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HOMOMORPHISM AND ANTI-HOMOMORPHISM OF SPHERICAL CUBIC BI-IDEALS OF GAMMA NEAR-RINGS

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Abstract: The purpose of the article is to study about homomorphism and anti-homomorphism of spherical cubic bi-ideals of Gamma near-rings $\mathcal{R}_1$ and $\mathcal{R}_2$. If $\phi : R_1 \longrightarrow R_2$ be a gamma homomorphism and $(\mathscr{C}\mathscr{U}_{s_1}, R_1)$, $(\mathscr{C}\mathscr{U}_{s_2}, R_2)$ are spherical cubic bi-ideals of gamma near-rings R_1 and R_2.Then the image $(\phi(\mathscr{C}\mathscr{U}_{s_1}), R_2)$ and pre-image $(\phi^{-1}(\mathscr{C}\mathscr{U}_{s_2}), R_1)$ are also spherical cubic bi-ideals of gamma near-rings R_2 and R_1. If $\phi : \mathcal{R}_1 \longrightarrow \mathcal{R}_2$ be an epimorphism of gamma near-rings $\mathcal{R}_1$ and $\mathcal{R}_2$ and $(\mathscr{C}\mathscr{U}_{s_2}, \mathcal{R}_2)$ is a SCS of $\mathcal{R}_2$ such that $(\phi^{-1}(\mathscr{C}\mathscr{U}_{s_2}), \mathcal{R}_1)$ is a SCBI of $\mathcal{R}_1$, then $(\mathscr{C}\mathscr{U}_{s_2}, \mathcal{R}_2)$ is a SCBI of $\mathcal{R}_2$.

Keywords and Phrases: Spherical set, cubic set, Γ-near-ring, bi-ideal, homomorphism, anti-homomorphism.

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1. Introduction

The notion of fuzzy set was introduced by Zadeh [18] in 1965. It is identified as a better tool for the scientific study of uncertainty, and came as a boost to the researchers working in the field of uncertainty. Many extensions and generalizations of fuzzy set was conceived by a number of researchers and a large number of real-life applications were developed in a variety of areas. In addition to this, parallel analysis of the classical results of many branches of Mathematics were also carried out in the fuzzy settings. Properties of fuzzy ideals in near-rings was studied by Hong et al. [9]. The monograph by Chinnadurai [1] gives a detailed discussion on fuzzy ideals in algebraic structures. Fuzzy ideals in Gamma near-ring $\mathcal{R}$ was discussed by Jun et al. [10, 11] and Satyanarayana [15]. Meenakumari and Tamizh chelvam[14] have defined fuzzy bi-ideals in gamma near-rings and established some properties of this structure. Srinivas and Nagaiah [16] have proved some results on T-fuzzy ideals of Γ-near-rings. Jun [12] introduced a new notion called a cubic set and investigated several properties. Thillaigovindan et al. [17] worked on interval valued fuzzy ideals of near-rings. Chinnadurai et al. [2, 3] discussed cubic ideals of Γ-near rings and homomorphism and anti- homomorphism of cubic ideals of near-rings. Kahraman and Gundogdu [13] introduced spherical fuzzy sets as an extension of picture fuzzy sets. Chinnadurai et al. [4] discussed interval-valued fuzzy ideals of gamma near-rings. Chinnadurai et al. [5, 6, 7, 8] discussed T-fuzzy, spherical fuzzy, spherical interval-valued fuzzy and spherical cubic bi-ideals of gamma near-rings. In this research work, we discuss the homomorphism and anti-homomorphism of spherical cubic bi-ideals of Gamma near-rings $\mathcal{R}_1$ and $\mathcal{R}_2$, establish some of its properties.

2. Preliminaries

In this section we present some definitions which are used in this research.
Let $\mathcal{R}$ be a near-ring and Γ be a non-empty set such that $\mathcal{R}$ is a Gamma near-ring.
A subgroup H of $(\mathcal{R}, +)$ is a bi-ideal if and only if $H\Gamma\mathcal{R}\Gamma H \subseteq H$.
Let $\mathcal{R}$ be a nonempty set. By a cubic set in $\mathcal{R}$ we mean a structure $\mathscr{A} = \{u, A(u), \lambda(u) | u \in \mathcal{R}\}$ in which A is an interval-valued fuzzy set in $\mathcal{R}$ and λ is a fuzzy set in $\mathcal{R}$. A cubic set is simply denoted by $\mathscr{A} =< A, \lambda >$.
A fuzzy set μ of $\mathcal{R}$ to be fuzzy bi-ideal of gamma near-ring $\mathcal{R}$ if the given conditions are satisfied

(i) $\mu(u - v) \geq min\{\mu(u), \mu(v)\}$,

(ii) $\mu(u\alpha v\beta w) \geq min\{\mu(u), \mu(w)\}$,

for all $u, v, w \in \mathcal{R}$ and $\alpha, \beta \in \Gamma$.
A spherical fuzzy set $\widetilde{A}_s$ of the universe of discourse U is given by, $\widetilde{A}_s = \{u, (\widetilde{\mu}(u), \widetilde{\nu}(u), \widetilde{\xi}(u)) | u \in U\}$ where $\widetilde{\mu}(u) : U \longrightarrow [0, 1]$, $\widetilde{\nu}(u) : U \longrightarrow [0, 1]$ and $\widetilde{\xi}(u) : U \longrightarrow [0, 1]$ and $0 \leq \widetilde{\mu}^2(u) + \widetilde{\nu}^2(u) + \widetilde{\xi}^2(u) \leq 1$, $u \in U$.
For each u, the numbers $\widetilde{\mu}(u)$, $\widetilde{\nu}(u)$ and $\widetilde{\xi}(u)$ are the degrees of membership,non-membership and hesitancy of u to $\widetilde{A}_s$, respectively.
A spherical fuzzy set(SFS) $A_s = (\mu, \nu, \xi)$, where $\mu : \mathcal{R} \longrightarrow [0, 1]$, $\nu : \mathcal{R} \longrightarrow [0, 1]$ and $\xi : \mathcal{R} \longrightarrow [0, 1]$ of $\mathcal{R}$ is said to be a spherical fuzzy bi-ideal of $\mathcal{R}$ if the following conditions are satisfied

(i) $\mu(u-v) \geq min\{\mu(u), \mu(v)\}$,
(ii) $\nu(u-v) \geq min\{\nu(u), \nu(v)\}$,
(iii) $\xi(u-v) \leq max\{\xi(u), \xi(v)\}$,
(iv) $\mu(u\alpha v\beta w) \geq min\{\mu(u), \mu(w)\}$,
(v) $\nu(u\alpha v\beta w) \geq min\{\nu(u), \nu(w)\}$,
(vi) $\xi(u\alpha v\beta w) \leq max\{\xi(u), \xi(w)\}$,
for all $u, v, w \in \mathcal{R}$ and $\alpha, \beta \in \Gamma$.

A spherical cubic set (SCS) in $\mathcal{R}$ is defined by $\mathscr{CU}_s = \{< u, \mathscr{A}_s(u), \mu(u) >, < u, \mathscr{B}_s(u), \nu(u) >, < u, \mathscr{C}_s(u), \xi(u) > | u \in \mathcal{R}\}$, where $\mathscr{A}_s$, $\mathscr{B}_s$, $\mathscr{C}_s$ are interval-valued spherical sets in $\mathcal{R}$ and μ, ν, ξ are spherical fuzzy sets in $\mathcal{R}$.

A spherical cubic set $\mathscr{CU}_s = \{< u, \mathscr{A}_s(u), \mu(u) >, < u, \mathscr{B}_s(u), \nu(u) >, < u, \mathscr{C}_s(u), \xi(u) > | u \in \mathcal{R}\}$ is simply denoted by $\mathscr{CU}_s = \{< \mathscr{A}_s, \mu >, < \mathscr{B}_s, \nu >, < \mathscr{C}_s, \xi >\}$.

A spherical cubic set $\mathscr{CU}_s = \{< u, \mathscr{A}_s(u), \mu(u) >, < u, \mathscr{B}_s(u), \nu(u) >, < u, \mathscr{C}_s(u), \xi(u) > | u \in \mathcal{R}\}$ is said to be a spherical cubic bi-ideal(SCBI) of gamma near-ring if the following conditions are satisfied

(i) $\mathscr{A}_s(u-v) \geq min^i\{\mathscr{A}_s(u), \mathscr{A}_s(v)\}$, $\mu(u-v) \leq max\{\mu(u), \mu(v)\}$,
(ii) $\mathscr{B}_s(u-v) \geq min^i\{\mathscr{B}_s(u), \mathscr{B}_s(v)\}$, $\nu(u-v) \leq max\{\nu(u), \nu(v)\}$,
(iii) $\mathscr{C}_s(u-v) \leq max^i\{\mathscr{C}_s(u), \mathscr{C}_s(v)\}$, $\xi(u-v) \geq min\{\xi(u), \xi(v)\}$,
(iv) $\mathscr{A}_s(u\alpha v\beta w) \geq min^i\{\mathscr{A}_s(u), \mathscr{A}_s(w)\}$, $\mu(u\alpha v\beta w) \leq max\{\mu(u), \mu(w)\}$,
(v) $\mathscr{B}_s(u\alpha v\beta w) \geq min^i\{\mathscr{B}_s(u), \mathscr{B}_s(w)\}$, $\nu(u\alpha v\beta w) \leq max\{\nu(u), \nu(w)\}$,
(vi) $\mathscr{C}_s(u\alpha v\beta w) \leq max^i\{\mathscr{C}_s(u), \mathscr{C}_s(w)\}$, $\xi(u\alpha v\beta w) \geq min\{\xi(u), \xi(w)\}$,
for all $u, v, w \in \mathcal{R}$ and $\alpha, \beta \in \Gamma$, where $\mathscr{A}_s : \mathcal{R} \longrightarrow D[0,1]$, $\mathscr{B}_s : \mathcal{R} \longrightarrow D[0,1]$ and $\mathscr{C}_s : \mathcal{R} \longrightarrow D[0,1]$. Here $D[0,1]$ denotes the family of closed subintervals of $[0,1]$ and $\mu : \mathcal{R} \longrightarrow [0,1]$, $\nu : \mathcal{R} \longrightarrow [0,1]$ and $\xi : \mathcal{R} \longrightarrow [0,1]$.

A gamma near-ring homomorphism is a mapping ϕ from a gamma near-ring R_1 into a gamma near-ring R_2, that is $\phi : R_1 \longrightarrow R_2$ such that

(i) $\phi(u-v) = \phi(u) - \phi(v)$, for all $u, v \in R_1$.
(ii) $\phi(u\alpha v\beta w) = \phi(u)\alpha\phi(v)\beta\phi(w)$, for all $u, v, w \in R_1$ and $\alpha, \beta \in \Gamma$.

A gamma near-ring anti-homomorphism is a mapping ϕ from a gamma near-ring R_1 into a gamma near-ring R_2, that is $\phi : R_1 \longrightarrow R_2$ such that

(i) $\phi(u-v) = \phi(v) - \phi(u)$, for all $u, v \in R_1$.
(ii) $\phi(u\alpha v\beta w) = \phi(w)\alpha\phi(v)\beta\phi(u)$, for all $u, v, w \in R_1$ and $\alpha, \beta \in \Gamma$.

3. Homomorphism of Spherical Cubic bi-ideals of Gamma near-rings

In this section, we study about the properties of spherical cubic bi-ideals of gamma near-rings using homomorphism.

Definition 3.1. *Let ϕ be a mapping from a set $\mathcal{R}_1$ to a set $\mathcal{R}_2$. Let* $\mathscr{CU}_{s_1} = \{< u, \mathscr{A}_{s_1}(u), \mu_1(u) >, < u, \mathscr{B}_{s_1}(u), \nu_1(u) >, < u, \mathscr{C}_{s_1}(u), \xi_1(u) > | u \in \mathcal{R}\}$

be a SCS in $\mathcal{R}_1$ *and* $\mathscr{CU}_{s_2} = \{< u, \mathscr{A}_{s_2}(u), \mu_2(u) >, < u, \mathscr{B}_{s_2}(u), \nu_2(u) >, < u, \mathscr{C}_{s_2}(u), \xi_2(u) > | u \in \mathcal{R}\}$ *be a SCS in* $\mathcal{R}_2$. *Then,*
(i) The image $\phi(\mathscr{CU}_{s_1}) = \{< \phi(\mathscr{A}_{s_1}), \phi(\mu_1) >, < \phi(\mathscr{B}_{s_1}), \phi(\nu_1) >, < \phi(\mathscr{C}_{s_1}), \phi(\xi_1) > \}$ *is a SCS in* $\mathcal{R}_2$ *defined by*

$$\phi(\mathscr{A}_{s_1})(u) = \begin{cases} \sup\limits_{v \in \phi^{-1}(u)} \mathscr{A}_{s_1}(v), & \text{if } \phi^{-1}(u) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}$$

$$\phi(\mu_1)(u) = \begin{cases} \inf\limits_{v \in \phi^{-1}(u)} \mu_1(v), & \text{if } \phi^{-1}(u) \neq \emptyset \\ 1, & \text{otherwise.} \end{cases}$$

$$\phi(\mathscr{B}_{s_1})(u) = \begin{cases} \sup\limits_{v \in \phi^{-1}(u)} \mathscr{B}_{s_1}(v), & \text{if } \phi^{-1}(u) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}$$

$$\phi(\nu_1)(u) = \begin{cases} \inf\limits_{v \in \phi^{-1}(u)} \nu_1(v), & \text{if } \phi^{-1}(u) \neq \emptyset \\ 1, & \text{otherwise.} \end{cases}$$

$$\phi(\mathscr{C}_{s_1})(u) = \begin{cases} \inf\limits_{v \in \phi^{-1}(u)} \mathscr{C}_{s_1}(v), & \text{if } \phi^{-1}(u) \neq \emptyset \\ 1, & \text{otherwise.} \end{cases}$$

$$\phi(\xi_1)(u) = \begin{cases} \sup\limits_{v \in \phi^{-1}(u)} \xi_1(v), & \text{if } \phi^{-1}(u) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}$$

(ii) The pre-image $\phi^{-1}(\mathscr{CU}_{s_2}) = \{< (\phi^{-1}(\mathscr{A}_{s_2}), \phi^{-1}(\mu_2)) >, < (\phi^{-1}(\mathscr{B}_{s_2}), \phi^{-1}(\nu_2)) >, < (\phi^{-1}(\mathscr{C}_{s_2}), \phi^{-1}(\xi_2)) >\}$ *is a SCS in* $\mathcal{R}_1$ *defined by*
$\phi^{-1}(\mathscr{CU}_{s_2})(u) = \{< (\phi^{-1}(\mathscr{A}_{s_2}(u)), \phi^{-1}(\mu_2(u))) >, < (\phi^{-1}(\mathscr{B}_{s_2}(u)), \phi^{-1}(\nu_2(u))) >, < (\phi^{-1}(\mathscr{C}_{s_2}(u)), \phi^{-1}(\xi_2(u))) >\} = \{< (\mathscr{A}_{s_2}(\phi(u)), \mu_2(\phi(u))) >, < (\mathscr{B}_{s_2}(\phi(u)), \nu_2(\phi(u))) >, < (\mathscr{C}_{s_2}(\phi(u)), \xi_2(\phi(u))) >\}$.

Example 3.2. Let $\mathcal{R} = \{0, 1, 2, 3\}$ with binary operation " $+$ " on $\mathcal{R}$, $\Gamma = \{0, 1\}$ and $\mathcal{R} \times \Gamma \times \mathcal{R} \longrightarrow \mathcal{R}$ be a mapping. We define SCS in $\mathcal{R}$ as

Table 3.1

$\mathcal{R}$	$\mathscr{A}_s$	μ
0	(0.3, 0.7)	0.9
1	(0.4, 0.6)	0.8
2	(0.5, 0.7)	0.4
3	(0.6, 0.8)	0.5

$\mathcal{R}$	$\mathscr{B}_s$	ν
0	(0.3, 0.7)	0.8
1	(0.2, 0.5)	0.6
2	(0.1, 0.4)	0.6
3	(0.4, 0.6)	0.7

$\mathcal{R}$	$\mathscr{C}_s$	ξ
0	(0.1,0.2)	0.4
1	(0.4, 0.7)	0.3
2	(0.3, 0.6)	0.7
3	(0.4, 0.8)	0.9

Then $\mathscr{CU}_s$ is a SCBI of $\mathcal{R}$ using homomorphism.

Theorem 3.3. *Let $\phi : \mathcal{R}_1 \longrightarrow \mathcal{R}_2$ be a gamma homomorphism and $(\mathscr{CU}_{s_1}, \mathcal{R}_1)$ be a SCBI of $\mathcal{R}_1$. Then the image $(\phi(\mathscr{CU}_{s_1}), \mathcal{R}_2)$ is also a SCBI of $\mathcal{R}_2$.*

Proof. Let $u, v, w \in \mathcal{R}_1$ and $\alpha, \beta \in \Gamma$. Since ϕ is a gamma homomorphism and $\mathscr{CU}_{s_1}$ is a SCBI of $\mathcal{R}_1$, we have

(i)
$$\begin{aligned}
\phi(\mathscr{A}_{s_1})(u-v) &= \sup_{w\in\phi^{-1}(u-v)} \mathscr{A}_{s_1}(w)\\
&= \sup_{\phi(w)=u-v} \mathscr{A}_{s_1}(w)\\
&= \sup_{\phi(u)=u,\phi(v)=v} \mathscr{A}_{s_1}(u-v)\\
&\geq \sup_{\phi(u)=u,\phi(v)=v} (min^i\{\mathscr{A}_{s_1}(u), \mathscr{A}_{s_1}(v)\})\\
&= min^i\{\sup_{\phi(u)=u} \mathscr{A}_{s_1}(u), \sup_{\phi(v)=v} \mathscr{A}_{s_1}(v)\}\\
&= min^i\{\phi(\mathscr{A}_{s_1})(u), \phi(\mathscr{A}_{s_1})(v)\},
\end{aligned}$$

$$\begin{aligned}
\phi(\mu_1)(u-v) &= \inf_{w\in\phi^{-1}(u-v)} \mu_1(w)\\
&= \inf_{\phi(w)=u-v} \mu_1(w)\\
&= \inf_{\phi(u)=u,\phi(v)=v} \mu_1(u-v)\\
&\leq \inf_{\phi(u)=u,\phi(v)=v} (max\{\mu_1(u), \mu_1(v)\})\\
&= max\{\inf_{\phi(u)=u} \mu_1(u), \inf_{\phi(v)=v} \mu_1(v)\}\\
&= max\{\phi(\mu_1)(u), \phi(\mu_1)(v)\},
\end{aligned}$$

(ii)
$$\begin{aligned}
\phi(\mathscr{B}_{s_1})(u-v) &= \sup_{w\in\phi^{-1}(u-v)} \mathscr{B}_{s_1}(w)\\
&= \sup_{\phi(w)=u-v} \mathscr{B}_{s_1}(w)\\
&= \sup_{\phi(u)=u,\phi(v)=v} \mathscr{B}_{s_1}(u-v)\\
&\geq \sup_{\phi(u)=u,\phi(v)=v} (min^i\{\mathscr{B}_{s_1}(u), \mathscr{B}_{s_1}(v)\})\\
&= min^i\{\sup_{\phi(u)=u} \mathscr{B}_{s_1}(u), \sup_{\phi(v)=v} \mathscr{B}_{s_1}(v)\}\\
&= min^i\{\phi(\mathscr{B}_{s_1})(u), \phi(\mathscr{B}_{s_1})(v)\},
\end{aligned}$$

$$\begin{aligned}
\phi(\nu_1)(u-v) &= \inf_{w\in\phi^{-1}(u-v)} \nu_1(w)\\
&= \inf_{\phi(w)=u-v} \nu_1(w)\\
&= \inf_{\phi(u)=u,\phi(v)=v} \nu_1(u-v)\\
&\leq \inf_{\phi(u)=u,\phi(v)=v} (max\{\nu_1(u), \nu_1(v)\})
\end{aligned}$$

$$= max\{\inf_{\phi(u)=u} \nu_1(u), \inf_{\phi(v)=v} \nu_1(v)\}$$
$$= max\{\phi(\nu_1)(u), \phi(\nu_1)(v)\},$$

(iii) $$\phi(\mathscr{C}_{s_1})(u-v) = \inf_{w\in\phi^{-1}(u-v)} \mathscr{C}_{s_1}(w)$$
$$= \inf_{\phi(w)=u-v} \mathscr{C}_{s_1}(w)$$
$$= \inf_{\phi(u)=u,\phi(v)=v} \mathscr{C}_{s_1}(u-v)$$
$$\leq \inf_{\phi(u)=u,\phi(v)=v} (max^i\{\mathscr{C}_{s_1}(u), \mathscr{C}_{s_1}(v)\})$$
$$= max^i\{\inf_{\phi(u)=u} \mathscr{C}_{s_1}(u), \inf_{\phi(v)=v} \mathscr{C}_{s_1}(v)\}$$
$$= max^i\{\phi(\mathscr{C}_{s_1})(u), \phi(\mathscr{C}_{s_1})(v)\},$$

$$\phi(\xi_1)(u-v) = \sup_{w\in\phi^{-1}(u-v)} \xi_1(w)$$
$$= \sup_{\phi(w)=u-v} \xi_1(w)$$
$$= \sup_{\phi(u)=u,\phi(v)=v} \xi_1(u-v)$$
$$\geq \sup_{\phi(u)=u,\phi(v)=v} (min\{\xi_1(u), \xi_1(v)\})$$
$$= min\{\sup_{\phi(u)=u} \xi_1(u), \sup_{\phi(v)=v} \xi_1(v)\}$$
$$= min\{\phi(\xi_1)(u), \phi(\xi_1)(v)\},$$

(iv) $$\phi(\mathscr{A}_{s_1})(u\alpha v\beta w) = \sup_{w\in\phi^{-1}(u\alpha v\beta w)} \mathscr{A}_{s_1}(w)$$
$$= \sup_{\phi(w)=u\alpha v\beta w} \mathscr{A}_{s_1}(w)$$
$$= \sup_{\phi(u)=u,\phi(w)=w} \mathscr{A}_{s_1}(u\alpha v\beta w)$$
$$\geq \sup_{\phi(u)=u,\phi(w)=w} (min^i\{\mathscr{A}_{s_1}(u), \mathscr{A}_{s_1}(w)\})$$
$$= min^i\{\sup_{\phi(u)=u} \mathscr{A}_{s_1}(u), \sup_{\phi(w)=w} \mathscr{A}_{s_1}(w)\}$$
$$= min^i\{\phi(\mathscr{A}_{s_1})(u), \phi(\mathscr{A}_{s_1})(w)\},$$

$$\phi(\mu_1)(u\alpha v\beta w) = \inf_{w\in\phi^{-1}(u\alpha v\beta w)} \mu_1(w)$$
$$= \inf_{\phi(w)=u\alpha v\beta w} \mu_1(w)$$
$$= \inf_{\phi(u)=u,\phi(w)=w} \mu_1(u\alpha v\beta w)$$
$$\leq \inf_{\phi(u)=u,\phi(w)=w} (max\{\mu_1(u), \mu_1(w)\})$$
$$= max\{\inf_{\phi(u)=u} \mu_1(u), \inf_{\phi(w)=w} \mu_1(w)\}$$
$$= max\{\phi(\mu_1)(u), \phi(\mu_1)(w)\},$$

$$
\begin{aligned}
\text{(v) } \phi(\mathscr{B}_{s_1})(u\alpha v\beta w) &= \sup_{w\in\phi^{-1}(u\alpha v\beta w)} \mathscr{B}_{s_1}(w) \\
&= \sup_{\phi(w)=u\alpha v\beta w} \mathscr{B}_{s_1}(w) \\
&= \sup_{\phi(u)=u,\phi(w)=w} \mathscr{B}_{s_1}(u\alpha v\beta w) \\
&\geq \sup_{\phi(u)=u,\phi(w)=w} (min^i\{\mathscr{B}_{s_1}(u), \mathscr{B}_{s_1}(w)\}) \\
&= min^i\{\sup_{\phi(u)=u} \mathscr{B}_{s_1}(u), \sup_{\phi(w)=w} \mathscr{B}_{s_1}(w)\} \\
&= min^i\{\phi(\mathscr{B}_{s_1})(u), \phi(\mathscr{B}_{s_1})(w)\},
\end{aligned}
$$

$$
\begin{aligned}
\phi(\nu_1)(u\alpha v\beta w) &= \inf_{w\in\phi^{-1}(u\alpha v\beta w)} \nu_1(w) \\
&= \inf_{\phi(w)=u\alpha v\beta w} \nu_1(w) \\
&= \inf_{\phi(u)=u,\phi(w)=w} \nu_1(u\alpha v\beta w) \\
&\leq \inf_{\phi(u)=u,\phi(w)=w} (max\{\nu_1(u), \nu_1(w)\}) \\
&= max\{\inf_{\phi(u)=u} \nu_1(u), \inf_{\phi(w)=w} \nu_1(w)\} \\
&= max\{\phi(\nu_1)(u), \phi(\nu_1)(w)\},
\end{aligned}
$$

$$
\begin{aligned}
\text{(vi) } \phi(\mathscr{C}_{s_1})(u\alpha v\beta w) &= \inf_{w\in\phi^{-1}(u\alpha v\beta w)} \mathscr{C}_{s_1}(w) \\
&= \inf_{\phi(w)=u\alpha v\beta w} \mathscr{C}_{s_1}(w) \\
&= \inf_{\phi(u)=u,\phi(w)=w} \mathscr{C}_{s_1}(u\alpha v\beta w) \\
&\leq \inf_{\phi(u)=u,\phi(w)=w} (max^i\{\mathscr{C}_{s_1}(u), \mathscr{C}_{s_1}(w)\}) \\
&= max^i\{\inf_{\phi(u)=u} \mathscr{C}_{s_1}(u), \inf_{\phi(w)=w} \mathscr{C}_{s_1}(w)\} \\
&= max^i\{\phi(\mathscr{C}_{s_1})(u), \phi(\mathscr{C}_{s_1})(w)\},
\end{aligned}
$$

$$
\begin{aligned}
\phi(\xi_1)(u\alpha v\beta w) &= \sup_{w\in\phi^{-1}(u\alpha v\beta w)} \xi_1(w) \\
&= \sup_{\phi(w)=u\alpha v\beta w} \xi_1(w) \\
&= \sup_{\phi(u)=u,\phi(w)=w} \xi_1(u\alpha v\beta w) \\
&\geq \sup_{\phi(u)=u,\phi(w)=w} (min\{\xi_1(u), \xi_1(w)\}) \\
&= min\{\sup_{\phi(u)=u} \xi_1(u), \sup_{\phi(w)=w} \xi_1(w)\} \\
&= min\{\phi(\xi_1)(u), \phi(\xi_1)(w)\}.
\end{aligned}
$$

Hence the image $(\phi(\mathscr{CU}_{s_1}), \mathcal{R}_2)$ is a SCBI of $\mathcal{R}_2$.

Theorem 3.4. *Let $\phi : \mathcal{R}_1 \longrightarrow \mathcal{R}_2$ be a gamma homomorphism and $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$*

be a SCBI of $\mathcal{R}_2$. Then the pre-image $(\phi^{-1}(\mathscr{CU}_{s_2}), \mathcal{R}_1)$ is also a SCBI of $\mathcal{R}_1$.

Proof. Let $u, v, w \in \mathcal{R}_1$ and $\alpha, \beta \in \Gamma$. Since ϕ is a gamma homomorphism and $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$ is a SCBI of $\mathcal{R}_2$, we have

(i) $\phi^{-1}(\mathscr{A}_{s_2})(u-v) = \mathscr{A}_{s_2}(\phi(u-v))$
$= \mathscr{A}_{s_2}(\phi(u) - \phi(v))$
$\geq min^i\{\mathscr{A}_{s_2}(\phi(u)), \mathscr{A}_{s_2}(\phi(v))\}$
$= min^i\{\phi^{-1}(\mathscr{A}_{s_2})(u), \phi^{-1}(\mathscr{A}_{s_2})(v)\}$,

$\phi^{-1}(\mu_2)(u-v) = \mu_2(\phi(u-v))$
$= \mu_2(\phi(u) - \phi(v))$
$\leq max\{\mu_2(\phi(u)), \mu_2(\phi(v))\}$
$= max\{\phi^{-1}(\mu_2)(u), \phi^{-1}(\mu_2)(v)\}$,

(ii) $\phi^{-1}(\mathscr{B}_{s_2})(u-v) = \mathscr{B}_{s_2}(\phi(u-v))$
$= \mathscr{B}_{s_2}(\phi(u) - \phi(v))$
$\geq min^i\{\mathscr{B}_{s_2}(\phi(u)), \mathscr{B}_{s_2}(\phi(v))\}$
$= min^i\{\phi^{-1}(\mathscr{B}_{s_2})(u), \phi^{-1}(\mathscr{B}_{s_2})(v)\}$,

$\phi^{-1}(\nu_2)(u-v) = \nu_2(\phi(u-v))$
$= \nu_2(\phi(u) - \phi(v))$
$\leq max\{\nu_2(\phi(u)), \nu_2(\phi(v))\}$
$= max\{\phi^{-1}(\nu_2)(u), \phi^{-1}(\nu_2)(v)\}$,

(iii) $\phi^{-1}(\mathscr{C}_{s_2})(u-v) = \mathscr{C}_{s_2}(\phi(u-v))$
$= \mathscr{C}_{s_2}(\phi(u) - \phi(v))$
$\leq max^i\{\mathscr{C}_{s_2}(\phi(u)), \mathscr{C}_{s_2}(\phi(v))\}$
$= max^i\{\phi^{-1}(\mathscr{C}_{s_2})(u), \phi^{-1}(\mathscr{C}_{s_2})(v)\}$,

$\phi^{-1}(\xi_2)(u-v) = \xi_2(\phi(u-v))$
$= \xi_2(\phi(u) - \phi(v))$
$\geq min\{\xi_2(\phi(u)), \xi_2(\phi(v))\}$
$= min\{\phi^{-1}(\xi_2)(u), \phi^{-1}(\xi_2)(v)\}$,

(iv) $\phi^{-1}(\mathscr{A}_{s_2})(u\alpha v\beta w) = \mathscr{A}_{s_2}(\phi(u\alpha v\beta w))$
$= \mathscr{A}_{s_2}(\phi(u)\alpha\phi(v)\beta\phi(w)$
$\geq min^i\{\mathscr{A}_{s_2}(\phi(u)), \mathscr{A}_{s_2}(\phi(w))\}$
$= min^i\{\phi^{-1}(\mathscr{A}_{s_2})(u), \phi^{-1}(\mathscr{A}_{s_2})(w)\}$,

$\phi^{-1}(\mu_2)(u\alpha v\beta w) = \mu_2(\phi(u\alpha v\beta w))$
$= \mu_2(\phi(u)\alpha\phi(v)\beta\phi(w)$
$\leq max\{\mu_2(\phi(u)), \mu_2(\phi(w))\}$
$= max\{\phi^{-1}(\mu_2)(u), \phi^{-1}(\mu_2)(w)\}$,

(v) $\phi^{-1}(\mathscr{B}_{s_2})(u\alpha v\beta w) = \mathscr{B}_{s_2}(\phi(u\alpha v\beta w))$
$= \mathscr{B}_{s_2}(\phi(u)\alpha\phi(v)\beta\phi(w)$
$\geq min^i\{\mathscr{B}_{s_2}(\phi(u)), \mathscr{B}_{s_2}(\phi(w))\}$

$$= min^i\{\phi^{-1}(\mathscr{B}_{s_2})(u), \phi^{-1}(\mathscr{B}_{s_2})(w)\},$$
$$\phi^{-1}(\nu_2)(u\alpha v\beta w) = \nu_2(\phi(u\alpha v\beta w))$$
$$= \nu_2(\phi(u)\alpha\phi(v)\beta\phi(w)$$
$$\leq max\{\nu_2(\phi(u)), \nu_2(\phi(w))\}$$
$$= max\{\phi^{-1}(\nu_2)(u), \phi^{-1}(\nu_2)(w)\},$$
(vi) $\phi^{-1}(\mathscr{C}_{s_2})(u\alpha v\beta w) = \mathscr{C}_{s_2}(\phi(u\alpha v\beta w))$
$$= \mathscr{C}_{s_2}(\phi(u)\alpha\phi(v)\beta\phi(w)$$
$$\leq max^i\{\mathscr{C}_{s_2}(\phi(u)), \mathscr{C}_{s_2}(\phi(w))\}$$
$$= max^i\{\phi^{-1}(\mathscr{C}_{s_2})(u), \phi^{-1}(\mathscr{C}_{s_2})(w)\},$$
$$\phi^{-1}(\xi_2)(u\alpha v\beta w) = \xi_2(\phi(u\alpha v\beta w))$$
$$= \xi_2(\phi(u)\alpha\phi(v)\beta\phi(w)$$
$$\geq min\{\xi_2(\phi(u)), \xi_2(\phi(w))\}$$
$$= min\{\phi^{-1}(\xi_2)(u), \phi^{-1}(\xi_2)(w)\}.$$
Hence the pre-image $(\phi^{-1}(\mathscr{CU}_{s_2}), \mathcal{R}_1)$ is a SCBI of $\mathcal{R}_1$.

Theorem 3.5. *Let $\phi : \mathcal{R}_1 \longrightarrow \mathcal{R}_2$ be an epimorphism of gamma near-rings $\mathcal{R}_1$ and $\mathcal{R}_2$. If $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$ is a SCS of $\mathcal{R}_2$ such that $(\phi^{-1}(\mathscr{CU}_{s_2}), \mathcal{R}_1)$ is a SCBI of $\mathcal{R}_1$, then $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$ is a SCBI of $\mathcal{R}_2$.*

Proof. Let $u, v, w \in \mathcal{R}_2$ and $\alpha, \beta \in \Gamma$, and take $\phi(x) = u, \phi(y) = v$, $\phi(z) = w$, for some $x, y, z \in \mathcal{R}_1$, we have

(i) $\mathscr{A}_{s_2}(u - v) = \mathscr{A}_{s_2}(\phi(x) - \phi(y)) = \mathscr{A}_{s_2}(\phi(x - y))$
$$= \phi^{-1}(\mathscr{A}_{s_2})(x - y)$$
$$\geq min^i\{\phi^{-1}(\mathscr{A}_{s_2})(x), \phi^{-1}(\mathscr{A}_{s_2})(y)\}$$
$$= min^i\{\mathscr{A}_{s_2}(\phi(x)), \mathscr{A}_{s_2}(\phi(y))\}$$
$$= min^i\{\mathscr{A}_{s_2}(u), \mathscr{A}_{s_2}(v)\},$$
$$\mu_2(u - v) = \mu_2(\phi(x) - \phi(y)) = \mu_2(\phi(x - y))$$
$$= \phi^{-1}(\mu_2)(x - y)$$
$$\leq max\{\phi^{-1}(\mu_2)(x), \phi^{-1}(\mu_2)(y)\}$$
$$= max\{\mu_2(\phi(x)), \mu_2(\phi(y))\}$$
$$= max\{\mu_2(u), \mu_2(v)\},$$
(ii) $\mathscr{B}_{s_2}(u - v) = \mathscr{B}_{s_2}(\phi(x) - \phi(y)) = \mathscr{B}_{s_2}(\phi(x - y))$
$$= \phi^{-1}(\mathscr{B}_{s_2})(x - y)$$
$$\geq min^i\{\phi^{-1}(\mathscr{B}_{s_2})(x), \phi^{-1}(\mathscr{B}_{s_2})(y)\}$$
$$= min^i\{\mathscr{B}_{s_2}(\phi(x)), \mathscr{B}_{s_2}(\phi(y))\}$$
$$= min^i\{\mathscr{B}_{s_2}(u), \mathscr{B}_{s_2}(v)\},$$
$$\nu_2(u - v) = \nu_2(\phi(x) - \phi(y)) = \nu_2(\phi(x - y))$$
$$= \phi^{-1}(\nu_2)(x - y)$$
$$\leq max\{\phi^{-1}(\nu_2)(x), \phi^{-1}(\nu_2)(y)\}$$
$$= max\{\nu_2(\phi(x)), \nu_2(\phi(y))\}$$

$$= max\{\nu_2(u), \nu_2(v)\},$$

(iii) $$\mathscr{C}_{s_2}(u - v) = \mathscr{C}_{s_2}(\phi(x) - \phi(y)) = \mathscr{C}_{s_2}(\phi(x - y))$$
$$= \phi^{-1}(\mathscr{C}_{s_2})(x - y)$$
$$\leq max^i\{\phi^{-1}(\mathscr{C}_{s_2})(x), \phi^{-1}(\mathscr{C}_{s_2})(y)\}$$
$$= max^i\{\mathscr{C}_{s_2}(\phi(x)), \mathscr{C}_{s_2}(\phi(y))\}$$
$$= max^i\{\mathscr{C}_{s_2}(u), \mathscr{C}_{s_2}(v)\},$$
$$\xi_2(u - v) = \xi_2(\phi(x) - \phi(y)) = \xi_2(\phi(x - y))$$
$$= \phi^{-1}(\xi_2)(x - y)$$
$$\geq min\{\phi^{-1}(\xi_2)(x), \phi^{-1}(\xi_2)(y)\}$$
$$= min\{\xi_2(\phi(x)), \xi_2(\phi(y))\}$$
$$= min\{\xi_2(u), \xi_2(v)\},$$

(iv) $$\mathscr{A}_{s_2}(u\alpha v\beta w) = \mathscr{A}_{s_2}(\phi(x)\alpha\phi(y)\beta\phi(z)) = \mathscr{A}_{s_2}(\phi(x\alpha y\beta z))$$
$$= \phi^{-1}(\mathscr{A}_{s_2})(x\alpha y\beta z)$$
$$\geq min^i\{\phi^{-1}(\mathscr{A}_{s_2})(x), \phi^{-1}(\mathscr{A}_{s_2})(z)\}$$
$$= min^i\{\mathscr{A}_{s_2}(\phi(x)), \mathscr{A}_{s_2}(\phi(z))\}$$
$$= min^i\{\mathscr{A}_{s_2}(u), \mathscr{A}_{s_2}(w)\},$$
$$\mu_2(u\alpha v\beta w) = \mu_2(\phi(x)\alpha\phi(y)\beta\phi(z))$$
$$= \mu_2(\phi(x\alpha y\beta z))$$
$$= \phi^{-1}(\mu_2)(x\alpha y\beta z)$$
$$\leq max\{\phi^{-1}(\mu_2)(x), \phi^{-1}(\mu_2)(z)\}$$
$$= max\{\mu_2(\phi(x)), \mu_2(\phi(z))\}$$
$$= max\{\mu_2(u), \mu_2(w)\},$$

(v) $$\mathscr{B}_{s_2}(u\alpha v\beta w) = \mathscr{B}_{s_2}(\phi(x)\alpha\phi(y)\beta\phi(z))$$
$$= \mathscr{B}_{s_2}(\phi(x\alpha y\beta z))$$
$$= \phi^{-1}(\mathscr{B}_{s_2})(x\alpha y\beta z)$$
$$\geq min^i\{\phi^{-1}(\mathscr{B}_{s_2})(x), \phi^{-1}(\mathscr{B}_{s_2})(z)\}$$
$$= min^i\{\mathscr{B}_{s_2}(\phi(x)), \mathscr{B}_{s_2}(\phi(z))\}$$
$$= min^i\{\mathscr{B}_{s_2}(u), \mathscr{B}_{s_2}(w)\},$$
$$\nu_2(u\alpha v\beta w) = \nu_2(\phi(x)\alpha\phi(y)\beta\phi(z))$$
$$= \nu_2(\phi(x\alpha y\beta z))$$
$$= \phi^{-1}(\nu_2)(x\alpha y\beta z)$$
$$\leq max\{\phi^{-1}(\nu_2)(x), \phi^{-1}(\nu_2)(z)\}$$
$$= max\{\nu_2(\phi(x)), \nu_2(\phi(z))\}$$
$$= max\{\nu_2(u), \nu_2(w)\},$$

(vi) $$\mathscr{C}_{s_2}(u\alpha v\beta w) = \mathscr{C}_{s_2}(\phi(x)\alpha\phi(y)\beta\phi(z))$$
$$= \mathscr{C}_{s_2}(\phi(x\alpha y\beta z))$$
$$= \phi^{-1}(\mathscr{C}_{s_2})(x\alpha y\beta z)$$
$$\leq max^i\{\phi^{-1}(\mathscr{C}_{s_2})(x), \phi^{-1}(\mathscr{C}_{s_2})(z)\}$$

$$= max^i\{\mathscr{C}_{s_2}(\phi(x)), \mathscr{C}_{s_2}(\phi(z))\}$$
$$= max^i\{\mathscr{C}_{s_2}(u), \mathscr{C}_{s_2}(w)\},$$
$$\xi_2(u\alpha v\beta w) = \xi_2(\phi(x)\alpha\phi(y)\beta\phi(z))$$
$$= \xi_2(\phi(x\alpha y\beta z))$$
$$= \phi^{-1}(\xi_2)(x\alpha y\beta z)$$
$$\geq min\{\phi^{-1}(\xi_2)(x), \phi^{-1}(\xi_2)(z)\}$$
$$= min\{\xi_2(\phi(x)), \xi_2(\phi(z))\} = min\{\xi_2(u), \xi_2(w)\}.$$

Hence $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$ is a SCBI of $\mathcal{R}_2$.

4. Anti-Homomorphism of Spherical Cubic Bi-ideals of Gamma Near-rings

In this section, we study the properties of SCBI of $\mathcal{R}$ using anti-homomorphism.

Theorem 4.1. *Let $\phi : \mathcal{R}_1 \longrightarrow \mathcal{R}_2$ be a gamma anti-homomorphism and $(\mathscr{CU}_{s_1}, \mathcal{R}_1)$ be a SCBI of $\mathcal{R}_1$, then the image $(\phi(\mathscr{CU}_{s_1}), \mathcal{R}_2)$ is also a SCBI of $\mathcal{R}_2$.*

Proof. Since ϕ is a gamma anti-homomorphism and $\mathscr{CU}_{s_1}$ is a SCBI of $\mathcal{R}_1$. Then we can easily seen that the image $(\phi(\mathscr{CU}_{s_1}), \mathcal{R}_2)$ is a SCBI of $\mathcal{R}_2$.

Theorem 4.2. *Let $\phi : \mathcal{R}_1 \longrightarrow \mathcal{R}_2$ be a gamma anti-homomorphism and $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$ be a SCBI of $\mathcal{R}_2$, then the pre-image $(\phi^{-1}(\mathscr{CU}_{s_2}), \mathcal{R}_1)$ is also a SCBI of $\mathcal{R}_1$.*

Proof. Since ϕ is a gamma anti-homomorphism and $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$ is a SCBI of $\mathcal{R}_2$. Then we can easily seen that the pre-image $(\phi^{-1}(\mathscr{CU}_{s_2}), \mathcal{R}_1)$ is a SCBI of $\mathcal{R}_1$.

Theorem 4.3. *Let $\phi : \mathcal{R}_1 \longrightarrow \mathcal{R}_2$ be an onto anti-homomorphism of gamma near-rings $\mathcal{R}_1$ and $\mathcal{R}_2$. If $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$ is a SCS of $\mathcal{R}_2$ such that $(\phi^{-1}(\mathscr{CU}_{s_2}), \mathcal{R}_1)$ is a SCBI of $\mathcal{R}_1$, then $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$ is a SCBI of $\mathcal{R}_2$.*

Proof. Let $u, v, w \in \mathcal{R}_2$ and take $\phi(x) = u, \phi(y) = v$ and $\phi(z) = w$, for some $x, y, z \in \mathcal{R}_1$. Then we can easily seen that $(\mathscr{CU}_{s_2}, \mathcal{R}_2)$ is a SCBI of $\mathcal{R}_2$.

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