)$	8	16

$$9\frac{m}{2}(m - 1)x^2 + 12(3m)x^2 + 16mx^2 + 4m(m)x^2 = \frac{m}{2}(17m + 95)x^2$$

For  $d(u, v) = 3$ , term of  $Sc(\mu(C_m), x)$  and  $Sc^*(\mu(C_m), x)$  is

$$[7m(m - 5) + 8m]x^3 = m(7m - 27)x^3$$

$$[12m(m - 5) + 16m]x^3 = 4m(3m - 11)x^3$$

For  $d(u, v) = 4$ , term of  $Sc(\mu(C_m), x)$  and  $Sc^*(\mu(C_m), x)$  will be  $4m(m - 7)x^4$  and  $8m(m - 7)x^4$ .

Hence from the following definitions the  $Sc(\mu(C_m), x)$  and  $Sc^*(\mu(C_m), x)$  is given by

$$Sc(\mu(C_m), x) = \sum_{u, v \in V(\mu(C_m))} (d_u + d_v)x^{d(u, v)}, \quad Sc^*(\mu(C_m), x) = \sum_{u, v \in V(\mu(C_m))} (d_u \times d_v)x^{d(u, v)}$$

$$Sc(\mu(C_m), x) = m(m + 25)x + 2m(2m + 15)x^2 + m(7m - 27)x^3 + 4m(m - 7)x^4.$$

$$Sc^*(\mu(C_m), x) = m(3m + 40)x + \frac{m}{2}(17m + 95)x^2 + 4m(3m - 11)x^3 + 8m(m - 7)x^4$$

From the definitions,  $Sc(\mu(C_m))$  and  $Sc^*(\mu(C_m))$  is given by

$$\begin{aligned} Sc(\mu(C_m)) &= \left. \frac{\partial Sc(\mu(C_m), x)}{\partial x} \right|_{x=1} \\ &= \frac{\partial}{\partial x} [m(m + 25)x + 2m(2m + 15)x^2 + m(7m - 27)x^3 + 4m(m - 7)x^4] \\ &= 46m^2 + 4m - 112 \end{aligned}$$

$$\begin{aligned} Sc^*(\mu(C_m)) &= \left. \frac{\partial Sc^*(\mu(C_m), x)}{\partial x} \right|_{x=1} \\ &= \frac{\partial}{\partial x} [m(3m + 40)x + \frac{m}{2}(17m + 95)x^2 + 4m(3m - 11)x^3 + 8m(m - 7)x^4] \\ &= 88m^2 + 3m - 224 \end{aligned}$$

**Corollary 3.2.**  $\mu(C_m)$  be the Mycielskian of cycle graph, for  $m = 3, 4, 5$

$$\begin{aligned}
Sc(\mu(C_3), x) &= 84x + 60x^2; Sc(\mu(C_3)) = 204 \\
Sc^*(\mu(C_3), x) &= 57x + 99x^2; Sc^*(\mu(C_3)) = 255 \\
Sc(\mu(C_4), x) &= 116x + 76x^2; Sc(\mu(C_4)) = 268 \\
Sc^*(\mu(C_4), x) &= 88x + 214x^2; Sc^*(\mu(C_4)) = 516 \\
Sc(\mu(C_5), x) &= 60x + 250x^2; Sc(\mu(C_5)) = 560 \\
Sc^*(\mu(C_5), x) &= 275x + 450x^2; Sc^*(\mu(C_5)) = 175
\end{aligned}$$

**Corollary 3.3.**  $\mu(C_m)$  be the Mycielskian of cycle graph, for  $m = 6, 7$  is,

$$\begin{aligned}
Sc(\mu(C_m), x) &= m(m + 25)x + 2m(2m + 15)x^2 + m(7m - 27)x^3 \\
Sc(\mu(C_m)) &= 30m^2 + 4m \\
Sc^*(\mu(C_m), x) &= m(3m + 40)x + \frac{m}{2}(17m + 95)x^2 + 4m(3m - 11)x^3 \\
Sc^*(\mu(C_m)) &= 56m^2 + 3m.
\end{aligned}$$

#### 4. Results on Mycielskian of Complete Bipartite Graph

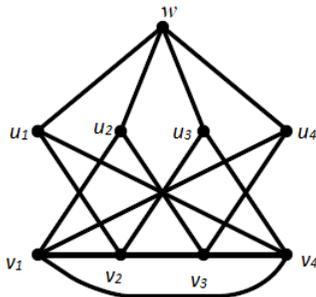


Figure 3. Mycielskian graph of  $K_{2,2}$

**Theorem 4.1.** Let Mycielskian of complete bipartite graph be denoted by  $\mu(K_{m,m})$ , for all  $m \geq 2$  is

$$\begin{aligned}
Sc(\mu(K_{m,m}), x) &= 2m(5m^2 + 4m + 1)x + 4m(2m^2 + 6m + 1)x^2 \\
Sc(\mu(K_{m,m})) &= 2m(13m^2 + 28m + 5) \\
Sc^*(\mu(K_{m,m}), x) &= 4m^2(2m^2 + 2m + 1)x + m(6m^3 + 19m^2 + 12m - 1)x^2 \\
Sc^*(\mu(K_{m,m})) &= 2m(10m^3 + 23m^2 + 14m - 1)
\end{aligned}$$

**Proof.** Consider Mycielskian of complete bipartite graph  $\mu(K_{m,m})$ ,  $\forall m \geq 2$  and it contains  $2m + 1$  vertices and  $3m^2 + 2m$  edges. From the structure of Mycielskian complete bipartite graph we could observe that  $2m + 1$  vertices have degree  $2m$  and  $2m$  vertices have degree  $m + 1$ . Thus we have two partitions of the vertex set  $V(\mu(K_{m,m}))$  as follows.

$$\begin{aligned}
V_1 &= \{v \in V(\mu(K_{m,m}))/d_v = 2m\} \\
V_2 &= \{v \in V(\mu(K_{m,m}))/d_v = m + 1\} \\
\mu(K_{m,m}) &= V_1 \cup V_2 \text{ and } V_1 \cap V_2 = \phi \text{ thus}
\end{aligned}$$

$$|E(\mu(K_{m,m}))| = \frac{1}{2}[(2m+1)|V_1| + 2m|V_2|] = 3m^2 + 2m$$

From the structure of Mycielskian complete bipartite graph  $\mu(P_m)$ , we see that  $\forall u, v \in V(\mu(K_{m,m})) \exists d(u, v) \in \{1, 2\}$  and the diameter  $D$  of mycielskian complete bipartite graph  $\mu(K_{m,m})$  is equal to  $D(\mu(K_{m,m})) = 2$ .

Now, we compute the cases of  $d(u, v)$  - edge paths  $d(u, v) = 1, 2$  of  $\mu(K_{m,m})$  in table 3.

Table 3: All cases of  $d(u, v)$ 

The distance $d(u, v) = i$	Degrees of $d_u, d_v$	Number of i-edge paths	Term of Schultz polynomial	Term of modified Schultz polynomial
1	$2m, 2m$	$m^2$	$4m$	$4m^2$
1	$2m, m+1$	$2m^2 + 2m$	$3m+1$	$2m^2 + 2m$
2	$2m, 2m$	$m^2 + m$	$4m$	$4m^2$
2	$m+1, m+1$	$m(2m-1)$	$2m+2$	$(m+1)^2$
2	$2m, m+1$	$6m$	$3m+1$	$2m^2 + 2m$

for  $d(u, v) = 1$ , term of  $Sc(\mu(K_{m,m}), x)$  and  $Sc^*(\mu(K_{m,m}), x)$  will be  
 $4m(m^2)x + (3m+1)(2m^2+2m)x = (10m^3 + 8m^2 + 2m)x$

and

$$4m^2(m^2)x + (2m^2+2m)(2m^2+2m)x = (8m^4 + 8m^3 + 4m^2)x$$

for  $d(u, v) = 2$ , term of  $Sc(\mu(K_{m,m}), x)$  and  $Sc^*(\mu(K_{m,m}), x)$  will be

$$4m(m^2+m)x^2 + (2m+2)(m(2m-1))x^2 + (3m+1)(6m)x^2 = (8m^3 + 24m^2 + 4m)x^2$$

and

$$4m^2(m^2+m)x^2 + (m+1)^2(m(2m-1))x^2 + (m^2+2m)(6m)x^2 = (6m^4 + 19m^3 + 12m^2 - m)x^2$$

Hence from the following definitions, the  $Sc(\mu(K_{m,m}))$  and  $Sc^*(\mu(K_{m,m}), x)$  is given by

$$Sc(\mu(K_{m,m}), x) = \sum_{u,v \in V(\mu(K_{m,m}))} (d_u + d_v)x^{d(u,v)}, \quad Sc^*(\mu(K_{m,m}), x) = \sum_{u,v \in V(\mu(K_{m,m}))} (d_u \times d_v)x^{d(u,v)}$$

$$Sc(\mu(K_{m,m}), x) = 2m(5m^2 + 4m + 1)x + 4m(2m^2 + 6m + 1)x^2$$

$$Sc^*(\mu(K_{m,m}), x) = 4m^2(2m^2 + 2m + 1)x + m(6m^3 + 19m^2 + 12m - 1)x^2$$

$$Sc(\mu(K_{m,m}), x) = \left. \frac{\partial Sc(\mu(K_{m,m}), x)}{\partial x} \right|_{x=1}$$

$$= \frac{\partial}{\partial x} [2m(5m^2 + 4m + 1)x + 4m(2m^2 + 6m + 1)x^2]$$

$$= 2m[13m^2 + 28m + 5]$$

$$Sc^*(\mu(K_{m,m})) = \left. \frac{\partial Sc^*(\mu(K_{m,m}), x)}{\partial x} \right|_{x=1}$$

$$\begin{aligned} &= \frac{\partial}{\partial x} [4m^2(2m^2 + 2m + 1)x + m(6m^3 + 19m^2 + 12m - 1)x^2] \\ &= 2m[10m^3 + 23m^2 + 14m - 1] \end{aligned}$$

### References

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