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ON SOME NEW GENERATING FUNCTIONS OF HYPERGEOMETRIC POLYNOMIALS

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Abstract: This paper contains mainly three theorems involving Kampé de Fériet's function $F^{(2)}$ and expressed in terms of single and double Laplace and Beta integrals. The theorems, in turn, yield, as special cases, a number of linear, bilinear and bilateral generating functions of generalized polynomials of Rice, Jacobi polynomials, Ultraspherical, Generalized Laguerre, Bedient polynomials and other polynomials hypergeometric in nature. One variable special cases of generalized polynomials are useful in several applied problems.

Keywords and Phrases: Linear, Bilinear and Bilateral Generating Functions, Eulerian integrals of first and second kind; Hankel's contour integral, Kampé de Fériet's double hypergeometric function $F^{(2)}[x, y]$, Srivastava's triple hypergeometric function $F^{(3)}[x, y, z]$ and Orthogonal polynomials.

2020 Mathematics Subject Classification: 33C05, 33C20, 33C45, 33C65, 33C70.

1. Introduction

Hypergeometric Polynomials occupy the pride place in the literature on special functions. One variable special functions namely generalized Rice polynomials, Jacobi polynomials, Ultraspherical polynomials, Legendre polynomials, generalized Laguerre polynomials and other polyomials hypergeometric in nature, are closely associated with problems of applied in nature. For example, Ultraspherical polynomials are deeply connected with axially symmetric potential in n dimensions and contain, Legendre and Chebyshev polynomials as special cases. The generalized Laguerre polynomials play an important role in finding the wave function associated with the electron in a hydrogen atom.Further Laguerre Polynomials are encountered in the solution of problems on the propagation of electromagnetic waves in long lines and in the analysis of the motion of electrons in Coulomb field as well as in certain other problems of Theoretical Physics. Further the Legendre polynomials are closely associated with physical phenomena for which spherical geometry is important.

Motivated by the work on generating functions by Khandekar [13], Manocha [17] and [18], Sharma and Mittal [27], Sharma, B. L. [26], Saran [24] and [25], Munot, Mathur and Kushwaha [21], Mathur, B. L. [20] and Chaudhary [5], the authors succeeded in establishing three theorems involving Kampé de Fériet's function $F^{(2)}$, expressed in terms of single and double Laplace and Beta integrals.

Theorem 1 establishes a generating function for Kampé de Fériet's double series $F^{(2)}$ [2, p. 150, eq. (29)] see also [4, p. 112], expressed in terms of beta transform of one variable. Theorem 2 is associated with obtaining a generating function involving Kampé de Fériet's double series $F^{(2)}$, expressed in terms of Laplace transform of one variable.Theorem 3 is devoted to proving a generating function for $F^{(2)}$ series, expressed in terms of Laplace transform of two variables.

The results established have been expressed in terms of Kampé de Fériet's double series $F^{(2)}$ as it is more general form of two variable functions, Further our results can also be applied to obtain generating relations involving Quadruple functions as well.

The above theorems yield, as special cases, some known results of Chaudhary [5, pp. 264-265], Srivastava and Joshi [35, p. 22], Srivastava and Manocha [37]. Many more known and unknown results can be obtained by specialising the parameters or variables and sometimes both.

The hypergeometric functions and polynomials used in our work are as given below:

The generalized Rice polynomial [13, p. 158, eq.(2.3)] is defined by:

$$H_n^{(\alpha,\beta)}[\nu,\sigma,x] = \frac{(1+\alpha)_n}{n!}\ {}_3F_2[-n, n+\alpha+\beta+1, \nu; 1+\alpha, \sigma; x] \tag{1.1}$$

has Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ [22, p. 254], Ultraspherical(or Gegenbauer) polynomials $C_n^{(\alpha)}$ [22, p. 277], Legendre polynomial $C_n^{\frac{1}{2}}(x)$ and the generalized Lagueree polynomials $L_n^{(\alpha)}(x)$ [22, p. 200] as special cases defined below.

$$P_n{}^{(\alpha,\beta)}(x) = H_n{}^{(\alpha,\beta)}\left[\sigma,\sigma,\frac{1-x}{2}\right] = (-1)^n P_n^{(\beta,\alpha)}(-x) =$$

$$= \frac{(1+\alpha)_n}{n!}\ {}_2F_1\ \left[-n,1+\alpha+\beta+n;1+\alpha;\frac{1-x}{2}\right]. \tag{1.2}$$

$$P_n{}^{(\alpha,\alpha)}(x) = \frac{(1+\alpha)_n}{(1+2\alpha)_n}\ C_n{}^{\alpha+\frac{1}{2}}(x) = \frac{(1+\alpha)_n}{n!}\ {}_2F_1\left[\begin{matrix}-n,1+2\alpha+n; \\ 1+\alpha;\end{matrix}\ \frac{1-x}{2}\right] \tag{1.3}$$

$$L_n^{(\alpha)}(x) = \frac{(1+\alpha)_n}{n!}{}_1F_1\left[\begin{matrix}-n; \\ 1+\alpha;\end{matrix}\ x\right] = \lim_{|\beta|\to\infty}\left(P_n^{(\alpha,\beta)}\left(1-\frac{2x}{\beta}\right)\right) \tag{1.4}$$

$$P_n(x) = P_n^{(0,0)}(x) = C_n^{\frac{1}{2}}(x) = (-1)^n\ P_n(-x) \tag{1.5}$$

$$P_n^{(\alpha-n,\beta-n)}(x) = \frac{1}{n!}\frac{\Gamma(1+\alpha)}{\Gamma(1+\alpha-n)}\left(\frac{1+x}{2}\right)^n\ {}_2F_1\left[\begin{matrix}-n,-\beta; \\ 1+\alpha-n;\end{matrix}\ \frac{x-1}{x+1}\right]$$

$$= \binom{\alpha+\beta}{n}\left(\frac{x-1}{2}\right)^n{}_2F_1\left[\begin{matrix}-n,-\alpha; \\ -\alpha-\beta;\end{matrix}\ \frac{2}{1-x}\right]$$

$$= \frac{(-\alpha-\beta)_n}{n!}\left(\frac{1-x}{2}\right)^n\ {}_2F_1\left[\begin{matrix}-n,-\alpha; \\ -\alpha-\beta;\end{matrix}\ \frac{2}{1-x}\right]. \tag{1.6}$$

Further Szegö [38, p. 64] recorded Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ as follows:

$$P_n^{(\alpha,\beta)}(x) = \left(\frac{1-x}{2}\right)^n P_n{}^{(-\alpha-\beta-2n-1,\beta)}\left(\frac{x+3}{x-1}\right) \tag{1.7}$$

and

$$P_n^{(\alpha-n,\beta-n)}(x) = \left(\frac{1-x}{2}\right)^n P_n{}^{(-\alpha-\beta-1,\beta-n)}\left(\frac{x+3}{x-1}\right), \tag{1.8}$$

where in each of the equations (1.1) to (1.8), $\Re(\alpha)>-1, \Re(\beta)>-1$ and n is a non-negative integer.

The functions F_E and F_G in the notation of Saran [23], see also [37, eqns.(26) and (28)] indicating also the numbering of Lauricella [16] on the left are as given below:

$$F_4 : F_E\left[a,a,a,b,c,c;d,e,f;x,y,z\right] = \sum_{m,n,p=0}^{\infty} \frac{(a)_{m+n+p}(b)_m(c)_{n+p}}{(d)_m(e)_n(f)_p} \frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!} \tag{1.9}$$

where $|x|<r, |y|<s, |z|<t$ such that $r + (\sqrt{s} + \sqrt{t})^2 = 1$.

$$F_8 : F_G[a,a,a;b,c,d;e,f,f;x,y,z] = \sum_{m,n,p=0}^{\infty} \frac{(a)_{m+n+p}(b)_m(c)_n(d)_p}{(e)_m(f)_{n+p}} \frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!} \tag{1.10}$$

where $(|x|<r, |y|<s, |z|<t)$ such that $r + s = 1 = r + t$.
The Bediant polynomials [3, p. 44, eq.(3.4)] are defined as under:

$$G_n(\alpha,\beta;x) = \frac{(\alpha)_n(\beta)_n}{(\alpha+\beta)_n n!}\ (2x)^n\ {}_3F_2\left[\begin{array}{c} \Delta(2;-n), 1-\alpha-\beta-n; \\ \\ 1-\alpha-n, \quad 1-\beta-n; \end{array} \frac{1}{x^2}\right]$$

$$= \frac{(\alpha)_n(\beta)_n}{(\alpha+\beta)_n n!}\ (2x)^n\ {}_3F_2\left[\begin{array}{c} \frac{-n}{2}, \frac{-n}{2}+\frac{1}{2}, 1-\alpha-\beta-n; \\ \\ 1-\alpha-n, 1-\beta-n \qquad ; \end{array} \frac{1}{x^2}\right]. \tag{1.11}$$

where $\Delta(N;\lambda)$ [37, p. 214] denotes the array of N parameters:
$\frac{\lambda}{N}, \frac{\lambda+1}{N}, \frac{\lambda+2}{N}, \ldots, \frac{\lambda+N-1}{N}$, $N \geq 1$ such that N is a positive integer.
The familiar Beta function $B(\alpha,\beta)$ [22, p. 18] is defined by

$$B(\alpha,\beta) = \begin{cases} \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\, dt; & (min\{\Re(\alpha),\Re(\beta)\}>0) \\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}; & (\alpha,\beta \in \mathbb{C}\backslash\mathbb{Z}_0) \end{cases} \tag{1.12}$$

The Kampé de Fériet's double hypergeometric function $F^{(2)}$ [2, p. 150] is defined as:

$$F^{(2)}\left[\begin{array}{c} (a_A):(b_B);(d_D); \\ \\ (f_F):(g_G);(h_H); \end{array} x,y\right] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{[(a_A)]_{m+n}}{[(f_F)]_{m+n}} \frac{[(b_B)]_m}{[(g_G)]_m} \frac{[(d_D)]_n}{[(h_H)]_n} \frac{x^m}{m!}\frac{y^n}{n!}. \tag{1.13}$$

where (a_A) and $[(a_A)]_{m+n}$ will mean the sequence of A parameters $a_1, a_2, ..., a_A$ and the product $(a_1)_{m+n}(a_2)_{m+n}, ..., (a_A)_{m+n}$ respectively.Thus $[(a_A)]_m$ is to be interpreted as

$$[(a_A)]_m = \prod_{i=1}^{A}(a_i)_m = (a_1)_m(a_2)_m...(a_A)_m = \prod_{i=1}^{A}\frac{\Gamma(a_i+m)}{\Gamma(a_i)}, \tag{1.14}$$

with similar interpretation for $[(b_B)], [(d_D)]$ etc.
$F^{(2)}$ function has Appell's functions F_1, F_2 and F_4 as special cases, defined as follows:

$$F_1[\alpha,\beta,\gamma;\delta;x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha)_{m+n}(\beta)_m(\gamma)_n x^m y^n}{(\delta)_{m+n} m! n!}, \quad \max\{|x|,|y|\}<1 \tag{1.15}$$

$$F_2[\alpha,\beta,\gamma;\delta,\xi;x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha)_{m+n}(\beta)_m(\gamma)_n x^m y^n}{(\delta)_m(\xi)_n m! n!}, \quad |x|+|y|<1 \tag{1.16}$$

$$F_4[\alpha,\beta,\gamma;\delta;x,y] = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha)_{m+n}(\beta)_{m+n} x^m y^n}{(\gamma)_m(\delta)_n m! n!}, \quad \sqrt{|x|}+\sqrt{|y|}<1 \tag{1.17}$$

Further, Appell's four double hypergeometric function $F_1,\ F_2,\ F_3$ and F_4 in the notation of Kampé de Fériet's are denoted by
$F^{\,1\,:\,1\,;\,1}_{\,1\,:\,0\,;\,0}\,,\ F^{\,1\,:\,1\,;\,1}_{\,0\,:\,1\,;\,1}\,,\ F^{0\,:\,2\,;2}_{1\,:0\,;\,0}$ and $F^{\,2\,:\,0\,;\,0}_{\,0\,:\,1\,;\,1}$ respectively.
Srivastava's generalized hypergeometric function $F^{(3)}$ [29, p. 428] of three variables is defined as

$$F^{(3)}\left[\begin{array}{ll} (a_A) :: (b_B); (b'_B); (b''_B) : (c_C); (c'_C); (c''_C); & \\ & x, y, z \\ (d_D) :: (e_E); (e'_E); (e''_E) : (f_F); (f'_F); (f''_F); & \end{array}\right] =$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\frac{[(a_A)]_{m+n+p}[(b_B)]_{m+n}[(b'_B)]_{n+p}[(b''_B)]_{p+m}[(c_C)]_m[(c'_C)]_n[(c''_C)]_p x^m y^n z^p}{[(d_D)]_{m+n+p}[(e_E)]_{m+n}[(e'_E)]_{n+p}[(e''_E)]_{p+m}[(f_F)]_m[(f'_F)]_n[(f''_F)]_p m! n! p!}. \tag{1.18}$$

where (a_A) and $[(a_A)]_m$ have their usual meaning as explained in (1.14) with similar interpretation for $(b_B), (b'_B), (b''_B)$ and so on. It is to be noted that A is the number of parameters in (a_A), B is the number of parameters in (b_B) with similar interpretation for $(b'_B), (b''_B)$ and so on.
It will be assumed throughout the paper that the absence of parameters shown by horizontal dashes mean that there exist no parameters and in that case, from (1.14), the conventional value of an empty product will be unity, that is $\prod_{i=1}^{0}(a_i)_m = 1$. Also, numerator parameters like $(a_A), (b_B), (b'_B), (b''_B)$ etc. may be zero or negative integers, but the denominator parameters like $(d_D), (e_E), (e'_E)$ etc. are not allowed to be zero or negative integers.
The region of convergence of the above triple power series (1.18) is given in the recent literature [8, p. 156], see also [9, p. 40].

Lauricella's function $F_A^{(n)}$ of n variables in the notation of Lauricella [16] is defined as under :

$$F_A^{(n)}[a,b_1,b_2,...,b_n;c_1,...,c_n;x_1,...,x_n] = \sum_{m_1,...,m_n}^{\infty} \frac{(a)_{m_1+...+m_n}(b_1)_{m_1}...(b_n)_{m_n}}{(c_1)_{m_1}...(c_n)_{m_n}} \frac{x_1^{m_1}}{m_1!}...\frac{x_n^{m_n}}{m_n!} \tag{1.19}$$

where $|x_1| + |x_2| + ... + |x_n| < l$.
Clearly, we have $F_A^{(2)} = F_2$ and $F_A^{(1)} = {}_2F_1$.

2. Main Results

Theorem 1. *Let $F(x,t)$ be a function having formal power series expansion in t, given by*

$$F(x,t) = \sum_{n=0}^{\infty} c_n \; f_n(x) \; t^n \tag{2.1}$$

where $\{c_n\}$ is a specified sequence of parameters ,independent of x and t, and $f_n(x)$; $n = 0,1,2\ldots$ are polynomials of degree n in x. Then with restrictions on y,z and t, such that the Kampé de Fériet's series $F^{(2)}[y,z]$ and $F\left(x, \frac{t\ p}{p-1}\right)$ remain uniformly convergent for $p \in (0,1)$,

$$\sum_{n=0}^{\infty} \frac{c_n(\lambda)_n}{(1+\lambda-\mu)_n} \; F^{(2)}\left[\begin{array}{lc} \lambda+n,(a_A):(d_D);(g_G); & \\ & y,z \\ \mu,(b_B):(e_E);(h_H); & \end{array}\right] f_n(x)\; t^n =$$

$$= \frac{\Gamma(\mu)}{\Gamma(\lambda)\Gamma(\mu-\lambda)} \int_0^1 p^{\lambda-1}(1-p)^{\mu-\lambda-1} \times$$

$$F^{(2)}\left[\begin{array}{lc} (a_A):(d_D);(g_G); & \\ & y\ p, z\ p \\ (b_B)\ :(e_E);(h_H); & \end{array}\right] F\left(x, \frac{t\ p}{p-1}\right)\; d\ p. \tag{2.2}$$

where $\Re(\mu) > \Re(\lambda) > 0$ *such that* $(\mu - \lambda \neq 0, -1, -2, -3\ldots)$.

Proof of Theorem 1. Let

$$T = \int_0^1 p^{\lambda-1}(1-p)^{\mu-\lambda-1} \; F^{(2)}\left[\begin{array}{lc} (a_A):(d_D);(g_G); & \\ & y\ p, z\ p \\ (b_B)\ :(e_E);(h_H); & \end{array}\right] \; F\left(x, \frac{t\ p}{p-1}\right)\; d\ p \tag{2.3}$$

Expanding functions $F^{(2)}$ and $F\left(x, \frac{tp}{p-1}\right)$ in series, we have

$$T = \int_0^1 p^{\lambda-1}(1-p)^{\mu-\lambda-1} \sum_{r,s=0}^{\infty} \frac{[(a_A)]_{r+s}}{[(b_B)]_{r+s}} \frac{[(d_D)]_r}{[(e_E)]_r} \frac{[(g_G)]_s}{[(h_H)]_s} \frac{(y\ p)^r}{r!} \frac{(z\ p)^s}{s!} \times$$

$$\sum_{n=0}^{\infty} c_n f_n(x) \left(\frac{t\ p}{p-1}\right)^n \ dp \tag{2.4}$$

or

$$T = \sum_{n=0}^{\infty} c_n \ f_n(x)\ t^n\ (-1)^n \sum_{r,s=0}^{\infty} \frac{[(a_A)]_{r+s}}{[(b_B)]_{r+s}} \frac{[(d_D)]_r}{[(e_E)]_r} \frac{[(g_G)]_s}{[(h_H)]_s} \frac{(y)^r}{r!} \frac{(z)^s}{s!} \times$$

$$\times \int_0^1 p^{\lambda+n+r+s-1}(1-p)^{\mu-\lambda-n-1}\ dp \tag{2.5}$$

$$= \sum_{n=0}^{\infty} c_n f_n(x) t^n (-1)^n \sum_{r,s=0}^{\infty} \frac{[(a_A)]_{r+s}}{[(b_B)]_{r+s}} \frac{[(d_D)]_r}{[(e_E)]_r} \frac{[(g_G)]_s}{[(h_H)]_s} \frac{(y)^r}{r!} \frac{(z)^s}{s!} \beta(\lambda+n+r+s, \mu-\lambda-n). \tag{2.6}$$

where $\beta(\lambda+n+r+s, \mu-\lambda-n)$ is a Beta function defined by (1.12).
Now on making little simplification, we get

$$T = \frac{\Gamma(\lambda)\Gamma(\mu-\lambda)}{\Gamma(\mu)} \sum_{n=0}^{\infty} \frac{(\lambda)_n}{(1+\lambda-\mu)_n} c_n F^{(2)} \left[\begin{array}{c} \lambda+n, (a_A) : (d_D); (g_G); \\ \\ \mu, (b_B) : (e_E); (h_H); \end{array} y, z \right] f_n(x) t^n. \tag{2.7}$$

Hence

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(1+\lambda-\mu)_n} c_n F^{(2)} \left[\begin{array}{c} \lambda+n, (a_A) : (d_D); (g_G); \\ \\ \mu, (b_B) : (e_E); (h_H); \end{array} y, z \right] f_n(x) t^n = \frac{\Gamma(\mu)}{\Gamma(\lambda)\Gamma(\mu-\lambda)} \times$$

$$\times \int_0^1 p^{\lambda-1}(1-p)^{\mu-\lambda-1}\ F^{(2)} \left[\begin{array}{c} (a_A) : (d_D); (g_G); \\ \\ (b_B) : (e_E); (h_H); \end{array} yp, zp \right]\ F\left(x, \frac{tp}{p-1}\right) dp. \tag{2.8}$$

Theorem 2. *Let*

$$G(x,t) = \sum_{n=0}^{\infty} g_n\ f_n(x) t^n \tag{2.9}$$

where $f_n(x); n = 0, 1, 2, 3, \ldots$ *are polynomials of degree* n *in* x *and* g_n *is a specified sequence of parameters, independent of* x *and* t*. Then*

$$\sum_{n=0}^{\infty} g_n (\lambda)_n \quad F^{(2)} \left[\begin{array}{c} \lambda+n, (a_A) : (d_D); (g_G); \\ \\ (b_B) : (e_E); (h_H); \end{array} y, z \right]\ f_n(x)\ t^n$$

$$= \frac{1}{\Gamma(\lambda)} \int_0^\infty e^{-p} p^{\lambda-1} \; F^{(2)} \left[\begin{array}{c} (a_A){:}(d_D);(g_G): \\ \\ (b_B){:}(e_E);(h_H); \end{array} \; y\;p, z\;p \right] \; G(x, t\;p)\;d\;p\;, \quad (2.10)$$

where $\Re(\lambda){>}0$.

Theorem 3. *Let*

$$H(x,t) = \sum_{n=0}^{\infty} \; h_n \; f_n(x) \; t^n \quad (2.11)$$

where $f_n(x); n = 0, 1, 2, 3, \ldots$ *are polynomials of degree* n *in* x *and* $\{h_n\}$ *is a specified sequence of parameters independent of* x *and* t. *Then*

$$\sum_{n=0}^{\infty} \; h_n(\alpha)_n(\beta)_n \;\; F^{(2)} \left[\begin{array}{c} \alpha+n, \beta+n; (a_A){:}(d_D);(g_G); \\ \\ (b_B){:}(e_E);(h_H); \end{array} \; y, z \right] \; f_n(x) t^n$$

$$= \frac{1}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\infty \int_0^\infty e^{-(p+q)} p^{\alpha-1} q^{\beta-1} \times$$

$$F^{(2)} \left[\begin{array}{c} (a_A){:}(d_D);(g_G); \\ \\ (b_B){:}(e_E);(h_H); \end{array} \; y\;p\;q, z\;p\;q \right] \; H(x, t\;p\;q)\;dp\;dq\;, \quad (2.12)$$

where $\Re(\alpha){>}0)$, $\Re(\beta{>}0)$ *such that* $\alpha, \beta \neq 0, -1, -2 \ldots$.

Note. The proofs of Theorem 2 and Theorem 3 are similar to that of Theorem 1.

3. Applications

Case I. Consider the generating function [33, p. 77, eq. (3.3)], see also [37, p. 146, eq. (33)]:

$$\sum_{n=0}^{\infty} \frac{[(p_P)]_n}{(\alpha+1)_n[(q_Q)]_n} \; H_n^{(\alpha,\beta-n)}[\nu,\sigma,x]\, t^n = F^{(2)} \left[\begin{array}{c} (p_P){:}-;\alpha+\beta+1,\nu; \\ \\ (q_Q){:}-;\alpha+1, \quad \sigma; \end{array} \; t, -xt \right]. \quad (3.1)$$

In (3.1), we take $F(x,t) = F^{(2)} \left[\begin{array}{c} (p_P) \;{:}-;\alpha+\beta+1,\nu; \\ \\ (q_Q) \;{:}-;\alpha+1, \quad \sigma; \end{array} \; t, -xt \right]$, combining (3.1) with (2.1) and using Theorem 1 with $z = 0$, we get bilateral generating relation:

$$\sum_{n=0}^{\infty} \frac{[(p_P)]_n}{(\alpha+1)_n[(q_Q)]_n} \frac{(\lambda)_n}{(1+\lambda-\mu)_n} \; {}_{1+A+D}F_{1+B+E}\left[\begin{array}{c} \lambda+n, (a_A), (d_D); \\ \\ \mu, (b_B), (e_E); \end{array} y\right] \times$$
$$\times H_n^{(\alpha,\beta-n)}[\nu,\sigma,x] \; t^n =$$

$$= F^{(3)}\left[\begin{array}{l} \lambda \;::-; \quad (p_P); - \quad :(a_A), (d_D); -; \alpha+\beta+1, \; \nu; \\ \\ -::-; 1+\lambda-\mu, (q_q); -:\mu, (b_B), (e_E); -; \alpha+1, \quad \sigma; \end{array} y, t, -xt\right]. \quad (3.2)$$

Now putting $P = Q = D = E = 0$, replacing ν by σ and x by $\frac{1-x}{2}$, equation (3.2) in the light of result (1.2) reduces to a bilateral generating relation involving special Jacobi polynomials:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(\alpha+1)_n(1+\lambda-\mu)_n} \; {}_{1+A}F_{1+B}\left[\begin{array}{l} \lambda+n, (a_A); \\ \\ \mu, \quad (b_B); \end{array} y\right] P_n^{(\alpha,\beta-n)}(x) \; t^n$$
$$= F^{(3)}\left[\begin{array}{l} \lambda::-; -; \quad - \quad :(a_A); -; \alpha+\beta+1; \\ \\ -::-; 1+\lambda-\mu; -: \; \mu, (b_B); -; \alpha+1; \end{array} y, t, \tfrac{1}{2}(x-1)t\right]. \quad (3.3)$$

Again equation (3.3) on using result (1.8) takes the form:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(\alpha+1)_n(1+\lambda-\mu)_n} {}_{1+A}F_{1+B}\left[\begin{array}{l} \lambda+n, (a_A); \\ \\ \mu, \quad (b_B); \end{array} y\right] \times$$
$$\left(\frac{1-x}{2}\right)^n P_n^{(-\alpha-\beta-n-1,\beta-n)}\left(\frac{x+3}{x-1}\right) t^n$$

$$= F^{(3)}\left[\begin{array}{l} \lambda \;::-; \quad - \quad ;- \quad :(a_A) \;;-; \; \alpha+\beta+1; \\ \\ -::-; 1+\lambda-\mu; -:\mu, (b_B) \;\; ;- \;; \; \alpha+1; \end{array} y, t, \tfrac{1}{2}(x-1)t\right]. \quad (3.4)$$

Also, equation (3.3) on taking $y = 0$ reduces to a linear generating relation involving Kampé de Fériet's function $F^{(2)}$ in the form:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(\alpha+1)_n(1+\lambda-\mu)_n} \; P_n^{(\alpha,\beta-n)} \; t^n =$$

$$= F^{(2)}\left[\begin{array}{l} \lambda \quad\quad\quad : - \quad ; \alpha+\beta+1; \\ \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad t, \frac{1}{2}(x-1)t \\ 1+\lambda-\mu : - \quad ; \quad \alpha+1; \end{array}\right]. \tag{3.5}$$

Further in equation (3.5), on replacing λ by $1+\lambda-\mu$, we get the relation:

$$\sum_{n=0}^{\infty} \frac{1}{(\alpha+1)_n} \, P_n^{(\alpha,\beta-n)}(x)\; t^n = e^t \, {}_1F_1\left[\begin{array}{l} \alpha+\beta+1; \\ \quad\quad\quad\quad\quad \frac{-1}{2}(1-x)t \\ \alpha+1 \quad ; \end{array}\right] \tag{3.6}$$

or equivalently

$$\sum_{n=0}^{\infty} \frac{1}{(\alpha+1)_n} P_n^{(\alpha,\beta-n)}(1-2x) t^n = e^t \, {}_1F_1\left[\begin{array}{l} \alpha+\beta+1; \\ \quad\quad\quad\quad\quad -xt \\ \alpha+1 \quad ; \end{array}\right], \tag{3.7}$$

which is a known result of Srivastava and Joshi [35, p. 22, eq. (4.6)], see also [37, p. 257, eq. (43)].

Case II. Consider the generating function [31, p. 69] see also [37, pp. 165-166] and [34, p. 969]:

$$\sum_{n=0}^{\infty} \frac{[(l_L)]_n}{[(m_M)]_n n!} \; {}_{P+1}F_Q\left[\begin{array}{l} -n, (p_P); \\ \quad\quad\quad\quad\quad x \\ (q_Q) \quad ; \end{array}\right] t^n = F^{(2)}\left[\begin{array}{l} (l_L) \; : - ; (p_P); \\ \quad\quad\quad\quad\quad\quad t, -xt \\ (m_M) : - ; (q_Q); \end{array}\right]. \tag{3.8}$$

In (3.8), we take $G(x,t) = \left[\begin{array}{l} (l_L) : -; (p_P); \\ \quad\quad\quad\quad\quad\quad t, -xt \\ (m_M) : -; (q_Q); \end{array}\right]$, combining (3.8) with (2.9) and using Theorem 2 with $L = M = 0$, we get bilateral generating relation:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{n!} \; F^{(2)}\left[\begin{array}{l} \lambda+n, (a_A) : (d_D); (g_G); \\ \quad\quad\quad\quad\quad\quad\quad\quad\quad y, z \\ \quad\quad\quad (b_B) : (e_E); (h_H); \end{array}\right] \; {}_{P+1}F_Q\left[\begin{array}{l} -n, (p_P); \\ \quad\quad\quad\quad\quad x \\ (q_Q) \quad ; \end{array}\right] t^n$$

$$= \sum_{u=0}^{\infty} \frac{(\lambda)_u (t)^u}{u!} F^{(3)}\left[\begin{array}{l} \lambda+u :: - ; (a_A); - : \; (p_P) \; ; (d_D); (g_G); \\ \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad -xt, y, z \\ - :: - ; (b_B); - : (q_Q); (e_E); \; (h_H); \end{array}\right]. \tag{3.9}$$

Above equation can also be expressed as under:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{n!} F^{(2)} \left[\begin{array}{c} \lambda+n, (a_A):(d_D);(g_G); \\ \\ (b_B):(e_E);(h_H); \end{array} y, z \right] {}_{P+1}F_Q \left[\begin{array}{c} -n, (p_P); \\ \\ (q_Q) \quad ; \end{array} x \right] t^n$$

$$= (1-t)^{-\lambda} F^{(3)} \left[\begin{array}{c} \lambda ::-;(a_A);-:(p_P);(d_D);(g_G); \\ \\ -::-;(b_B);-:(q_Q);(e_E);(h_H); \end{array} \tfrac{xt}{t-1}, \tfrac{y}{1-t}, \tfrac{z}{1-t} \right]. \tag{3.10}$$

Now on putting $P = 0, Q = 1$ such that replacing q_1 by $1 + \alpha - n$, equation (3.10) gives a bilateral generating relation in the form:

$$\sum_{n=0}^{\infty} \frac{(-1)^n(\lambda)_n}{(-\alpha)_n} F^{(2)} \left[\begin{array}{c} \lambda+n, (a_A):(d_D);(g_G); \\ \\ (b_B):(e_E);(h_H); \end{array} y, z \right] L_n^{(\alpha-n)}(x)\ t^n$$

$$= (1-t)^{-\lambda} F^{(3)} \left[\begin{array}{c} \lambda::-;(a_A);-:- \quad ;(d_D) \quad ;(g_G) \quad ; \\ \\ -::-;(b_B);-:1+\alpha-n;(e_E);(h_H); \end{array} \tfrac{xt}{t-1}, \tfrac{y}{1-t}, \tfrac{z}{1-t} \right] \tag{3.11}$$

where $L_n^{(\alpha-n)}(x)$ is the generalized Laguerre polynomials defined by equation (1.4). Again, Kampé de Fériet's function $F^{(2)}$ in (3.11) can be specialized to yield the generating functions for Appell's functions F_1 to F_4.

For example, on taking $A = E = H = 0, B = D = G = 1$, then replacing b_1 by μ, d_1 by β_1 and g_1 by β_2, equation (3.11) reduces to a bilateral generating function involving Appell's function F_1 in the form:

$$\sum_{n=0}^{\infty} \frac{(-1)^n(\lambda)_n}{(-\alpha)_n} F_1 [\lambda+n, \beta_1, \beta_2; \mu; y, z]\ L_n^{(\alpha-n)}(x)\ t^n$$

$$= (1-t)^{-\lambda}\ F^{(3)} \left[\begin{array}{c} \lambda::-;-;-:- \quad ; \quad \beta_1 \quad ;\beta_2 \quad ; \\ \\ -::-;\mu;-:1+\alpha-n;-;-; \end{array} \tfrac{xt}{t-1}, \tfrac{y}{1-t}, \tfrac{z}{1-t} \right]. \tag{3.12}$$

Again, on putting $A = B = 0, D = E = G = H = 1$, then replacing d_1 by β_1, g_1 by β_2, e_1 by μ_1 and h_1 by μ_2, equation (3.11) reduces to a bilateral generating relation involving Appell's function F_2 in the form:

$$\sum_{n=0}^{\infty} \frac{(-1)^n(\lambda)_n}{(-\alpha)_n} F_2 [\lambda+n, \beta_1, \beta_2; \mu_1, \mu_2; y, z]\ L_n^{(\alpha-n)}(x)\ t^n$$

$$= (1-t)^{-\lambda}\ F^{(3)}\left[\begin{array}{l} \lambda{::}-;-;-{:}-\ ;\ \beta_1\ ;\beta_2\ ; \\ -{::}-;-;-{:}1+\alpha-n;\mu_1;\mu_2; \end{array} \frac{xt}{t-1},\frac{y}{1-t},\frac{z}{1-t}\right]. \tag{3.13}$$

Similarly, on putting $B = D = G = 0, A = E = H = 1$ and then replacing a_1 by μ, e_1 by β_1 and h_1 by β_2, equation (3.11) gives the bilateral generating relation:

$$\sum_{n=0}^{\infty}\frac{(-1)^n(\lambda)_n}{(-\alpha)_n}\ F_4\left[\lambda+n,\mu,\beta_1,\beta_2;y,z\right]\ L_n^{(\alpha-n)}(x)\ t^n$$

$$= (1-t)^{-\lambda}\ F^{(3)}\left[\begin{array}{l} \lambda{::}-;\mu;\ -{:}\ -\ ;-;-; \\ -{::}-;-;-\ {:}\ 1+\alpha-n;\beta_1;\ \beta_2\ ; \end{array} \frac{xt}{t-1},\frac{y}{1-t},\frac{z}{1-t}\right]. \tag{3.14}$$

Further equation (3.11) for $y = 0$ gives the relation:

$$\sum_{n=0}^{\infty}\frac{(-1)^n(\lambda)_n}{(\alpha)_n}\ {}_{1+A+G}F_{B+H}\left[\begin{array}{l} \lambda+n,(a_A),(g_G); \\ (b_B)\ ,\ (h_H)\ ; \end{array} z\right]\ L_n^{(\alpha-n)}(x)t^n$$

$$= (1-t)^{-\lambda}\ F^{(2)}\left[\begin{array}{l} \lambda{:}-\ ;\ (a_A),\ (g_G)\ ; \\ -{:}1+\alpha-n;(b_B),(h_H); \end{array} \frac{xt}{t-1},\frac{z}{1-t}\right]. \tag{3.15}$$

Case III. Consider the generating function [37, p. 109, eq. (20)], see also [22, p. 271, eq.(1)]:

$$\sum_{n=0}^{\infty}\frac{(\gamma)_n(\delta)_n}{(\alpha+1)_n(\beta+1)_n}P_n^{(\alpha,\beta)}(x)\ t^n = F_4\left[\gamma,\delta;\alpha+1,\beta+1;\frac{1}{2}(x-1)t,\frac{1}{2}(x+1)t\right] \tag{3.16}$$

where $P_n^{(\alpha,\beta)}$ are Jacobi polynomials defined by equation (1.2).
In (3.16), we take $H(x,t) = F_4\left[\gamma,\delta;\alpha+1,\beta+1;\frac{1}{2}(x-1)t,\frac{1}{2}(x+1)t\right]$, Combining (3.16) with (2.11) and using Theorem 3 with $z = 0$, we obtain the bilateral generating relation:

$$\sum_{n=0}^{\infty}\frac{(\gamma)_n(\delta)_n(\lambda)_n(\mu)_n}{(\alpha+1)_n(\beta+1)_n}\ {}_{A+D+2}F_{B+E}\left[\begin{array}{l} \lambda+n,\mu+n,(a_A),(d_D); \\ (b_B),\ \ (e_E)\ ; \end{array} y\right]\ P_n^{(\alpha,\beta)}(x)\ t^n$$

$$= F^{(3)}\left[\begin{array}{l} \lambda,\mu::-;\gamma,\delta;-:\ (a_A),(d_D)\ ;-\ ;-\ ;\\ -\ ::-;-;-\ :(b_B),(e_E);\alpha+1;\beta+1; \end{array} y,\tfrac{1}{2}(x-1)t,\tfrac{1}{2}(x+1)t\right] \tag{3.17}$$

which for $A = D = E = 0, B = 1$ and then replacing b_1 by ν gives a bilateral generating relation:

$$\sum_{n=0}^{\infty}\frac{(\gamma)_n(\delta)_n(\lambda)_n(\mu)_n}{(\alpha+1)_n(\beta+1)_n}\,{}_2F_1\left[\begin{array}{l}\lambda+n,\mu+n;\\ \nu\qquad ;\end{array} y\right] P_n^{(\alpha,\beta)}(x)\ t^n$$

$$= F^{(3)}\left[\begin{array}{l} \lambda,\mu::-;\gamma,\delta;-:\ -\ ;-\ ;-\ ;\\ -::-;-;\ -:\nu;\alpha+1;\beta+1; \end{array} y,\tfrac{1}{2}(x-1)t,\tfrac{1}{2}(x+1)t\right]. \tag{3.18}$$

Again equation (3.18) in view of the result [37, p. 91, eq. (17)] gives the generating relation:

$$\sum_{n=0}^{\infty}\frac{(\gamma)_n(\delta)_n(\mu)_n(\lambda)_n}{(\alpha+1)_n(\beta+1)_n}\,{}_2F_1\left[\begin{array}{l}\lambda+n,\mu+n;\\ \nu\qquad ;\end{array} y\right]\left(\frac{1-x}{2}\right)^n \times$$

$$\times\ P_n^{(-\alpha-\beta-2n-1,\beta)}\left(\frac{x+3}{x-1}\right)\ t^n$$

$$= F^{(3)}\left[\begin{array}{l} \lambda,\mu::-;\gamma,\delta;-:\ -;\ -\ ;\ -\ ;\\ -::-;-;\ -:\nu;\alpha+1;\beta+1; \end{array} y,\tfrac{1}{2}(x-1)t,\tfrac{1}{2}(x+1)t\right]. \tag{3.19}$$

Now multiplying equation (3.18) by $e^k\ k^{-\sigma}$, replacing t by $\frac{t}{k}$ and evaluating the result thus obtained with the help of Hankel's Contour integral for Gamma function [12, p. 32, eq. (1.5.1.5)]:

$$\frac{1}{2\pi\omega}\int_C e^t\ t^{-a-m}\ dt = \frac{1}{\Gamma(a+m)}\ , \tag{3.20}$$

where $\omega = \sqrt{-1}$, m is a non-negative integer and a does not take a non-negative integer values , we get the relation:

$$\sum_{n=0}^{\infty}\frac{(\gamma)_n(\delta)_n(\lambda)_n(\mu)_n}{(\sigma)_n(\alpha+1)_n(\beta+1)_n}\,{}_2F_1\left[\begin{array}{l}\lambda+n,\mu+n;\\ \nu\qquad ;\end{array} y\right] P_n^{(\alpha,\beta)}(x)\ t^n$$

$$= F^{(3)} \left[\begin{array}{l} \lambda, \mu{::}-; \gamma, \delta; -{:}\, - \;;\; - \;;\; - \;; \\ \\ - {::}-; \sigma; - \;{:}\nu; \alpha+1; \beta+1; \end{array} y, \tfrac{1}{2}(x-1)t, \tfrac{1}{2}(x+1)t \right]. \quad (3.21)$$

Further equation (3.18) for $y = 0$ reduces to a linear generating involving Kampé de Fériet's function $F^{(2)}$ in the form:

$$\sum_{n=0}^{\infty} \frac{(\gamma)_n(\delta)_n(\lambda)_n(\mu)_n}{(\alpha+1)_n(\beta+1)_n} \; P_n^{(\alpha,\beta)}(x)\; t^n \;=$$

$$= F^{(2)} \left[\begin{array}{l} \lambda, \mu, \gamma, \delta{:}-; - \;; \\ \\ - \;{:}\alpha+1; \beta+1; \end{array} \tfrac{1}{2}(x-1)t, \tfrac{1}{2}(x+1)t \right]. \quad (3.22)$$

Case IV. Consider the generating function [3, p. 44], see also [37, p. 186]:

$$\sum_{n=0}^{\infty} \frac{(\alpha+\beta)_n}{(\gamma)_n} \; G_n(\alpha, \beta; x)\; t^n = F_3\left[\alpha, \alpha, \beta, \beta; \gamma; u\, t, v\, t\right] \quad (3.23)$$

where $u = x - \sqrt{x^2-1}$ and $v = x + \sqrt{x^2+1}$, and $G(\alpha, \beta; x)$ are Bediant polynomials given by equation (1.11).
In (2.1) we take $F(x,t) = F_3\left[\alpha, \alpha, \beta, \beta; \gamma; u\, t, v\, t\right]$, Combining (3.23) with (2.1) and using (2.2) of Theorem 1 with $z = 0$, we get the bilateral generating function:

$$\sum_{n=0}^{\infty} \frac{(\alpha+\beta)_n(\lambda)_n}{(\gamma)_n(1+\lambda-\sigma)_n} \; {}_{1+A+D}F_{1+B+E} \left[\begin{array}{l} \lambda+n, (a_A), (d_D); \\ \\ \sigma, (b_B), (e_E) \;; \end{array} y \right] G_n(\alpha, \beta; x)\; t^n$$

$$= F^{(3)} \left[\begin{array}{l} \lambda{::}-; -; \quad - {:}(a_A), (d_D); \alpha, \beta; \alpha, \beta; \\ \\ -{::}-; \gamma, 1+\lambda-\sigma; -{:}\; \sigma, (b_B), (e_E); -; -; \end{array} y, u\, t, v\, t \right]. \quad (3.24)$$

Now on taking $D = B = E = 0, A = 1$ and then replacing a_1 by μ, we obtain a bilateral generating relation for Bediant polynomials in the form:

$$\sum_{n=0}^{\infty} \frac{(\alpha+\beta)_n(\lambda)_n}{(\gamma)_n(1+\lambda-\sigma)_n} \; {}_2F_1 \left[\begin{array}{l} \lambda+n, \mu; \\ \\ \sigma \;; \end{array} y \right] G_n(\alpha, \beta; x)\; t^n =$$

$$= F^{(3)} \left[\begin{array}{l} \lambda :: - ; - ; - : \mu; \alpha, \beta; \alpha, \beta; \\ \qquad\qquad\qquad\qquad\qquad\qquad y, u\, t, v\, t \\ - :: -; \gamma, 1+\lambda-\sigma; - : \sigma; - ; - ; \end{array} \right], \tag{3.25}$$

which on putting $y = 0$ gives a linear generating relation involving Kampé de Fériet's double hypergeometric function $F^{(2)}$ in the form:

$$\sum_{n=0}^{\infty} \frac{(\alpha+\beta)_n (\lambda)_n}{(\gamma)_n (1+\lambda-\sigma)_n} G_n(\alpha, \beta; x)\, t^n = F^{(2)} \left[\begin{array}{l} \lambda : \alpha, \beta; \alpha, \beta ; \\ \qquad\qquad\qquad u\, t, v\, t \\ 1+\lambda-\sigma, \gamma : -; -; \end{array} \right]. \tag{3.26}$$

Further in (3.26), replacing y into $\frac{y}{\mu}$ and letting $\mu \to \infty$, we obtain a bilateral generating relation for Bediant polynomials involving Kumar's function ${}_1F_1$ in the form:

$$\sum_{n=0}^{\infty} \frac{(\alpha+\beta)_n (\lambda)_n}{(\gamma)_n (1+\lambda-\sigma)_n} {}_1F_1 \left[\begin{array}{l} \lambda+n; \\ \qquad\quad y \\ \sigma; \end{array} \right] G_n(\alpha, \beta; x)\, t^n =$$

$$= F^{(3)} \left[\begin{array}{l} \lambda :: -; -; - : -; \alpha, \beta; \alpha, \beta ; \\ \qquad\qquad\qquad\qquad\qquad\qquad y, u\, t, v\, t \\ - :: -; 1+\lambda-\sigma, \gamma; - : \sigma ; -; - ; \end{array} \right]. \tag{3.27}$$

Case V. Consider the generating function [37, p. 202, eq. (1)], see also [11, p. 267, eq. (25)]:

$$\sum_{n=0}^{\infty} \frac{P_n^{(\alpha-n, \beta-n)}}{(-\alpha-\beta)_n}(x)\, t^n = e^{\left(\frac{1}{2}(1-x)t\right)} {}_1F_1 \left[\begin{array}{l} -\alpha ; \\ \qquad\qquad -t \\ -\alpha-\beta; \end{array} \right]. \tag{3.28}$$

In (3.28), we take $G(x,t) = e^{\left(\frac{1}{2}(1-x)t\right)} {}_1F_1 \left[\begin{array}{l} -\alpha ; \\ \qquad\qquad -t \\ -\alpha-\beta; \end{array} \right]$, Combining (3.28) with (2.9) and then using Theorem 2, we obtain the bilateral generating relation in the form:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(-\alpha-\beta)_n} F^{(2)} \left[\begin{array}{l} \lambda+n, (a_A) : (d_D) ; (g_G); \\ \qquad\qquad\qquad\qquad y, z \\ (b_B) : (e_E); (h_H) ; \end{array} \right] P_n^{(\alpha-n, \beta-n)}(x)\, t^n =$$

$$= \sum_{r=0}^{\infty} \frac{(\lambda)_r (\frac{1}{2}(1-x)t)^r}{r!} F^{(3)} \left[\begin{array}{l} \lambda+r :: -; (a_A); - : -\alpha ; (d_D) ; (g_G); \\ \qquad\qquad\qquad\qquad\qquad\qquad -t, y, z \\ - :: -; (b_B); - : -\alpha-\beta; (e_E); (h_H); \end{array} \right]. \tag{3.29}$$

Also, equation (3.29) can be expressed equivalently in the form:

$$\sum_{n=0}^{\infty}\frac{(\lambda)_n}{(-\alpha-\beta)_n}\ F^{(2)}\left[\begin{array}{l} \lambda+n,(a_A):(d_D);(g_G);\\ \\ (b_B)\qquad\quad :(e_E);(h_H);\end{array} y,z\right]\ P_n^{(\alpha-n,\beta-n)}(x)\ t^n$$

$$=\left(1-\frac{1}{2}(1-x)t\right)^{-\lambda}\times$$

$$F^{(3)}\left[\begin{array}{l} \lambda::-;\ \ (a_A);-:-\alpha\ \ ;(d_D)\ \ ;\ \ (g_G);\\ \\ -::-;(b_B);-:-\alpha-\beta;\ \ (e_E);\ (h_H);\end{array} \frac{-2t}{2-(1-x)t},\frac{2y}{2-(x-1)t},\frac{2z}{2-(x-1)t}\right]. \quad (3.30)$$

Again, Kampé de Fériet's function $F^{(2)}$ in (3.30) can be specialized to yield the generating function for Appell's function F_1 to F_4.
For example, on taking $A=E=H=0, B=D=G=1$ and replacing b_1 by γ,d_1 by μ_1 and g_1 by μ_2, we get a bilateral generating function involving Appell's function F_1 in the form:

$$\sum_{n=0}^{\infty}\frac{(\lambda)_n}{(-\alpha-\beta)_n}\ F_1\left[\lambda+n,\mu_1,\mu_2;\gamma;y,z\right]\ P_n^{(\alpha-n,\beta-n)}(x)\ t^n$$

$$=(\omega)^{-\lambda}\ F_G\left[\lambda,\lambda,\lambda,-\alpha,\mu_1,\mu_2;-\alpha-\beta,\mu,\mu;\frac{-t}{\omega},\frac{y}{\omega},\frac{z}{\omega}\right] \quad (3.31)$$

where for convenience, we put

$$\omega=1-\tfrac{1}{2}(1-x)t$$

and F_G is Saran's triple hypergeometric function given by equation (1.10).

Similarly on putting $B=D=G=0, A=E=H=1$ and then replacing a_1 by γ , e_1 by μ_1 and h_1 by μ_2, equation (3.30) reduces to a generating relation involving Appell's function F_4 in the form:

$$\sum_{n=0}^{\infty}\frac{(\lambda)_n}{(-\alpha-\beta)_n}\ F_4\left[\lambda+n,\gamma;\mu_1,\mu_2;y,z\right]\ P_n^{(\alpha-n,\beta-n)}(x)\ t^n$$

$$=(\omega)^{-\lambda}\ F_E\left[\lambda,\lambda,\lambda,-\alpha,\gamma,\gamma;-\alpha-\beta,\mu_1,\mu_2;\frac{-t}{\omega},\frac{y}{\omega},\frac{z}{\omega}\right]. \quad (3.32)$$

where F_E is Saran's triple hypergeometric function defined by equation (1.9).
Again putting $A=B=0, D=E=G=H=1$ and then replacing d_1 by b, g_1 by

c, e_1 by d and h_1 by e , we get a bilateral generating relation involving Lauricella function $F_A^{(3)}$ in the form:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(-\alpha-\beta)_n} \; F_2\left[\lambda+n, b, c; d, e; y, z\right] \; P_n^{(\alpha-n,\beta-n)}(x) \; t^n$$

$$= (\omega)^{-\lambda} \; F_A^{(3)} \left[\lambda, -\alpha, b, c; -\alpha-\beta, d, e; \frac{-t}{\omega}, \frac{y}{\omega}, \frac{z}{\omega}\right]. \qquad (3.33)$$

where $F_A^{(3)}$ is Lauricella function of three variables defined by (1.19).
It is to be noted that equations (3.31) to (3.33) are known results of Chaudhary [6, pp. 264-265, eqns. (4.3) to (4.5)]. Some more bilinear and bilateral generating relations involving special Jacobi polynomials follow immediately from above results upon reducing F_1, F_2 and F_4 to Gauss function ${}_2F_1$.
Further equation (3.30) in the light of result (1.8) gives the generating relation:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(-\alpha-\beta)_n} F^{(2)} \left[\begin{array}{lll} \lambda+n, (a_A) : (d_D); (g_G); & & \\ & & y.z \\ (b_B) \qquad : (e_E); (h_H); & & \end{array}\right] \left(\frac{1-x}{2}\right)^n P_n^{(-\alpha-\beta-1,\beta-n)} \left(\frac{x+3}{x-1}\right) t^n$$

$$= (\omega)^{-\lambda} \; F^{(3)} \left[\begin{array}{ll} \lambda :: -; (a_A); - : -\alpha \quad ; (d_D) \quad ; (g_G); & \\ & \frac{-t}{\omega}, \frac{y}{\omega}, \frac{z}{\omega} \\ - :: -; (b_B); - : -\alpha-\beta; (e_E); (h_H); & \end{array}\right]. \qquad (3.34)$$

Finally in equation (3.30), putting $G = H = D = E = z = 0, A = B = 1$ and then replacing a_1 by ν and b_1 by σ, we get a generating relation involving Kampé de Fériet's function $F^{(2)}$ in the form:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(-\alpha-\beta)_n} \; {}_2F_1 \left[\begin{array}{ll} \lambda+n, \nu; & \\ & y \\ \sigma \qquad ; & \end{array}\right] \; P_n^{(\alpha-n,\beta-n)}(x) \; t^n =$$

$$= (\omega)^{-\lambda} \; F^{(2)} \left[\begin{array}{ll} \lambda : -\alpha \quad ; \nu \;\; ; & \\ & \frac{-t}{\omega}, \frac{y}{\omega} \\ - : -\alpha-\beta; \sigma; & \end{array}\right]. \qquad (3.35)$$

4. Conclusion

In present paper three theorems in terms of Kampé de Fériet's function $F^{(2)}[x, y]$ have been obtained. These theorems have been used to obtain linear, bilinear and bilateral generating relations for a variety of hypergeometric polynomials. These results can further be extended as well as used to obtain generating relations for the hypergeometric functions and polynomials available in the literature on special functions.

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