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MULTIPLICATION AND TRANSLATION OF CUBIC $\beta-$IDEALS

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Abstract: Cubic set is a structure with two components which has been applied in the conditions of $\beta-$ideals. This paper presents the notion of cubic fuzzy $\beta-$ideal of a $\beta-$algebra. In addition that, the notion of cubic $(\overline{a}, b)$-translation, cubic μ-multiplication were presented. Further, some engrossing results of cubic $\beta-$ideals with the combination of multiplication and translation were investigated.

Keywords and Phrases: $\beta-$algebra, $\beta-$ideals, Cubic $\beta-$ideal, cubic ideals, Cubic translation, Cubic multiplication.

2020 Mathematics Subject Classification: 06F35, 03G25, 08A72, 03E72.

1. Introduction

The concept of fuzzy sets, a generalisation of the classical notion of set and its characteristic functions, was first developed by Zadeh [13] in 1965. The thought of

$\beta-$algebras has been initiated by Neggers and Kim [12] which is a generalization of $BCK-$algebras and $BCI-$algebras. Atanassov [2] proposed the idea of intuitionistic fuzzy sets as an extension of fuzzy set which incorporate the degrees of membership and non-membership. Abu Ayub Ansari et al. [1] presented the notion of fuzzy $\beta-$ideals of $\beta-$algebras. Hemavathi et al. [5] discussed about $\beta-$ideals which is applied in interval valued fuzzy set. The notion of cubic sets have been introduced by Jun et al. [8]. Different kinds of union and intersection of cubic sets have been explored. The notion of cubic subalgebras and ideals of $BCK/BCI-$algebras has been depicted by Jun et al. [6, 7]. Also the authors applied the cubic structures in to ideals of $BCI-$algebras. Furthermore, they have discussed about the characterizations of cubic $a-$ideal and the relations between cubic $a-$ideal and cubic $p-$ideal.

Lee et al. [10] presented the concept of fuzzy translations and fuzzy multiplications of BCK/BCI algebras, where the relationships between fuzzy translations, fuzzy extensions, and fuzzy multiplications were explored. Chandramouleeswaran et al. [3] depicted some interesting results on fuzzy translations and fuzzy multiplications in BF/BG-algebras. The notion of translation and multiplication of cubic subalgebras and cubic ideals of BCK/BCI-algebras introduced by Dutta et al. [4] and few of their properties were examined. A number of related features are examined along with the concept of cubic extension of cubic subalgebras and cubic extension cubic ideals. Khalid et al [9] initiated the perception on translation and multiplication of a neutrosophic cubic set. Recently, Muralikrishna et al. [11] exhibited some aspects on cubic fuzzy $\beta-$subalgebra of $\beta-$algebra. With all these inspiration and motivation, this work presents the notion of $\mu-$multiplication and $(\overline{a}, b)-$translation of cubic $\beta-$ideal and few of its associated results have been studied.

2. Preliminaries

This section reveals the necessary definitions required for the work.

Definition 2.1. [12] *A $\beta-$ algebra is a non-empty set X with a constant 0 and two binary operations $+$ and $-$ are satisfying the following axioms:*
(i) $x - 0 = x$
(ii) $(0 - x) + x = 0$
(iii) $(x - y) - z = x - (z + y) \;\; \forall\, x, y, z \in X.$

Definition 2.2. [1] *A non-empty subset I of a $\beta-$algebra $(X, +, -, 0)$ is called a $\beta-$ideal of X, if*
(i) $0 \in I$
(ii) $x + y \in I$

(iii) $x - y$ *&* $y \in I$ *then* $x \in I \quad \forall x, y \in X.$

Example 2.3. The following Cayley table shows $(X = \{0, 1, 2, 3\}, +, -, 0)$ is a $\beta-$algebra.

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	1	0
3	3	2	0	1

−	0	1	2	3
0	0	1	3	2
1	1	0	2	3
2	2	3	0	1
3	3	2	1	0

Definition 2.4. [8] *Let X be a non empty set. By a cubic set in X we mean a structure $C = \{\langle x, \overline{\zeta}_C(x), \eta_C(x)\rangle :\ x \in\ X\}$ in which $\overline{\zeta}_C$ is an interval valued fuzzy set in X and η_C is a fuzzy set in X.*

Definition 2.5. [1] *Let $C = \{x, \overline{\zeta}_C(x), \eta_C(x) : x \in X\}$ be a cubic fuzzy set in a $\beta-$algebra of X. C is called a cubic fuzzy $\beta-$ideal of X, if $\forall x, y \in X$*
(i) $\overline{\zeta}_C(0) \geq \overline{\zeta}_C(x)$ & $\eta_C(0) \leq \eta_C(x)$
(ii) $\overline{\zeta}_C(x + y) \geq rmin\{\overline{\zeta}_C(x), \overline{\zeta}_C(y)\}$ & $\eta_C(x + y) \leq max\{\eta_C(x), \eta_C(y)\}$
(iii) $\overline{\zeta}_C(x) \geq rmin\{\overline{\zeta}_C(x - y), \overline{\zeta}_C(y)\}$ & $\eta_C(x) \leq max\{\eta_C(x - y), \eta_C(y)\}$

Definition 2.6. [10] *Let μ be a fuzzy subset of X and $\alpha \in [0, T]$ where $T = 1 - sup\{\mu(x)/x \in X\}$. A mapping $\mu^T_\alpha : X \to [0, 1]$ is said to be a fuzzy $\alpha-$translation of μ if it satisfies $\mu^T_\alpha(x) = \mu(x) + \alpha \quad \forall x \in X$*

Definition 2.7. [10] *Let μ be a fuzzy subset of X and $\alpha \in [0, 1]$. A mapping $\mu^M_\alpha : X \to [0, 1]$ is said to be a fuzzy α-multiplication of μ if it satisfies $\mu^M_\alpha(x) = \alpha.\mu(x), \quad \forall x \in X$.*

3. Multiplications of Cubic β-ideals

This section gives the notion of multiplications of cubic fuzzy β-ideal and some of its results are investigated.

Definition 3.1. *Let $C = \left\{\left\langle x, \overline{\zeta}_C(x), \eta_C(x)\right\rangle : x \in X\right\}$ be a cubic fuzzy set of X and $\mu \in (0, 1]$. An object having the form $C^M_\mu = \left\{\left(\overline{\zeta}_C\right)^M_\mu, (\eta_C)^M_\mu\right\}$ is said to be cubic μ-multiplication of C if it satisfies $\left(\overline{\zeta}_C\right)^M_\mu(x) = \mu.\overline{\zeta}_C(x)$ and $(\eta_C)^M_\mu(x) = \mu.\eta_C(x)$, for all $x \in X$.*

Example 3.2. For the cubic β-ideal given in example 3.4, consider $\mu = 0.6 \in (0, 1]$. Then the μ-multiplication $\left(\left(\overline{\zeta}_C\right)^T_{0.6}, (\eta_C)^T_{0.6}\right)$ of cubic set C is given by

$$\left(\overline{\zeta}_C\right)^T_{0.6} = \begin{cases} [0.24, 0.3], & x = 0 \\ [0.18, 0.24], & x = b \\ [0.12, 0.18], & x = a, c \end{cases} \quad \text{and } (\eta_C)^T_{0.6} = \begin{cases} 0.3, & x = 0 \\ 0.24, & x = b \\ 0.18, & x = a, c \end{cases}$$

Theorem 3.3. *If* $C = \left\{\left\langle x, \overline{\zeta}_C(x), \eta_C(x)\right\rangle : x \in X\right\}$ *is a cubic* β*-ideal of* X *and let* $\mu \in [0,1]$.
Then the cubic μ-multiplication C_μ^M of C is cubic β-ideal of X.
Proof. Suppose $C = \left\{\left\langle x, \overline{\zeta}_C(x), \eta_C(x)\right\rangle : x \in X\right\}$ is a cubic β-ideal of X. Then

$$\begin{aligned}
\left(\overline{\zeta}_C\right)_\mu^M(0) &= \mu.\overline{\zeta}_C(0) \\
&\geq \mu.\overline{\zeta}_C(x) \\
&= \left(\overline{\zeta}_C\right)_\mu^M(x) \\
(\eta_C)_\mu^M(0) &= \mu.\eta_C(0) \\
&\leq \mu.\eta_C(x) \\
&= (\eta_C)_\mu^M(x) \\
\left(\overline{\zeta}_C\right)_\mu^{MT}(x+y) &= \mu.\overline{\zeta}_C(x+y) \\
&\geq \mu.rmin\{\overline{\zeta}_C(x), \overline{\zeta}_C(y)\} \\
&= rmin\{\mu.\overline{\zeta}_C(x)\ , \mu.\overline{\zeta}_C(y)\} \\
&= rmin\left\{\left(\overline{\zeta}_C\right)_\mu^M(x), \left(\overline{\zeta}_C\right)_\mu^M(y)\right\} \\
(\eta_C)_\mu^M(x+y) &= \mu.\eta_C(x+y) \\
&\leq \mu.max\{\eta_C(x), \eta_C(y)\} \\
&= max\{\mu.\eta_C(x), \mu.\eta_C(y)\} \\
&= max\left\{(\eta_C)_\mu^M(x), (\eta_C)_\mu^M(y)\right\} \\
\left(\overline{\zeta}_C\right)_\mu^M(x) &= \mu.\overline{\zeta}_C(x) \\
&\geq \mu.rmin\{\overline{\zeta}_C(x-y), \overline{\zeta}_C(y)\} \\
&= rmin\{\mu.\overline{\zeta}_C(x-y), \mu.\overline{\zeta}_C(y)\} \\
&= rmin\left\{\mu.\overline{\zeta}_C(x-y), \mu.\overline{\zeta}_C(y)\right\} \\
&= rmin\left\{\left(\overline{\zeta}_C\right)_\mu^M(x-y), \left(\overline{\zeta}_C\right)_\mu^M(y)\right\} \\
(\eta_C)_\mu^M(x) &= \mu.\eta_C(x) \\
&\leq \mu.max\{\eta_C(x-y), \eta_C(y)\} \\
&= max\{\mu.\eta_C(x-y), \mu.\eta_C(y)\} \\
&= max\left\{(\eta_C)_\mu^M(x-y), (\eta_C)_\mu^M(y)\right\}
\end{aligned}$$

For all $x, y \in X$ and $\mu \in (0,1]$. Hence C_{μ}^{M} of C is cubic β-ideal of X.

Theorem 3.4. *If C is a cubic set of X such that cubic μ-multiplication C_{μ}^{M} of C is cubic β-ideal of X and $\mu \in [0,1]$ then C is cubic β-ideal of X.*

Proof. Assume that $C_{\mu}^{M}(x)$ of C be a cubic β-ideal of X, $\mu \in (0,1]$. Then

$$\begin{aligned}\mu.\overline{\zeta}_C(0) &= \left(\overline{\zeta}_C\right)_{\mu}^{M}(0)\\ &\geq \left(\overline{\zeta}_C\right)_{\mu}^{M}(x)\\ &= \mu.\overline{\zeta}_C(x)\end{aligned}$$

In the same manner, we have $\mu.\eta_C(0) \leq \mu.\eta_C(x)$

$$\begin{aligned}\mu.\overline{\zeta}_C(x+y) &= \left(\overline{\zeta}_C\right)_{\mu}^{M}(x+y)\\ &\geq rmin\{\left(\overline{\zeta}_C\right)_{\mu}^{M}(x), \left(\overline{\zeta}_C\right)_{\mu}^{M}(y)\}\\ &= rmin\{\mu.\overline{\zeta}_C(x), \mu.\overline{\zeta}_C(y)\}\\ &= \mu.rmin\left\{\overline{\zeta}_C(x), \overline{\zeta}_C(y)\right\}\end{aligned}$$

Likewise we get $\mu.\eta_C(x+y) \leq \mu.max\{\eta_C(x), \eta_C(y)\}$

$$\begin{aligned}\mu.\overline{\zeta}_C(x) &= \left(\overline{\zeta}_C\right)_{\mu}^{M}(x)\\ &\geq rmin\{\left(\overline{\zeta}_C\right)_{\mu}^{M}(x-y), \left(\overline{\zeta}_C\right)_{\mu}^{M}(y)\}\\ &= rmin\{\mu.\overline{\zeta}_C(x-y), \mu.\overline{\zeta}_C(y)\}\\ &= \mu.rmin\left\{\overline{\zeta}_C(x-y), \overline{\zeta}_C(y)\right\}\end{aligned}$$

In a similar way we may have $\mu.\eta_C(x) \leq \mu.max\{\eta_C(x-y), \eta_C(y)\}$

For all $x, y \in X$ and $\mu \in (0,1]$. Hence C is cubic β-ideal of X.

4. Translation of Cubic $\beta-$ideals

In this section, the notion of translation of Cubic $\beta-$ideals is presented and examined some delightful results based on union and intersection. We use $\mathfrak{B} = \inf\{\eta_C(x)/x \in X\}$ and $\widetilde{\sigma} = \left(\sigma^L, \sigma^U\right)$ where $\sigma^U = 1 - sup\left\{\zeta_C^U(x)/x \in X\right\}$ for any cubic set $C = \left\{\left\langle x, \overline{\zeta}_C(x), \eta_C(x)\right\rangle : x \in X\right\}$.

Definition 4.1. *Let $C = \left\{\left\langle x, \overline{\zeta}_C(x), \eta_C(x)\right\rangle : x \in X\right\}$ be a cubic fuzzy set of X and $0 \leq a^U \leq \sigma^U$ where $\overline{a} = \left(a^L, a^U\right) \in D[0, \sigma^U]$ and $b \in [0, \mathfrak{B}]$. An object having the form $C_{\overline{a},b}^{T} = \left\{\left(\overline{\zeta}_C\right)_{\overline{a}}^{T}, (\eta_C)_b^T\right\}$ is said to be cubic $(\overline{a}, b)$-translation of C if it satisfies $\left(\overline{\zeta}_C\right)_{\overline{a}}^{T}(x) = \overline{\zeta}_C(x) + \overline{a}$, $(\eta_C)_b^T(x) = \eta_C(x) - b$, for all $x \in X$.*

Example 4.2. For the cubic β-ideal given in example 3.2, consider $\sigma^U = 1 - sup\left\{\zeta_C^U(x)/x \in X\right\}$=1-0.5=0.5 and $\mathfrak{B} = \inf\{\eta_C(x)/x \in X\}$=0.3. Let $\overline{a} = [0.15, 0.25] \in D[0, \sigma^U]$ and $b = 0.2 \in [0, \mathfrak{B}]$. Then the $(\overline{a}, b)$-translation $\left((\overline{\zeta}_C)^T_{[0.15,0.25]}, (\eta_C)^T_{0.2}\right)$ of cubic set C is given by

$$\left(\overline{\zeta}_C\right)^T_{[0.15,0.25]} = \begin{cases} [0.55, 0.75], & x = 0 \\ [0.45, 0.65], & x = b \\ [0.35, 0.55], & x = a, c \end{cases} \quad \text{and } (\eta_C)^T_{0.2} = \begin{cases} 0.3, & x = 0 \\ 0.2, & x = b \\ 0.1, & x = a, c \end{cases}$$

Theorem 4.3. *Let* $C = \left\{\left\langle x, \overline{\zeta}_C(x), \eta_C(x)\right\rangle : x \in X\right\}$ *be a cubic* β*-ideal of* X *and let* $\overline{a} \in D[0, \sigma^U]$, $b \in [o, \mathfrak{B}]$ *if and only if the cubic* $(\overline{a}, b)$*-translation* $C^T_{\overline{a},b}$ *of* C *is cubic* β*-ideal of* X*.*

Proof. Suppose $C = \left\{\left\langle x, \overline{\zeta}_C(x), \eta_C(x)\right\rangle : x \in X\right\}$ is a cubic β-ideal of X. Then

$$\begin{aligned}
\left(\overline{\zeta}_C\right)^T_{\overline{a}}(0) &= \overline{\zeta}_C(0) + \overline{a} \\
&\geq \overline{\zeta}_C(x) + \overline{a} \\
&= \left(\overline{\zeta}_C\right)^T_{\overline{a}}(x) \\
(\eta_C)^T_b(0) &= \eta_C(0) - b \\
&\leq \eta_C(x) - b \\
&= (\eta_C)^T_b(x) \\
\left(\overline{\zeta}_C\right)^T_{\overline{a}}(x+y) &= \overline{\zeta}_C(x+y) + \overline{a} \\
&\geq rmin\{\overline{\zeta}_C(x), \overline{\zeta}_C(y)\} + \overline{a} \\
&= rmin\{\overline{\zeta}_C(x) + \overline{a}, \overline{\zeta}_C(y) + \overline{a}\} \\
&= rmin\left\{\left(\overline{\zeta}_C\right)^T_{\overline{a}}(x), \left(\overline{\zeta}_C\right)^T_{\overline{a}}(y)\right\} \\
(\eta_C)^T_b(x+y) &= \eta_C(x+y) - b \\
&\leq max\{\eta_C(x), \eta_C(y)\} - b \\
&= max\{\eta_C(x) - b, \eta_C(y) - b\} \\
&= max\left\{(\eta_C)^T_b(x), (\eta_C)^T_b(y)\right\} \\
\left(\overline{\zeta}_C\right)^T_{\overline{a}}(x) &= \overline{\zeta}_C(x) + \overline{a} \\
&\geq rmin\{\overline{\zeta}_C(x-y), \overline{\zeta}_C(y)\} + \overline{a} \\
&= rmin\{\overline{\zeta}_C(x-y) + \overline{a}, \overline{\zeta}_C(y) + \overline{a}\} \\
&= rmin\left\{\overline{\zeta}_C(x-y) + \overline{a}, \overline{\zeta}_C(y) + \overline{a}\right\} \\
&= rmin\left\{\left(\overline{\zeta}_C\right)^T_{\overline{a}}(x-y), \left(\overline{\zeta}_C\right)^T_{\overline{a}}(y)\right\}
\end{aligned}$$

$$\begin{aligned}(\eta_C)_b^T(x) &= \eta_C(x) - b\\ &\leq max\left\{\eta_C(x-y), \eta_C(y)\right\} - b\\ &= max\left\{\eta_C(x-y) - b, \eta_C(y) - b\right\}\\ &= max\left\{(\eta_C)_b^T(x-y), (\eta_C)_b^T(y)\right\}\end{aligned}$$

Hence $C_{\overline{a},b}^T$ of C is cubic β-ideal of X. Conversely, assume that $C_{\overline{a},b}^T$ be a cubic β-ideal of X. Then

$$\overline{\zeta}_C(0) + \overline{a} = \left(\overline{\zeta}_C\right)_{\overline{a}}^T(0) \qquad \geq \left(\overline{\zeta}_C\right)_{\overline{a}}^T(x) = \overline{\zeta}_C(x) + \overline{a}$$

Similarly we have $\eta_C(0) - b \leq \eta_C(x) - b$

$$\begin{aligned}\overline{\zeta}_C(x+y) + \overline{a} &= \left(\overline{\zeta}_C\right)_{\overline{a}}^T(x+y)\\ &\geq rmin\{\left(\overline{\zeta}_C\right)_{\overline{a}}^T(x), \left(\overline{\zeta}_C\right)_{\overline{a}}^T(y)\}\\ &= rmin\{\overline{\zeta}_C(x) + \overline{a}, \overline{\zeta}_C(y) + \overline{a}\}\\ &= rmin\left\{\overline{\zeta}_C(x), \overline{\zeta}_C(y)\right\} + \overline{a}\end{aligned}$$

In a same manner, we may have $\eta_C(x+y) - b \leq max\left\{\eta_C(x), \eta_C(y)\right\} - b$

$$\begin{aligned}\overline{\zeta}_C(x) + \overline{a} &= \left(\overline{\zeta}_C\right)_{\overline{a}}^T(x)\\ &\geq rmin\{\left(\overline{\zeta}_C\right)_{\overline{a}}^T(x-y), \left(\overline{\zeta}_C\right)_{\overline{a}}^T(y)\}\\ &= rmin\{\overline{\zeta}_C(x-y) + \overline{a}, \overline{\zeta}_C(y) + \overline{a}\}\\ &= rmin\left\{\overline{\zeta}_C(x-y), \overline{\zeta}_C(y)\right\} + \overline{a}\end{aligned}$$

Likewise we get $\eta_C(x) - b \leq max\left\{\eta_C(x-y), \eta_C(y)\right\} - b$

For all $x, y \in X$ and $\overline{a} \in D[0, \sigma^U]$, $b \in [o, \mathfrak{B}]$. Hence C is cubic β-ideal of X.

5. Conclusion

This study depicts the enhancement of $\mu-$multiplication and $(\overline{a}, b)$-translation incorporated with cubic $\beta-$ideals. Some of the astonishing outcomes of the same has been established. In future, this can be applied into various algebraic substructures.

References

[1] Aub Ayub Anasri, M. and Chandramouleeswaran, M., Fuzzy $\beta-$ideals of $\beta-$algebras, International Journal of Mathematical Science and Engineering applications, 5, (1) (2014), 1-10.

[2] Atanassov, K. T., Intuitionistic fuzzy sets, Fuzzy sets and systems, 20, (1) (1986), 87-96.

[3] Chandramouleeswaran, M., Muralikrishna, P. and Srinivasan, S., Fuzzy translation and fuzzy multiplication in BF/BG-algebras, Indian Journal of Science and Technology, 6, (2013) (9), 5216-5219.

[4] Dutta, A. K. Barbhuiya, S. R. and Dutta Choudhury, K., Translations and multiplications of cubic subalgebras and cubic ideals of BCK/BCI-algebras, Sohag Journal of Mathematics, 4, (3) (2017), 75-86.

[5] Hemavathi, P., Muralikrishna, P. and Palanivel, K., i-v-f beta-ideals of beta-Algebras, Materials Science and Engineering Conference Series, 263, (4) (2017), 1-10.

[6] Jun, Y. B., Kim, C. S. and Kang, M. S., Cubic subalgebras and ideals of BCK/BCI-algebras, Far East Journal of Mathematical Sciences, 44, (2) (2010), 239-250.

[7] Jun, Y. B., Kyoung Ja Lee. and Kang, M. S., Cubic structures applied to ideals of BCI-algebras, Computers & Mathematics with Applications, 62, (9) (2011), 3334-3342.

[8] Jun, Y. B., Kim, C. S., & Yang, K. O., Cubic sets, Annals of Fuzzy Mathematics and Informatics, 4, (1) (2012), 83-98.

[9] Khalid, M., Smarandache, F., Khalid, N. A. and Broumi, S., Translative and Multiplicative Interpretation of Neutrosophic Cubic Set, Infinite Study, (2020).

[10] Lee, K. J., Jun, Y. B. and Doh, M. I., Fuzzy translations and fuzzy multiplications of BCK/BCI-algebras, Commun. Korean Math. Soc, 24, (3) (2009), 353-360.

[11] Muralikrishna, P., Vinodkumar, R. and Palani, G., Some aspects on cubic fuzzy $\beta-$subalgebra of $\beta-$algebra, Journal of Physics: Conference Series, 1597, (1) (2020), 012-018.

[12] Neggers, J. Kim Hee Sik, On $\beta-$algebras, Mathematica Slovaca, 52, (5) (2002), 517-530.

[13] Zadeh, L. A., Fuzzy sets, Information Control, 8, (3) (1965), 338-353.