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CERTAIN GENERALIZED FRACTIONAL CALCULUS FORMULAS AND INTEGRAL TRANSFORMS OF (p, q)-EXTENDED τ-HYPERGEOMETRIC FUNCTION

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Abstract: In this paper, we established certain image formulas of the (p, q)–extended τ hypergeometric function $R^{\tau}_{p,q}(a, b; c; z)$ by employing Marichev-Sigo-Maeda(M-S-M) fractional integration and differentation formulas. Corresponding special cases for the Saigo's, Riemann-Liouville and Erdelyi-Kober fractional integral and differential operators are also deduced which are earlier obtained by Solanki et al. [23]. Further certain integral transforms of the (p, q)–extended τ hypergeometric function $R^{\tau}_{p,q}(a, b; c; z)$ are established. All the results are represented in terms of the Hadamard product of the (p, q)–extended τ hypergeometric function $R^{\tau}_{p,q}(a, b; c; z)$ and the Fox-Wright function.

Keywords and Phrases: (p, q)-extended τ–hypergeometric function; (p, q) - extended hypergeometric function; Fractional calculus operators.

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1. Introduction and Preliminaries

The general pair of fractional integral operators so-called Marichev-Saigo-Maeda(M-S-M) involving the third Appell's function of two-variables $F_3(.)$ are defined by (see, for details, [1, 11, 18, 19, 20]).

Definition 1. *Let $\xi, \xi', \varpi, \varpi', \mu \in \mathbb{C}$ and $x > 0$. Then for $\Re(\mu) > 0$,*

$$\left(I_{0,x}^{\xi,\xi',\varpi,\varpi',\mu} f\right)(x) = \frac{x^{-\xi}}{\Gamma(\mu)} \int_0^x (x-t)^{\mu-1}\, t^{-\xi'} \times F_3\left(\xi, \xi', \varpi, \varpi'; \mu; 1 - \frac{t}{x}, 1 - \frac{x}{t}\right) f(t)\, \mathrm{d}t, \quad (1.1)$$

and

$$\left(I_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu} f\right)(x) = \frac{x^{-\xi'}}{\Gamma(\mu)} \int_x^\infty (t-x)^{\mu-1}\, t^{-\xi} \times F_3\left(\xi, \xi', \varpi, \varpi'; \mu; 1 - \frac{x}{t}, 1 - \frac{t}{x}\right) f(t)\, \mathrm{d}t. \quad (1.2)$$

Here $F_3(.)$ denotes the Appell's hypergeometric function of two variables [24].

Definition 2. *Let $\xi, \xi', \varpi, \varpi', \mu \in \mathbb{C}$ and $x > 0$. Then for $\Re(\eta) > 0$,*

$$\begin{aligned}\left(D_{0,x}^{\xi,\xi',\varpi,\varpi',\mu} f\right)(x) &= \left(I_{0+}^{-\xi',-\xi,-\varpi',-\varpi,-\mu} f\right)(x) \\ &= \left(\frac{d}{dx}\right)^n \left(I_{0+}^{-\xi',-\xi,-\varpi'+n,-\varpi,-\mu+n} f\right)(x) \quad (n = [\Re(\mu)] + 1) \\ &= \frac{1}{\Gamma(n-\mu)} \left(\frac{d}{dx}\right)^n x^{\xi'} \int_0^x (x-t)^{n-\eta-1}\, t^{\sigma} \\ &\quad \times F_3\left(-\xi', -\xi, n - \varpi', -\varpi; n - \mu; 1 - \frac{t}{x}, 1 - \frac{x}{t}\right) f(t)\, \mathrm{d}t, \quad (1.3)\end{aligned}$$

and

$$\begin{aligned}\left(D_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu} f\right)(x) &= \left(I_{-}^{-\xi',-\xi,-\varpi',-\varpi,-\mu} f\right)(x) \\ &= \left(-\frac{d}{dx}\right)^n \left(I_{-}^{-\xi',-\xi,-\varpi',-\varpi'+n,-\mu+n} f\right)(x) \quad (n = [\Re(\eta)] + 1) \\ &= \frac{1}{\Gamma(n-\mu)} \left(-\frac{d}{dx}\right)^n x^{\xi'} \int_x^\infty (t-x)^{n-\mu-1}\, t^{\sigma'} \\ &\quad \times F_3\left(-\xi', -\xi, \varpi', n - \varpi; n - \mu; 1 - \frac{x}{t}, 1 - \frac{t}{x}\right) f(t)\, \mathrm{d}t. \quad (1.4)\end{aligned}$$

respectively.
In recent years, extension of a number of well-known special function have been investigated, for example, the (p,q)–variant, and in turn, when $p = q$ the p–variant together with the set of related higher transcendental hypergeometric type special functions (see, for details, [2, 3, 4, 5, 6, 7, 10, 13, 15, 16]). In particular, Parmar *et al.* [14] introduced and studied the (p,q)–extended τ hypergeometric function $R^{\tau}_{p,q}(a,b;c;z)$ with $\min\{\Re(p),\Re(q)\} > 0, \tau \geq 0, |z| < 1$ while $\Re(c) > \Re(b) > 0$ when $p = q = 0$ in the form:

$$R^{\tau}_{p,q}(a,b;c;z) = \sum_{n=0}^{\infty} (a)_n \, \frac{\mathrm{B}(b+\tau n,\, c-b\,;\, p,q)}{\mathrm{B}(b,\, c-b)} \, \frac{z^n}{n!} \tag{1.5}$$

where $\mathrm{B}(x,y;p,q)$ is the (p,q)–extended Beta function introduced by Choi *et al.* [8]

$$\mathrm{B}(x,y;p,q) = \int_0^1 t^{x-1}(1-t)^{y-1}\, \mathrm{e}^{-\frac{p}{t}-\frac{q}{1-t}} \mathrm{d}t\,, \tag{1.6}$$

when $\min\{\Re(x),\Re(y)\} > 0; \min\{\Re(p),\Re(q)\} \geq 0$. They established its various properties as integral representations, Mellin transforms, complete monotonicity, Turán type inequality and associated non-homogeneous differential-difference equations. The case $p = 0 = q$ reduces for series to Virchenko's τ–hypergeometric function

$${}_2R^{\tau}_1(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)} \sum_{n=0}^{\infty} (a)_n \, \frac{\Gamma(b+\tau n)}{\Gamma(c+\tau n)} \, \frac{z^n}{n!}\,.$$

The concept of the Hadamard product (or convolution) of two analytic functions is required in our current investigation. It can aid in the decomposition of a newly emerged function into two known functions. If one of the power series, in particular, describes an entire function, then the Hadamard product series also defines an entire function. If we assume

$$g(z) := \sum_{n=0}^{\infty} c_n z^n \;\; (|z| < R_f) \;\text{ and }\; h(z) := \sum_{n=0}^{\infty} d_n z^n \;\; (|z| < R_g)$$

two given power series and whose radii of convergence are given by R_f and R_g, respectively. Then their Hadamard product(or convolution) is the power series defined by

$$(g \,*\, h)(z) := \sum_{n=0}^{\infty} c_n \, d_n z^n = (h \,*\, g)(z) \;\; (|z| < R) \tag{1.7}$$

where

$$R = \lim_{n\to\infty}\left|\frac{c_n\, d_n}{c_{n+1}\, d_{n+1}}\right| = \left(\lim_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right|\right)\cdot\left(\lim_{n\to\infty}\left|\frac{d_n}{d_{n+1}}\right|\right) = R_f\, .R_g,$$

so that, in general, we have $R \geqq R_f \cdot R_g$.

The Fox-Wright function${}_r\Psi_s(z)$ $(r,\ s \in \mathbb{N}_0)$, which is a generalization of hypergeometric function, is defined as follows (see, for details, [9, 12]; see also [21, 24]):

$${}_r\Psi_s\left[\begin{array}{l}(a_1, A_1), \cdots, (a_s, A_s);\\ (b_1, B_1), \cdots, (b_s, B_s);\end{array} z\right] = \sum_{n=0}^{\infty} \frac{\Gamma(a_1 + A_1 n)\cdots\Gamma(a_r + A_r n)}{\Gamma(b_1 + B_1 n)\cdots\Gamma(b_s + B_s n)}\, \frac{z^n}{n!} \tag{1.8}$$

$$\left(A_\ell \in \mathbb{R}^+\ (\ell = 1, \ldots, r);\ B_\ell \in \mathbb{R}^+\ (\ell = 1, \ldots, s);\ 1 + \sum_{\ell=1}^{s} B_\ell - \sum_{\ell=1}^{r} A_\ell \geqq 0\right),$$

where the equality in the convergence condition holds true for

$$|z| < \nabla := \left(\prod_{\ell=1}^{r} A_\ell^{-A_\ell}\right)\ .\ \left(\prod_{\ell=1}^{s} B_\jmath^{B_\ell}\right).$$

The paper contains the study of the compositions formulas of the generalized fractional integration and differentiation operators (1.1), (1.2), (1.3) and (1.4) involving (p, q)–extended τ–hypergeometric function $R^{\tau}_{p,q}(a, b; c; z)$. Corresponding assertions for the Saigo's, Riemann-Liouville(R-L) and Erdélyi-Kober(E-K) fractional integral and differential operators are deduced which are earlier obtained by Solanki et al. [23]. Further certain integral transforms of the (p, q)–extended τ hypergeometric function $R^{\tau}_{p,q}(a, b; c; z)$ are established. All the results are represented in terms of the Hadamard product of the (p, q)–extended τ hypergeometric function $R^{\tau}_{p,q}(a, b; c; z)$ and the Fox-Wright function.

2. Fractional operators of the $R^{\tau}_{p,q}(a, b; c; z)$

We begin the main results exposition with showing a composition formulas of generalized fractional integrals (1.1) and (1.2) and differentiation (1.3) and (1.4) involving Appell's function for the (p, q)–extended τ hypergeometric function $R^{\tau}_{p,q}(a, b; c; z)$.

Lemma 1. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho \in \mathbb{C}$. *Then for* $x > 0$, *the following relation holds*
(a) *If* $\Re(\mu) > 0$ *and* $\Re(\varrho) > \max\{0, \Re(\xi + \xi' + \varpi - \mu), \Re(\xi' - \varpi')\}$, *then*

$$\left(I_{0,x}^{\xi,\xi',\varpi,\varpi',\mu} t^{\varrho-1}\right)(x) = \frac{\Gamma(\varrho)\Gamma(\varrho + \mu - \xi - \xi' - \varpi)\Gamma(\varrho + \varpi' - \xi')}{\Gamma(\varrho + \varpi')\Gamma(\varrho + \mu - \xi - \xi')\Gamma(\varrho + \mu - \xi' - \varpi)} x^{\varrho+\mu-\xi-\xi'-1}. \tag{2.1}$$

(b) *If* $\Re(\mu) > 0$ *and* $\Re(\varrho) < 1 + \min\{\Re(-\varpi), \Re(\xi + \xi' - \mu), \Re(\xi + \varpi' - \mu)\}$, *then*

$$\left(I_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu} t^{\varrho-1}\right)(x) = \frac{\Gamma(1-\varrho-\varpi)\Gamma(1-\varrho-\mu+\xi+\xi')\Gamma(1-\varrho-\mu+\xi+\varpi')}{\Gamma(1-\varrho)\Gamma(1-\varrho-\mu+\xi+\xi'+\varpi')\Gamma(1-\varrho+\xi-\varpi)} x^{\varrho+\mu-\xi-\xi'-1}. \tag{2.2}$$

Lemma 2. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho \in \mathbb{C}$ *and* $x > 0$. *Then for* $x > 0$, *the following relation holds*

(a) *If* $\Re(\mu) > 0$ *and* $\Re(\varrho) > \max\{0, \Re(\mu - \xi - \xi' + \varpi'), \Re(\varpi - \xi)\}$, *then*

$$\left(D_{0,x}^{\xi,\xi',\varpi,\varpi',\mu} t^{\varrho-1}\right)(x) = \frac{\Gamma(\varrho)\Gamma(\varrho-\mu+\xi+\xi'+\varpi')\Gamma(\varrho-\varpi+\xi)}{\Gamma(\varrho-\varpi)\Gamma(\varrho-\mu+\xi+\xi')\Gamma(\varrho-\mu+\xi+\varpi')} x^{\varrho-\mu+\xi+\xi'-1}. \tag{2.3}$$

(b) *If* $\Re(\mu) > 0$ *and* $\Re(\varrho) < 1 + \min\{\Re(\varpi'), \Re(\mu - \xi - \xi', \Re(\eta - \xi' - \varpi)\}$, *then*

$$\left(D_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu} t^{\varrho-1}\right)(x) = \frac{\Gamma(1-\varrho-\varpi')\Gamma(1-\varrho+\mu-\xi-\xi')\Gamma(1-\varrho+\mu-\xi'-\varpi)}{\Gamma(1-\varrho)\Gamma(1-\varrho+\mu-\xi-\xi'-\nu)\Gamma(1-\varrho-\xi'-\varpi')} x^{\varrho-\mu+\xi+\xi'-1}. \tag{2.4}$$

Theorem 1. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(\mu) > 0$ *and* $\Re(\varrho) > \max[0, \Re(\xi + \xi' + \varpi - \mu), \Re(\xi' - \varpi')]$. *Then the below mentioned fractional integration formula holds true:*

$$\begin{aligned} \left(I_{0,x}^{\xi,\xi',\varpi,\varpi',\mu} \left\{t^{\varrho-1} R_{p,q}^{\tau}(a,b;c;t^{\gamma})\right\}\right)(x) &= x^{\varrho+\mu-\xi-\xi'-1} R_{p,q}^{\tau}(a,b;c;x^{\gamma}) \\ &* {}_4\Psi_3\left[\begin{array}{c} (1,1), (\varrho,\gamma), (\varrho+\mu-\xi-\xi'-\varpi,\gamma), (\varrho+\varpi'-\xi',\gamma); \\ (\varrho+\varpi',\gamma), (\varrho+\mu-\xi-\xi',\gamma)(\varrho+\mu-\xi'-\varpi,\gamma); \end{array} (x^{\gamma})\right]. \end{aligned} \tag{2.5}$$

Proof. Applying definition (1.5), using (1.1) and (2.1) and changing the orders of integration and summation, we find for $x > 0$

$$\begin{aligned} &\left(I_{0,x}^{\xi,\xi',\varpi,\varpi',\mu} \left\{t^{\varrho-1} R_{p,q}^{\tau}(a,b;c;t^{\gamma})\right\}\right)(x) \\ &= \sum_{k=0}^{\infty} (a)_k \frac{\mathrm{B}(b+\tau k,\, c-b\,;\, p,q)}{\mathrm{B}(b,\, c-b) k!} \left(I_{0,x}^{\xi,\xi',\varpi,\varpi',\mu} t^{\varrho+\gamma k-1}\right)(x) \\ &= x^{\varrho+\mu-\xi-\xi'-1} \sum_{k=0}^{\infty} (a)_k \frac{\mathrm{B}(b+\tau n,\, c-b\,;\, p,q)}{\mathrm{B}(b,\, c-b)\, k!} \times \frac{\Gamma(1+k)\Gamma(\varrho+\gamma k)}{\Gamma(\varrho+\varpi'+\gamma k)} \\ &\quad \times \frac{\Gamma(\varrho+\mu-\xi-\xi'+\varpi+\gamma k)\Gamma(\varrho+\varpi'-\xi'+\gamma\nu+\gamma k)}{\Gamma(\varrho+\mu-\xi-\xi'+\gamma k)\Gamma(\varrho+\mu-\xi'-\varpi+\gamma k)\, k!} \frac{(x^{\gamma})^k}{k!}. \end{aligned} \tag{2.6}$$

By applying the Hadamard product (1.7) in (2.6), which in view of (1.5) and (1.8), yields the desired formula (2.5).

Theorem 2. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$,

$\Re(\mu) > 0$ *and* $\Re(\varrho) < 1 + \min\{\Re(-\varpi), \Re(\xi + \xi' - \mu), \Re(\xi + \varpi' - \mu)\}$. *Then the below mentioned fractional integration holds true:*

$$\left(I_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{t^{\gamma}}\right))\right\}\right)(x) = x^{\varrho+\mu-\xi-\xi'-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{x^{\gamma}}\right))$$
$$* \ {}_4\Psi_3\left[\begin{array}{c}(1,1),(1-\varrho-\varpi,\gamma),(1-\varrho-\mu+\xi+\xi',\gamma),(1-\varrho-\mu+\varpi'+\xi,\gamma);\\(1-\varrho,\gamma),(1-\varrho-\mu+\xi+\xi'+\varpi',\gamma)(1-\varrho+\xi-\varpi,\gamma);\end{array}\ \frac{1}{x^{\gamma}}\right]. \tag{2.7}$$

Proof. Applying definition (1.5), using (1.2) and (2.2) and changing the orders of integration and summation, we find for $x > 0$

$$\left(I_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{t^{\gamma}}\right))\right\}\right)(x)$$
$$= \sum_{k=0}^{\infty}(a)_k\frac{\mathrm{B}(b+\tau k,\ c-b\,;\,p,q)}{\mathrm{B}(b,\ c-b)k!}\ \left(I_{x\infty}^{\xi,\xi',\varpi,\varpi',\mu}t^{\varrho-\gamma k-1}\right)(x)$$
$$= x^{\varrho+\mu-\xi-\xi'-1}\ \sum_{k=0}^{\infty}(a)_k\frac{\mathrm{B}(b+\tau n,\ c-b\,;\,p,q)}{\mathrm{B}(b,\ c-b)\,k!}\times\frac{\Gamma(1+k)\Gamma(1-\varrho-\varpi+\gamma k)}{\Gamma(1-\varrho+\gamma k)}$$
$$\times\frac{\Gamma(1-\varrho-\mu+\xi+\xi'+\gamma k)\Gamma(1-\varrho-\mu+\xi+\varpi'+\gamma k)}{\Gamma(1-\varrho-\mu+\xi+\xi'+\varpi'+\gamma k)\Gamma(1-\varrho+\xi-\varpi+\gamma k)}\frac{(x^{-\gamma k})}{k!}. \tag{2.8}$$

By applying the Hadamard product (1.7) in (2.8), which in view of (1.5) and (1.8), yields the desired formula (2.7).

Theorem 3. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(\mu) > 0$ *and* $\Re(\varrho) > \max[0, \Re(\mu - \xi + \xi' - \varpi'), \Re(\varpi - \xi)]$. *Then the below mentioned fractional differentiation formula holds true:*

$$\left(D_{0,x}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;t^{\gamma})\right\}\right)(x) = x^{\varrho-\mu+\xi+\xi'-1}R_{p,q}^{\tau}(a,b;c;x^{\gamma})$$
$$* \ {}_4\Psi_3\left[\begin{array}{c}(1,1),(\varrho,\gamma),(\varrho-\mu+\xi+\xi'+\varpi',\gamma),(\varrho-\varpi+\xi,\gamma);\\(\varrho-\varpi,\gamma),(\varrho-\mu+\xi+\xi',\gamma)(\varrho-\mu+\xi+\varpi',\gamma);\end{array}\ (x^{\gamma})\right]. \tag{2.9}$$

Proof. By virtue of the formulas (1.3) and (1.5), the term-by-term fractional differentiation and the application of the relation (2.3), yields for $x > 0$

$$\left(D_{0,x}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;t^{\gamma})\right\}\right)(x)$$
$$= \sum_{k=0}^{\infty}(a)_k\frac{\mathrm{B}(b+\tau k,\ c-b\,;\,p,q)}{\mathrm{B}(b,\ c-b)k!}\ \left(D_{0,,x}^{\xi,\xi',\varpi,\varpi',\mu}t^{\varrho+\gamma k-1}\right)(x)$$

$$= x^{\varrho-\mu+\xi+\xi'-1} \sum_{k=0}^{\infty} (a)_k \frac{\mathrm{B}(b+\tau n,\, c-b\,;\, p,q)}{\mathrm{B}(b,\, c-b)\, k!}$$

$$\times \frac{\Gamma(1+k)\Gamma(\varrho+\gamma k)\Gamma(\varrho-\mu+\xi+\xi'+\varpi'\gamma k)\Gamma(\varrho+\xi-\varpi+\gamma k)}{\Gamma(\varrho-\varpi+\gamma k)\Gamma(\varrho-\mu+\xi+\xi'+\gamma k)\Gamma(\varrho-\mu+\xi+\varpi'+\gamma k)} \frac{(x^{\gamma k})}{k!}. \tag{2.10}$$

By applying the Hadamard product (1.7) in (2.10), which in view of (1.5) and (1.8), yields the desired formula 2.9.

Theorem 4. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(c) > 0$ *and* $\Re(\varrho) < 1 + \min\{\Re(\varpi'), \Re(\mu-\xi-\xi'), \Re(\mu-\xi'-\varpi)\}$. *Then the below mentioned fractional differentiation holds true:*

$$\left(D_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{t^{\gamma}}\right))\right\}\right)(x) = x^{\varrho-\mu+\xi+\xi'-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{x^{\gamma}}\right))$$

$$* \;{}_4\Psi_3\left[\begin{matrix}(1,1),(1-\varrho-\varpi',\gamma),(1-\varrho+\mu-\xi-\xi',\gamma),(1-\varrho+\mu-\varpi-\xi',\gamma); \\ (1-\varrho,\gamma),(1-\varrho+\mu-\xi-\xi'-\varpi,\gamma)(1-\varrho-\xi'-\varpi',\gamma);\end{matrix}\ \frac{1}{x^{\gamma}}\right]. \tag{2.11}$$

Proof. By virtue of the formulas (1.4) and (1.5), the term-by-term fractional differentiation and the application of the relation (2.4), yields for $x > 0$

$$\left(D_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{t^{\gamma}}\right))\right\}\right)(x)$$

$$= \sum_{k=0}^{\infty} (a)_k \frac{\mathrm{B}(b+\tau k,\, c-b\,;\, p,q)}{\mathrm{B}(b,\, c-b) k!} \left(D_{x\infty}^{\xi,\xi',\varpi,\varpi',\mu} t^{\varrho-\gamma k-1}\right)(x)$$

$$= x^{\varrho-\mu+\xi+\xi'-1} \sum_{k=0}^{\infty} (a)_k \frac{\mathrm{B}(b+\tau n,\, c-b\,;\, p,q)}{\mathrm{B}(b,\, c-b)\, k!} \times \frac{\Gamma(1+k)\Gamma(1-\varrho-\varpi'+\gamma k)}{\Gamma(1-\varrho+\gamma k)}$$

$$\times \frac{\Gamma(1-\varrho+\mu-\xi-\xi'+\gamma k)\Gamma(1-\varrho+\mu-\xi'-\varpi+\gamma k)}{\Gamma(1-\varrho+\mu-\xi-\xi'-\varpi+\gamma k)\Gamma(1-\varrho-\xi'-\varpi'+\gamma k)} \frac{(x^{-\gamma k})}{k!}. \tag{2.12}$$

By applying the Hadamard product (1.7) in (2.12), which in view of (1.5) and (1.8), yields the desired formula (2.11).

3. Certain Integral Transforms of the $R_{p,q}^{\tau}(a,b;c;z)$

Here , in this section, first we give definition of the certain integral transforms as Beta, Laplace and Whittaker transforms. Then we apply to the composition formulas for generalized fractional integrals and differential operators.

Definition 3. *The Euler - Beta transformation [22] of the function f(z) is defined,* *as usual, by*

$$\mathrm{B}(f(z);a,b)=\int_0^1 z^{\alpha-1}(1-z)^{b-1}\,f(z)\mathrm{d}t\,. \tag{3.1}$$

Definition 4. *The Laplace transformation (see, e.g., [22]) of the function f(z) is defined , as usual, by*

$$\mathrm{L}(f(z);t)=\int_0^\infty \mathrm{e}^{-tz}\,f(z)\mathrm{d}z \qquad (\Re(t)>0). \tag{3.2}$$

The below integral containing Whittaker function $W_{\kappa,\nu}$

$$\int_0^\infty t^{x-1}\mathrm{e}^{-\frac{1}{2}at}\,W_{\kappa,\nu}(at)\mathrm{d}z=a^{-\rho}\frac{\Gamma(\frac{1}{2}\pm\nu+\rho)}{\Gamma(1-\kappa+\rho)} \quad (\Re(\alpha)>0,\Re(\rho\pm\nu)>-\frac{1}{2}) \tag{3.3}$$

is well known.

Theorem 5. *Let* $\xi,\xi',\varpi,\varpi',\mu,\varrho,p,q\in\mathbb{C}$ *be such that* $\min\{\Re(p),\Re(q)\}>0$, $\Re(c)>0$ *and* $\Re(\varrho)>\max[0,\Re(\xi+\xi'+\varpi-\mu),\Re(\xi'-\varpi')]$ *. Then the below mentioned Euler - Beta transformation holds true:*

$$\begin{aligned}&(\mathrm{B}\left(I_{0,x}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;(tz)^{\gamma})\right\}\right)(x):l,m)\\&=x^{\varrho+\mu-\xi-\xi'-1}\Gamma(m)R_{p,q}^{\tau}(a,b;c;(x)^{\gamma})\\&*\ {}_5\Psi_4\left[\begin{matrix}(1,1),(l,\gamma),(\varrho,\gamma),(\varrho+\mu-\xi-\xi'-\varpi,\gamma)(\varrho+\varpi'-\xi',\gamma);\\(l+m,\gamma),(\varrho+\varpi',\gamma)(\varrho+\mu-\xi-\xi',\gamma)(\varrho+\mu-\xi'-\varpi,\gamma);\end{matrix}\ (x^{\gamma})\right].\end{aligned} \tag{3.4}$$

Theorem 6. *Let* $\xi,\xi',\varpi,\varpi',\mu,\varrho,p,q\in\mathbb{C}$ *be such that* $\min\{\Re(p),\Re(q)\}>0$, $\Re(c)>0$ *and* $\Re(\varrho)<1+\min\{\Re(-\varpi),\Re(\xi+\xi'-c),\Re(\xi+\varpi'-\mu)\}$*. Then the below mentioned Euler - Beta transformation holds true:*

$$\begin{aligned}&(\mathrm{B}\left(I_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{z}{t}\right)^{\gamma})\right\}\right)(x):l,m)\\&=x^{\varrho+\mu-\xi-\xi'-1}\Gamma(m)R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{x^{\gamma}}\right))\\&*\ {}_5\Psi_4\left[\begin{matrix}(1,1),(l,\gamma),(1-\varrho-\varpi,\gamma),(1-\varrho-\mu+\xi+\xi',\gamma),\\(1-\varrho-\mu+\xi+\varpi',\gamma);\\(l+m,\gamma),(1-\varrho,\gamma)(1-\varrho-\mu+\xi+\xi'+\varpi',\gamma)(1-\varrho+\xi-\varpi,\gamma);\end{matrix}\ \frac{1}{x^{\gamma}}\right].\end{aligned} \tag{3.5}$$

Theorem 7. *Let* $\xi,\xi',\varpi,\varpi',\mu,\varrho,p,q\in\mathbb{C}$ *be such that* $\min\{\Re(p),\Re(q)\}>0$, $\Re(\mu)>0$ *and* $\Re(\varrho)>\max[0,\Re(\mu-\xi-\xi'-\varpi'),\Re(\varpi-\xi)]$ *. Then the below* *mentioned Euler - Beta transformation holds true:*

$$
\begin{aligned}
&(\mathrm{B}\left(D_{0,x}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;(tz)^{\gamma})\right\}\right)(x):l,m) \\
&= x^{\varrho-\mu+\xi+\xi'-1}\Gamma(m)R_{p,q}^{\tau}(a,b;c;(x)^{\gamma}) \\
&*\ {}_5\Psi_4\left[\begin{array}{c}(1,1),(l,\gamma),(\varrho,\gamma),(\varrho-\mu+\xi+\xi'+\varpi',\gamma)(\varrho-\varpi+\xi,\gamma);\\ (l+m,\gamma),(\varrho-\varpi,\gamma)(\varrho-\mu+\xi+\xi',\gamma)(\varrho-\mu+\xi+\varpi',\gamma);\end{array}(x^{\gamma})\right]. \quad (3.6)
\end{aligned}
$$

Theorem 8. *Let $\xi,\xi',\varpi,\varpi',\mu,\varrho,p,q\in\mathbb{C}$ be such that $\min\{\Re(p),\Re(q)\}>0$, $\Re(\mu)>0$ and $\Re(\varrho)<1+\min\{\Re(\varpi'),\Re(\mu-\xi-\xi'),\Re(\mu-\xi'-\varpi)\}$. Then the below mentioned Euler - Beta transformation holds true:*

$$
\begin{aligned}
&(\mathrm{B}\left(D_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{z}{t}\right)^{\gamma})\right\}\right)(x):l,m) \\
&= x^{\varrho-\mu+\xi+\xi'-1}\Gamma(m)R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{x^{\gamma}}\right)) \\
&*\ {}_5\Psi_4\left[\begin{array}{c}(1,1),(l,\gamma),(1-\varrho-\varpi',\gamma),(1-\varrho+\mu-\xi-\xi',\gamma),\\ (1-\varrho+\mu-\xi'-\varpi,\gamma);\\ (l+m,\gamma),(1-\varrho,\gamma)(1-\varrho+\mu-\xi-\xi'-\varpi,\gamma)(1-\varrho-\xi'-\varpi',\gamma);\end{array}\frac{1}{x^{\gamma}}\right]. \quad (3.7)
\end{aligned}
$$

Theorem 9. *Let $\xi,\xi',\varpi,\varpi',\mu,\varrho,p,q\in\mathbb{C}$ be such that $\min\{\Re(p),\Re(q)\}>0$, $\Re(c)>0$ and $\Re(\varrho)>\max[0,\Re(\xi+\xi'+\varpi-\mu),\Re(\xi'-\varpi')]$. Then the below mentioned Laplace-transformation holds true:*

$$
\begin{aligned}
&\mathrm{L}\left(z^{l-1}(I_{0,x}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;(tz)^{\gamma})\right\}\right)(x)=\frac{x^{\varrho+\mu-\xi-\xi'-1}}{s^{l}}R_{p,q}^{\tau}(a,b;c;\left(\frac{x}{s}\right)^{\gamma}) \\
&*\ {}_5\Psi_3\left[\begin{array}{c}(1,1),(l,\gamma),(\varrho,\gamma),(\varrho+\mu-\xi-\xi'-\varpi,\gamma)\\ (\varrho+\varpi'+\xi',\gamma);\\ (\varrho+\varpi',\gamma),(\varrho+\mu-\xi-\xi',\gamma)(\varrho+\mu-\xi'-\varpi,\gamma);\end{array}\left(\frac{x}{s}\right)^{\gamma}\right]. \quad (3.8)
\end{aligned}
$$

Theorem 10. *Let $\xi,\xi',\varpi,\varpi',\mu,\varrho,p,q\in\mathbb{C}$ be such that $\min\{\Re(p),\Re(q)\}>0$, $\Re(c)>0$ and $\Re(\varrho)<1+\min\{\Re(-\varpi),\Re(\xi+\xi'-\mu),\Re(\xi+\varpi'-)\}$. Then the below mentioned Laplace-transformation holds true:*

$$
\begin{aligned}
&\mathrm{L}\left(z^{l-1}(I_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{z}{t}\right)^{\gamma})\right\}\right)=\frac{x^{\varrho+\mu-\xi-\xi'-1}}{s^{l}}R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{xs}\right)^{\gamma}) \\
&*\ {}_5\Psi_3\left[\begin{array}{c}(1,1),(l,\gamma),(1-\varrho-\varpi,\gamma),(1-\varrho-\mu+\xi+\xi',\gamma)\\ (1-\varrho-\mu+\xi+\varpi',\gamma);\\ (1-\varrho,\gamma),(1-\varrho-\mu+\xi+\xi'+\varpi',\gamma)(1-\varrho+\xi-\varpi,\gamma);\end{array}\left(\frac{1}{xs}\right)^{\gamma}\right]. \quad (3.9)
\end{aligned}
$$

Theorem 11. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(\mu) > 0$ *and* $\Re(\varrho) > \max[0, \Re(\mu - \xi - \xi' - \varpi'), \Re(\varpi - \xi)]$ *. Then the below mentioned Laplace-transformation holds true:*

$$\mathrm{L}\left(z^{l-1}(D_{0,x}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;(tz)^{\gamma})\right\}\right) = \frac{x^{\varrho-\mu+\xi+\xi'-1}}{s^{l}}R_{p,q}^{\tau}(a,b;c;\left(\frac{x}{s}\right)^{\gamma})$$
$$*\ {}_5\Psi_3\left[\begin{array}{c}(1,1),(l,\gamma),(\varrho,\gamma),(\varrho-\mu+\xi+\xi'+\varpi',\gamma)\\(\varrho-\varpi+\xi,\gamma);\\(\varrho-\varpi,\gamma),(\varrho-\mu+\xi+\xi',\gamma)(\varrho-\mu+\xi+\varpi',\gamma);\end{array}\left(\frac{x}{s}\right)^{\gamma}\right]. \quad (3.10)$$

Theorem 12. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(c) > 0$ *and* $\Re(\varrho) < 1 + \min\{\Re(\varpi'), \Re(\mu - \xi - \xi'), \Re(\mu - \xi' - \varpi)\}$ *. Then the below mentioned Laplace-transformation holds true:*

$$\mathrm{L}\left(z^{l-1}(D_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{z}{t}\right)^{\gamma})\right\}\right) = \frac{x^{\varrho-\mu+\xi+\xi'-1}}{s^{l}}R_{p,q}^{\tau}(a,b;c;\left(\frac{1}{xs}\right)^{\gamma})$$
$$*\ {}_5\Psi_3\left[\begin{array}{c}(1,1),(l,\gamma),(1-\varrho-\varpi',\gamma),(1-\varrho+\mu-\xi-\xi',\gamma)\\(1-\varrho+\mu-\xi'-\varpi,\gamma);\\(1-\varrho,\gamma),(1-\varrho+\mu-\xi-\xi'-\varpi,\gamma)(1-\varrho-\xi'-\varpi',\gamma);\end{array}\left(\frac{1}{xs}\right)^{\gamma}\right]. \quad (3.11)$$

Theorem 13. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(\mu) > 0$ *and* $\Re(\varrho) > \max[0, \Re(\xi + \xi' + \varpi - \mu), \Re(\xi' - \varpi')]$ *. Then the below mentioned integral holds true:*

$$\int_0^{\infty} z^{l-1}\mathrm{e}^{-\frac{1}{2}\delta z}\,W_{\tau,\varsigma}(\delta z)\left(I_{0,x}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;(wtz)^{\gamma})\right\}\right)(x)\mathrm{d}z$$
$$= \frac{x^{\varrho+\mu-\xi-\xi'-1}}{\delta^{l}}R_{p,q}^{\tau}(a,b;c;\left(\frac{wx}{\delta}\right)^{\gamma})$$
$$*\ {}_6\Psi_4\left[\begin{array}{c}(1,1),(\frac{1}{2}+\varsigma+l+\gamma\nu,\gamma),(\frac{1}{2}-\varsigma+l,\gamma),(\varrho,\gamma)(\varrho+\mu-\xi-\xi'-\varpi,\gamma)\\(\varrho+\varpi'-\xi',\gamma);\\(\frac{1}{2}-\varsigma+l,\gamma)(\varrho+\varpi',\gamma),(\varrho+\mu-\xi-\xi',\gamma)(\varrho+\mu-\xi'-\varpi,\gamma);\end{array}\left(\frac{wx}{\delta}\right)^{\gamma}\right]. \quad (3.12)$$

Theorem 14. *Let*$\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(c) > 0$ *and* $\Re(\varrho) < 1 + \min\{\Re(-b), \Re(\xi + \xi' - \mu), \Re(\xi + \varpi' - \mu)\}$ *. Then the below mentioned integral holds true:*

$$\int_0^{\infty} z^{l-1}\mathrm{e}^{-\frac{1}{2}\delta z}\,W_{\tau,\varsigma}(\delta z)\left(I_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu}\left\{t^{\varrho-1}R_{p,q}^{\tau}(a,b;c;\left(\frac{wz}{t}\right)^{\gamma})\right\}\right)(x)\mathrm{d}z$$
$$= \frac{x^{\varrho+\mu-\xi-\xi'-1}}{\delta^{l}}R_{p,q}^{\tau}(a,b;c;\left(\frac{w}{x\delta}\right)^{\gamma})$$

$$* \,{}_6\Psi_4 \left[\begin{array}{c} (1,1), (\frac{1}{2}+\varsigma+l,\gamma), (\frac{1}{2}-\varsigma+l,\gamma), (1-\varrho-\varpi,\gamma)(1-\varrho-\mu+\xi+\xi',\gamma) \\ (1-\varrho-\mu+\varpi'+\xi,\gamma); \\ (\frac{1}{2}-\varsigma+l,\gamma)(1-\varrho,\gamma), (1-\varrho-\mu+\xi+\xi'+\varpi',\gamma)(1-\varrho+\xi-\varpi,\gamma); \end{array} \left(\frac{w}{x\delta}\right)^{\gamma} \right]. \tag{3.13}$$

Theorem 15. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(c) > 0$ *and* $\Re(\varrho) > \max[0, \Re(\mu-\xi-\xi'-\varpi'), \Re(\varpi-\xi)]$. *Then the below mentioned integral holds true:*

$$\int_0^\infty z^{l-1} e^{-\frac{1}{2}\delta z} W_{\tau,\varsigma}(\delta z) \left(D_{0,x}^{\xi,\xi',\varpi,\varpi',\mu} \left\{ t^{\varrho-1} R^{\tau}_{p,q}(a,b;c;(wtz)^{\gamma}) \right\} \right)(x) dz$$
$$= \frac{x^{\varrho-\mu+\xi+\xi'-1}}{\delta^l} R^{\tau}_{p,q}(a,b;c;\left(\frac{wx}{\delta}\right)^{\gamma})$$
$$* \,{}_6\Psi_4 \left[\begin{array}{c} (1,1), (\frac{1}{2}+\varsigma+l,\gamma), (\frac{1}{2}-\varsigma+l,\gamma), (\varrho,\gamma)(\varrho-\mu+\xi+\xi'+\varpi',\gamma) \\ (\varrho-\varpi+\xi,\gamma); \\ (\frac{1}{2}-\varsigma+l,\gamma)(\varrho-\varpi,\gamma), (\varrho-\mu+\xi+\xi',\gamma)(\varrho-\mu+\xi-\varpi',\gamma); \end{array} \left(\frac{wx}{\delta}\right)^{\gamma} \right]. \tag{3.14}$$

Theorem 16. *Let* $\xi, \xi', \varpi, \varpi', \mu, \varrho, p, q \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(c) > 0$ *and* $\Re(\varrho) < 1 + \min\{\Re(b'), \Re(c-\alpha-\alpha'), \Re(c-\alpha'-b)\}$. *Then the below mentioned integral holds true:*

$$\int_0^\infty z^{l-1} e^{-\frac{1}{2}\delta z} W_{\tau,\varsigma}(\delta z) \left(D_{x,\infty}^{\xi,\xi',\varpi,\varpi',\mu} \left\{ t^{\varrho-1} R^{\tau}_{p,q}(a,b;c;\left(\frac{wz}{t}\right)^{\gamma}) \right\} \right)(x) dz$$
$$= \frac{x^{\varrho-\mu+\xi+\xi'-1}}{\delta^l} R^{\tau}_{p,q}(a,b;c;\left(\frac{w}{x\delta}\right)^{\gamma})$$
$$* \,{}_6\Psi_4 \left[\begin{array}{c} (1,1), (\frac{1}{2}+\varsigma+l,\gamma), (\frac{1}{2}-\varsigma+l,\gamma), (1-\varrho-\varpi',\gamma)(1-\varrho+\mu-\xi-\xi',\gamma) \\ (1-\varrho+\mu-\varpi-\xi',\gamma); \\ (\frac{1}{2}-\varsigma+l,\gamma)(1-\varrho,\gamma), (1-\varrho+\mu-\xi-\xi'-\varpi,\gamma)(1-\varrho-\xi'-\varpi',\gamma); \end{array} \left(\frac{w}{x\delta}\right)^{\gamma} \right]. \tag{3.15}$$

4. Concluding Remarks

In this paper, we obtain certain image formulas of the (p,q)–extended τ–hypergeometric function $R^{\tau}_{p,q}(a,b;c;z)$ by using by employing Marichev-Sigo-Maeda (M-S-M) fractional (integral and differential) operators (1.1), (1.2), (1.3) and (1.4) involving the third Appell's function of two-variables $F_3(.)$. Corresponding assertions for the Saigo's, Riemann-Liouville (R-L) and Erdélyi-Kober(E-K) fractional integral and differential operators are deduced which are earlier obtained by Solanki et al. [23]. All the results are represented in terms of the Hadamard product of the (p,q)–extended τ–hypergeometric function $R^{\tau}_{p,q}(a,b;c;z)$ and Fox-Wright function

${}_r\Psi_s(z)$. We also established certain integral transforms as Beta, Laplace and Whittaker transforms of the (p, q)–extended τ–hypergeometric function $R^{\tau}_{p,q}(a, b; c; z)$.

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