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SOME OPERATIONS ON BIPOLAR INTUITIONISTIC FUZZY IDEAL AND BIPOLAR INTUITIONISTIC ANTI FUZZY IDEAL OF A BP-ALGEBRA

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Abstract: The concept of a bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are a new algebraic structure of BP-algebra and to use necessity and possibility operator. The purpose of this study is to implement the fuzzy set theory and ideal theory of a BP-algebra. The relation between the operation on bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are established.

Keywords and Phrases: BP-algebra, fuzzy ideal, bipolar fuzzy ideal, bipolar intuitionistic fuzzy ideal, bipolar intuitionistic anti fuzzy ideal, necessity and possibility operator.

2020 Mathematics Subject Classification: 08A72.

1. Introduction

The concept of fuzzy sets was initiated by L. A. Zadeh [17] then it has become a vigorous area of research in engineering, medical science, graph theory. S. S. Ahn [2] gave the idea of BP-algebra. Bipolar valued fuzzy sets was commenced by K. J. Lee [7] are an extension of fuzzy sets whose positive membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the positive membership degree (0, 1] indicates that elements somewhat satisfy the property and the negative membership degree [-1, 0) indicates that elements somewhat satisfy the implicit counter property. The author W. R. Zhang [18] introduced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1998. K. Chakrabarthy and Biswas R. Nanda [3] investigated note on union and intersection of intuitionistic fuzzy sets. A. Rajeshkumar [16] was analyzed fuzzy groups and level subgroups. M. Palanivelrajan and S. Nandakumar [15] introduced the definition and some operations of intuitionistic fuzzy primary and semiprimary ideal. K. Gunasekaran, S. Nandakumar and S. Sivakaminathan [4] introduced the definition of bipolar intuitionistic fuzzy ideal of a BP-algebra.

2. Preliminaries

Definition 2.1. *Let A and B be any two bipolar intuitionistic fuzzy set $A = (\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N)$ and $B = (\mu_B^P, \mu_B^N, \nu_B^P, \nu_B^N)$ in X, we define*
(i) $A \cap B = \{(x, \min(\mu_A^P(x), \mu_B^P(x)), \max(\mu_A^N(x), \mu_B^N(x)), \max(\nu_A^P(x), \nu_B^P(x)),$ $min(\nu_A^N(x), \nu_B^N(x)))/x \in X\}$
(ii) $A \cup B = \{(x, \max(\mu_A^P(x), \mu_B^P(x)), \min(\mu_A^N(x), \mu_B^N(x)), \min(\nu_A^P(x), \nu_B^P(x)),$ $max(\nu_A^N(x), \nu_B^N(x)))/x \in X\}$
(iii) $\bar{A} = \{(x, \nu_A^P(x), \nu_A^N(x), \mu_A^P(x), \mu_A^N(x))/x \in X\}$.

Definition 2.2. *A bipolar intuitionistic fuzzy set $A = \{\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N/x \in X\}$ of BP-algebra X is called a bipolar intuitionistic fuzzy ideal of X if it satisfies the following conditions:*
(i) $\mu_A^P(0) \geq \mu_A^P(x)$ *and* $\mu_A^N(0) \leq \mu_A^N(x)$
(ii) $\mu_A^P(x) \geq \min\{\mu_A^P(x * y), \mu_A^P(y)\}$
(iii) $\mu_A^N(x) \leq \max\{\mu_A^N(x * y), \mu_A^N(y)\}$ *(iv)* $\nu_A^P(0) \leq \nu_A^P(x)$ *and* $\nu_A^N(0) \geq \nu_A^N(x)$ *(v)* $\nu_A^P(x) \leq \max\{\nu_A^P(x * y), \nu_A^P(y)\}$ *(vi)* $\nu_A^N(x) \geq \min\{\nu_A^N(x * y), \nu_A^N(y)\}$, *for all* $x, y \in X$.

Definition 2.3. *A bipolar intuitionistic fuzzy set $A = \{\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N/x \in X\}$ of BP-algebra X is called a bipolar intuitionistic anti fuzzy ideal of X if it satisfies the following conditions:*

(i) $\mu_A^P(0) \leq \mu_A^P(x)$ *and* $\mu_A^N(0) \geq \mu_A^N(x)$
(ii) $\mu_A^P(x) \leq \max\{\mu_A^P(x*y), \mu_A^P(y)\}$
(iii) $\mu_A^N(x) \geq \min\{\mu_A^N(x*y), \mu_A^N(y)\}$
(iv) $\nu_A^P(0) \geq \nu_A^P(x)$ *and* $\nu_A^N(0) \leq \nu_A^N(x)$
(v) $\nu_A^P(x) \geq \min\{\nu_A^P(x*y), \nu_A^P(y)\}$
(vi) $\nu_A^N(x) \leq \max\{\nu_A^N(x*y), \nu_A^N(y)\}$, *for all* $x, y \in X$.

Definition 2.4. *Let A is a bipolar intuitionistic fuzzy set of X, then the necessity operator $\Box$ is defined by* $\Box A = \{(x, \mu_A^P(x), \mu_A^N(x), 1-\mu_A^P(x), -1-\mu_A^N(x))/x \in X\}$.

Definition 2.5. *Let A is a bipolar intuitionistic fuzzy set of X, then the possibility operator $\Diamond$ is defined by* $\Diamond A = \{(x, 1-\nu_A^P(x), -1-\nu_A^N(x), \nu_A^P(x), \nu_A^N(x))/x \in X\}$.

3. Operations on Bipolar Intuitionistic Fuzzy Ideal

Theorem 3.1. *If A is a bipolar intuitionistic fuzzy ideal of X, then $\Box A$ is a bipolar intuitionistic fuzzy ideal of X.*

Proof. Given A is a bipolar intuitionistic fuzzy ideal of X. Consider 0, $x, y \in A$.

(i) Now $\mu_{\Box A}^P(0) = \mu_A^P(0) \geq \mu_A^P(x) = \mu_{\Box A}^P(x)$. Therefore $\mu_{\Box A}^P(0) \geq \mu_{\Box A}^P(x)$.
Now $\mu_{\Box A}^N(0) = \mu_A^N(0) \leq \mu_A^N(x) = \mu_{\Box A}^N(x)$. Therefore $\mu_{\Box A}^N(0) \leq \mu_{\Box A}^N(x)$.

(ii) Now $\mu_{\Box A}^P(x) = \mu_A^P(x) \geq \min\{\mu_A^P(x*y), \mu_A^P(y)\} = \min\{\mu_{\Box A}^P(x*y), \mu_{\Box A}^P(y)\}$.
Therefore $\mu_{\Box A}^P(x) \geq \min\{\mu_{\Box A}^P(x*y), \mu_{\Box A}^P(y)\}$.

(iii) Now $\mu_{\Box A}^N(x) = \mu_A^N(x) \leq \max\{\mu_A^N(x*y), \mu_A^N(y)\} = \max\{\mu_{\Box A}^N(x*y), \mu_{\Box A}^N(y)\}$.
Therefore $\mu_{\Box A}^N(x) \leq \max\{\mu_{\Box A}^N(x*y), \mu_A^N(y)\}$.

(iv) Now $\nu_{\Box A}^P(0) = 1-\mu_A^P(0) \leq 1-\mu_A^P(x) = \nu_{\Box A}^P(x)$. Therefore $\nu_{\Box A}^P(0) \leq \nu_{\Box A}^P(x)$.
Now $\nu_{\Box A}^N(0) = -1-\mu_A^N(0) \geq -1-\mu_A^N(x) = \nu_{\Box A}^N(x)$.
Therefore $\nu_{\Box A}^N(0) \geq \nu_{\Box A}^N(x)$.

(v) Now $\nu_{\Box A}^P(x) = 1-\mu_A^P(x) \leq \max\{1-\mu_A^P(x*y), 1-\mu_A^P(y)\}$
$= \max\{\nu_{\Box A}^P(x*y), \nu_{\Box A}^P(y)\}$.
Therefore $\nu_{\Box A}^P(x) \leq \max\{\nu_{\Box A}^P(x*y), \nu_{\Box A}^P(y)\}$.

(vi) Now $\nu_{\Box A}^N(x) = -1-\mu_A^N(x) \geq \min\{-1-\mu_A^N(x*y), -1-\mu_A^N(y)\}$
$= \min\{\nu_{\Box A}^N(x*y), \nu_{\Box A}^N(y)\}$.
Therefore $\nu_{\Box A}^N(x) \geq \min\{\nu_{\Box A}^N(x*y), \nu_{\Box A}^N(y)\}$.

Therefore $\Box A$ is a bipolar intuitionistic fuzzy ideal of X.

Theorem 3.2. *If A and B are bipolar intuitionistic fuzzy ideal of X, then $\Box(A \cap B) = \Box A \cap \Box B$ is also a bipolar intuitionistic fuzzy ideal of X.*

Proof. Let A and B are bipolar intuitionistic fuzzy ideal of X. Consider $0, x, y \in A \cap B$ then $0, x, y \in A$ and $0, x, y \in B$.

(i) Now $\mu^P_{\square(A\cap B)}(0) = \mu^P_{(A\cap B)}(0) = \min\{\mu^P_A(0), \mu^P_B(0)\}$
$\geq \min\{\mu^P_A(x), \mu^P_B(x)\} = \min\{\mu^P_{\square A}(x), \mu^P_{\square B}(x)\}$
$= \mu^P_{\square A\cap\square B}(x)$.
Therefore $\mu^P_{\square(A\cap B)}(0) \geq \mu^P_{\square A\cap\square B}(x)$
Now $\mu^N_{\square(A\cap B)}(0) = \mu^N_{A\cap B}(0) = \max\{\mu^N_A(0), \mu^N_B(0)\}$
$\leq \max\{\mu^N_A(x), \mu^N_B(x)\} = \max\{\mu^N_{\square A}(x), \mu^N_{\square B}(x)\}$
$= \mu^N_{\square A\cap\square B}(x)$.
Therefore $\mu^N_{\square(A\cap B)}(0) \leq \mu(\square A \cap \square B)^N(x)$.

(ii) Now $\mu^P_{\square(A\cap B)}(x) = \mu^P_{(A\cap B)}(x) = \min\{\mu^P_A(x), \mu^P_B(x)\}$
$\geq \min\{\min\{\mu^P_A(x*y), \mu^P_A(y)\}, \min\{\min^P_B(x*y), \mu^P_B(y)\}\}$
$= \min\{\min\{\mu^P_A(x*y), \mu^P_B(x*y)\}, \min\{\mu^P_A(y), \mu^P_B(y)\}\}$
$= \min\{\min\{\mu^P_{\square A}(x*y), \mu^P_{\square B}(x*y)\}, \min\{\mu^P_{\square A}(y), \mu^P_{\square B}(y)\}\}$
$= \min\{\mu^P_{\square A\cap\square B}(x*y), \mu^P_{\square A\cap\square B}(y)\}$.
Therefore $\mu^P_{\square(A\cap B)}(x) \geq \min\{\mu^P_{\square A\cap\square B}(x*y), \mu^P_{\square A\cap\square B}(y)\}$.

(iii) Now $\mu^N_{\square(A\cap B)}(x) = \mu^N_{A\cap B}(x) = \max\{\mu^N_A(x), \mu^N_B(x)\}$
$\leq \max\{\max\{\mu^N_A(x*y), \mu^N_A(y)\}, \max\{\mu^N_B(x*y), \mu^N_B(y)\}\}$
$= \max\{\max\{\mu^N_A(x*y), \mu^N_B(x*y)\}, \max\{\mu^N_A(y), \mu^N_B(y)\}\}$
$= \max\{\max\{\mu^N_{\square A}(x*y), \mu^N_{\square B}(x*y)\}, \max\{\mu^N_{\square A}(y), \mu^N_{\square B}(y)\}\}$
$= \max\{\mu^N_{\square A\cap\square B}(x*y), \mu^N_{\square A\cap\square B}(y)\}$.
Therefore $\mu^N_{\square(A\cap B)}(x) \leq \max\{\mu^N_{\square A\cap\square B}(x*y), \mu^N_{\square A\cap\square B}(y)\}$.

(iv) Now $\nu^P_{\square(A\cap B)}(0) = 1 - \mu^P_{A\cap B}(0) = 1 - \mu^P_{\square(A\cap B)}(0)$
$\leq 1 - \mu^P_{\square A\cap\square B}(x) = \nu^P_{\square A\cap\square B}(x)$.
Therefore $\nu^P_{\square(A\cap B)}(0) \leq \nu^P_{\square A\cap\square B}(x)$.
Now $\nu^N_{\square(A\cap B)}(0) = -1 - \mu^N_{A\cap B}(0) = -1 - \mu^N_{\square(A\cap B)}(0)$
$\geq -1 - \mu^N_{\square A\cap\square B}(x) = \nu^N_{\square A\cap\square B}(x)$.
Therefore $\nu^N_{\square(A\cap B)}(0) \geq \nu^N_{\square A\cap\square B}(x)$.

(v) Now $\nu^P_{\square(A\cap B)}(x) = 1 - \mu^P_{A\cap B}(x) = 1 - \mu^P_{\square(A\cap B)}(x)$
$\leq \max\{1 - \mu^P_{\square A\cap\square B}(x*y), 1 - \mu^P_{\square A\cap\square B}(y)\}$
$= \max\{\nu^P_{\square A\cap\square B}(x*y), \nu^P_{\square A\cap\square B}(y)\}$.
Therefore $\nu^P_{\square(A\cap B)}(x) \leq \max\{\nu^P_{\square A\cap\square B}(x*y), \nu^P_{\square A\cap\square B}(y)\}$.

(vi) Now $\nu^N_{\Box(A\cap B)}(x) = -1 - \mu^N_{A\cap B}(x) = -1 - \mu^N_{\Box(A\cap B)}(x)$
$\geq \min\{-1 - \mu^N_{\Box A\cap \Box B}(x * y), -1 - \mu^N_{\Box A\cap \Box B}(y)\}$
$= \min\{\nu^N_{\Box A\cap \Box B}(x * y), \nu^N_{\Box A\cap \Box B}(y)\}.$
Therefore $\nu^N_{\Box(A\cap B)}(x) \geq \min\{\nu^N_{\Box A\cap \Box B}(x * y), \nu^N_{\Box A\cap \Box B}(y)\}.$

Therefore $\Box(A \cap B) = \Box A \cap \Box B$ is a bipolar intuitionistic fuzzy ideal of X.

Theorem 3.3. *If A is a bipolar intuitionistic fuzzy ideal of X, then $\Diamond A$ is a bipolar intuitionistic fuzzy ideal of X.*
Proof. Given A is a bipolar intuitionistic fuzzy ideal of X. Consider $0, x, y \in A$.

(i) Now $\mu^P_{\Diamond A}(0) = 1 - \nu^P_A(0) \geq 1 - \nu^P_A(x) = \mu^P_{\Diamond A}(x)$.
Therefore $\mu^P_{\Diamond A}(0) \geq \mu^P_{\Diamond A}(x)$.
Now $\mu^N_{\Diamond A}(0) = -1 - \nu^N_A(0) \leq -1 - \nu^N_A(x) = \mu^N_{\Diamond A}(x)$.
Therefore $\mu^N_{\Diamond A}(0) \leq \mu^N_{\Diamond A}(x)$.

(ii) Now $\mu^P_{\Diamond A}(x) = 1 - \nu^P_A(x) \geq \min\{1 - \nu^P_A(x * y), 1 - \nu^P_A(y)\}$.
$= \min\{\mu^P_{\Diamond A}(x * y), \mu^P_{\Diamond A}(y)\}$.
Therefore $\mu^P_{\Diamond A}(x) \geq \min\{\mu^P_{\Diamond A}(x * y), \mu^P_{\Diamond A}(y)\}$.

(iii) Now $\mu^N_{\Diamond A}(x) = -1 - \nu^N_A(x) \leq \max\{-1 - \nu^N_A(x * y), -1 - \nu^N_A(y)\}$.
$= \max\{\mu^N_{\Diamond A}(x * y), \mu^N_{\Diamond A}(y)\}$.
Therefore $\mu^N_{\Diamond A}(x) \leq \max\{\mu^N_{\Diamond A}(x * y), \mu^N_{\Diamond A}(y)\}$.

(iv) Now $\nu^P_{\Diamond A}(0) = \nu^P_A(0) \leq \nu^P_A(x) = \nu^P_{\Diamond A}(x)$.
Therefore $\nu^P_{\Diamond A}(0) \leq \nu^P_{\Diamond A}(x)$.
Now $\nu^N_{\Diamond A}(0) = \nu^N_A(0) \geq \nu^N_A(x) = \nu^N_{\Diamond A}(x)$.
Therefore $\nu^N_{\Diamond A}(0) \geq \nu^N_{\Diamond A}(x)$.

(v) Now $\nu^P_{\Diamond A}(x) = \nu^P_A(x) \leq \max\{\nu^P_A(x * y), \nu^P_A(y)\}$.
$= \max\{\nu^P_{\Diamond A}(x * y), \nu^P_{\Diamond A}(y)\}$.
Therefore $\nu^P_{\Diamond A}(x) \leq \max\{\nu^P_{\Diamond A}(x * y), \nu^P_{\Diamond A}(y)\}$.

(vi) Now $\nu^N_{\Diamond A}(x) = \nu^N_A(x) \geq \min\{\nu^N_A(x * y), \nu^N_A(y)\}$.
$= \min\{\nu^N_{\Diamond A}(x * y), \nu^N_{\Diamond A}(y)\}$.
Therefore $\nu^N_{\Diamond A}(x) \geq \min\{\nu^N_{\Diamond A}(x * y), \nu^N_{\Diamond A}(y)\}$.

Therefore $\Diamond A$ is a bipolar intuitionistic fuzzy ideal of X.

Theorem 3.4. *If A and B are bipolar intuitionistic fuzzy ideal of X, then $\Diamond(A \cap B) = \Diamond A \cap \Diamond B$ is also a bipolar intuitionistic fuzzy ideal of X.*
Proof. Let A and B are bipolar intuitionistic fuzzy ideal of X. Consider $0, x, y \in A \cap B$ then $0, x, y \in A$ and $0, x, y \in B$.

(i) Now $\mu^P_{\Diamond(A\cap B)}(0) = 1-\nu^P_{A\cap B}(0) = 1-\max\{\nu^P_A(0),\nu^P_B(0)\}$
$\geq 1-\max\{\nu^P_A(x),\nu^P_B(x)\} = 1-\max\{\nu^P_{\Diamond A}(x),\nu^P_{\Diamond B}(x)\}$
$= 1-\nu^P_{\Diamond A\cap\Diamond B}(x) = \mu^P_{\Diamond A\cap\Diamond B}(x)$.

Therefore $\mu^P_{\Diamond(A\cap B)}(0) \geq \mu^P_{\Diamond A\cap\Diamond B}(x)$.

Now $\mu^N_{\Diamond(A\cap B)}(0) = -1-\nu^N_{A\cap B}(0) = -1-\min\{\nu^N_A(0),\nu^N_B(0)\}$
$\leq -1-\min\{\nu^N_A(x),\nu^N_B(x)\} = -1-\min\{\nu^N_{\Diamond A}(x),\nu^N_{\Diamond B}(x)\}$
$= -1-\nu^N_{\Diamond A\cap\Diamond B}(x) = \mu^N_{\Diamond A\cap\Diamond B}(x)$.

Therefore $\mu^N_{\Diamond(A\cap B)}(0) \leq \mu^N_{\Diamond A\cap\Diamond B}(x)$.

(ii) Now $\mu^P_{\Diamond(A\cap B)}(x) = 1-\nu^P_{A\cap B}(x) = 1-\max\{\nu^P_A(x),\nu^P_B(x)\}$
$\geq 1-\max\{\min\{\nu^P_A(x*y),\nu^P_A(y)\},\min\{\nu^P_B(x*y),\nu^P_B(y)\}\}$
$= 1-\min\{\max\{\nu^P_{\Diamond A}(x*y),\nu^P_{\Diamond B}(x*y)\},\max\{\nu^P_{\Diamond A}(y),\nu^P_{\Diamond B}(y)\}$
$= 1-\min\{\nu^P_{\Diamond A\cap\Diamond B}(x*y),\nu^P_{\Diamond A\cap\Diamond B}(y)\}$
$= \min\{1-\nu^P_{\Diamond A\cap\Diamond B}(x*y),1-\nu^P_{\Diamond A\cap\Diamond B}(y)\}$
$= \min\{\mu^P_{\Diamond A\cap\Diamond B}(x*y),\mu^P_{\Diamond A\cap\Diamond B}(y)\}$

Therefore $\mu^P_{\Diamond(A\cap B)}(x) \geq \min\{\mu^P_{\Diamond A\cap\Diamond B}(x*y),\mu^P_{\Diamond A\cap\Diamond B}(y)\}$.

(iii) Now $\mu^N_{\Diamond(A\cap B)}(x) = -1-\nu^N_{A\cap B}(x) = -1-\min\{\nu^N_A(x),\nu^N_B(x)\}$
$\leq -1-\min\{\max\{\nu^N_A(x*y),\nu^N_A(y)\},\max\{\nu^N_B(x*y),\nu^N_B(y)\}\}$
$= -1-\max\{\min\{\nu^N_{\Diamond A}(x*y),\nu^N_{\Diamond B}(x*y)\},\min\{\nu^N_{\Diamond A}(y),\nu^N_{\Diamond B}(y)\}$
$= -1-\max\{\nu^N_{\Diamond A\cap\Diamond B}(x*y),\nu^N_{\Diamond A\cap\Diamond B}(y)\}$
$= \max\{-1-\nu^N_{\Diamond A\cap\Diamond B}(x*y),-1-\nu^N_{\Diamond A\cap\Diamond B}(y)\}$
$= \max\{\mu^N_{\Diamond A\cap\Diamond B}(x*y),\mu^N_{\Diamond A\cap\Diamond B}(y)\}$

Therefore $\mu^N_{\Diamond(A\cap B)}(x) \leq \min\{\mu^N_{\Diamond A\cap\Diamond B}(x*y),\mu^N_{\Diamond A\cap\Diamond B}(y)\}$.

(iv) Now $\nu^P_{\Diamond(A\cap B)}(0) = \nu^P_{\Diamond A\cap B}(0) = \max\{\nu^P_A(0),\nu^P_B(0)\}$
$\leq \max\{\nu^P_A(x),\nu^P_B(x)\} = \max\{\nu^P_{\Diamond A}(x),\nu^P_{\Diamond B}(x)\}$
$= \nu^P_{\Diamond A\cap\Diamond B}(x)$.

Therefore $\nu^P_{\Diamond(A\cap B)}(0) \leq \nu^P_{\Diamond A\cap\Diamond B}(x)$.

Now $\nu^N_{\Diamond(A\cap B)}(0) = \nu^N_{\Diamond A\cap B}(0) = \min\{\nu^N_A(0),\nu^N_B(0)\}$
$\geq \min\{\nu^N_A(x),\nu^N_B(x)\} = \min\{\nu^N_{\Diamond A}(x),\nu^N_{\Diamond B}(x)\}$
$= \nu^N_{\Diamond A\cap\Diamond B}(x)$.

Therefore $\nu^N_{\Diamond(A\cap B)}(0) \geq \nu^N_{\Diamond A\cap\Diamond B}(x)$.

(v) Now $\nu^P_{\Diamond(A\cap B)}(x) = \nu^P_{\Diamond A\cap B}(x) = \max\{\nu^P_A(x),\nu^P_B(x)\}$
$\leq \max\{\max\{\nu^P_A(x*y),\nu^P_A(y)\},\max\{\nu^P_B(x*y),\nu^P_B(y)\}\}$
$= \max\{\max\{\nu^P_{\Diamond A}(x*y),\nu^P_{\Diamond B}(x*y)\},\max\{\nu^P_{\Diamond A}(y),\nu^P_{\Diamond B}(y)\}\}$

$= \max\{\nu^P_{\Diamond A\cap\Diamond B}(x*y), \nu^P_{\Diamond A\cap\Diamond B}(y)\}$

Therefore $\nu^P_{\Diamond(A\cap B)}(x) \leq \max\{\nu^P_{\Diamond A\cap B}(x*y), \nu^P_{\Diamond A\cap B}(y)\}$.

(vi) Now $\nu^N_{\Diamond(A\cap B)}(x) = \nu^N_{\Diamond A\cap B}(x) = \min\{\nu^N_A(x), \nu^N_B(x)\}$
$\geq \min\{\min\{\nu^N_A(x*y), \nu^N_A(y)\}, \min\{\nu^N_B(x*y), \nu^N_B(y)\}\}$
$= \min\{\min\{\nu^N_{\Diamond A}(x*y), \nu^N_{\Diamond B}(x*y)\}, \min\{\nu^N_{\Diamond A}(y), \nu^N_{\Diamond B}(y)\}\}$
$= \min\{\nu^N_{\Diamond A\cap\Diamond B}(x*y), \nu^N_{\Diamond A\cap\Diamond B}(y)\}$

Therefore $\nu^N_{\Diamond(A\cap B)}(x) \geq \min\{\nu^N_{\Diamond A\cap B}(x*y), \nu^N_{\Diamond A\cap B}(y)\}$.

Therefore $\Diamond(A\cap B) = \Diamond A\cap\Diamond B$ is also a bipolar intuitionistic fuzzy ideal of X.

Theorem 3.5. *If A is a bipolar intuitionistic fuzzy ideal of X, then $\Box\Box A = \Box A$ is a bipolar intuitionistic fuzzy ideal of X.*

Proof. Given A is a bipolar intuitionistic fuzzy ideal of X. Consider $0, x, y \in A$.

(i) Now $\mu^P_{\Box\Box A}(0) = \mu^P_{\Box A}(0) = \mu^P_A(0) \geq \mu^P_A(x) = \mu^P_{\Box A}(x)$.

Therefore $\mu^P_{\Box\Box A}(0) \geq \mu^P_{\Box A}(x)$.

Now $\mu^N_{\Box\Box A}(0) = \mu^N_{\Box A}(0) = \mu^N_A(0) \leq \mu^N_A(x) = \mu^N_{\Box A}(x)$.

Therefore $\mu^N_{\Box\Box A}(0) \leq \mu^N_{\Box A}(x)$.

(ii) Now $\mu^P_{\Box\Box A}(x) = \mu^P_{\Box A}(x) = \mu^P_A(x) \geq \min\{\mu^P_A(x*y), \mu^P_A(y)\}$
$= \min\{\mu^P_{\Box A}(x*y), \mu^P_{\Box A}(y)\}$.

Therefore $\mu^P_{\Box\Box A}(x) \geq \min\{\mu^P_{\Box A}(x*y), \mu^P_{\Box A}(y)\}$.

(iii) Now $\mu^N_{\Box\Box A}(x) = \mu^N_{\Box A}(x) = \mu^N_A(x) \leq \max\{\mu^N_A(x*y), \mu^N_A(y)\}$
$= \max\{\mu^N_{\Box A}(x*y), \mu^N_{\Box A}(y)\}$.

Therefore $\mu^N_{\Box\Box A}(x) \leq \max\{\mu^N_{\Box A}(x*y), \mu^N_{\Box A}(y)\}$.

(iv) Now $\nu^P_{\Box\Box A}(0) = 1 - \mu^P_{\Box A}(0) = 1 - \mu^P_A(0)$
$\leq 1 - \mu^P_A(x) = \nu^P_{\Box A}(x)$.

Therefore $\nu^P_{\Box\Box A}(0) \leq \nu^P_{\Box A}(x)$.

Now $\nu^N_{\Box\Box A}(0) = -1 - \mu^N_{\Box A}(0) = -1 - \mu^N_A(0)$
$\geq 1 - \mu^N_A(x) = \nu^N_{\Box A}(x)$.

Therefore $\nu^N_{\Box\Box A}(0) \geq \nu^N_{\Box A}(x)$.

(v) Now $\nu^P_{\Box\Box A}(x) = 1 - \mu^P_{\Box A}(x) = 1 - \mu^P_A(x)$
$\leq \max\{1 - \mu^P_A(x*y), 1 - \mu^P_A(y)\}$
$= \max\{\nu^P_{\Box A}(x*y), \nu^P_{\Box A}(y)\}$.

Therefore $\nu^P_{\Box\Box A}(x) \leq \max\{\nu^P_{\Box A}(x*y), \nu^P_{\Box A}(y)\}$.

(vi) Now $\nu^N_{\Box\Box A}(x) = -1 - \mu^N_{\Box A}(x) = -1 - \mu^N_A(x)$
$\geq \min\{-1 - \mu^N_A(x * y), 1 - \mu^N_A(y)\}$
$= \min\{\nu^N_{\Box A}(x * y), \nu^N_{\Box A}(y)\}$.
Therefore $\nu^N_{\Box\Box A}(x) \geq \min\{\nu^N_{\Box A}(x * y), \nu^N_{\Box A}(y)\}$.

Therefore $\Box\Box A = \Box A$ is a bipolar intuitionistic fuzzy ideal of X.

Theorem 3.6. *If A is a bipolar intuitionistic fuzzy ideal of X, then $\Box\Diamond A = \Diamond A$ is a bipolar intuitionistic fuzzy ideal of X.*

Proof. Given A is a bipolar intuitionistic fuzzy ideal of X. Consider $0, x, y \in A$.

(i) Now $\mu^P_{\Box\Diamond A}(0) = \mu^P_{\Diamond A}(0) = 1 - \nu^P_A(0) \geq 1 - \nu^P_A(x) = \mu^P_{\Diamond A}(x)$.
Therefore $\mu^P_{\Box\Diamond A}(0) \geq \mu^P_{\Diamond A}(x)$.
Now $\mu^N_{\Box\Diamond A}(0) = \mu^N_{\Diamond A}(0) = -1 - \nu^N_A(0) \leq -1 - \nu^N_A(x) = \mu^N_{\Diamond A}(x)$.
Therefore $\mu^N_{\Box\Diamond A}(0) \leq \mu^N_{\Diamond A}(x)$.

(ii) Now $\mu^P_{\Box\Diamond A}(x) = \mu^P_{\Diamond A}(x) = 1 - \nu^P_A(x)$
$\geq \min\{1 - \nu^P_A(x * y), 1 - \nu^P_A(y)\}$
$= \min\{\mu^P_{\Diamond A}(x * y), \mu^P_{\Diamond A}(y)\}$.
Therefore $\mu^P_{\Box\Diamond A}(x) \geq \min\{\mu^P_{\Diamond A}(x * y), \mu^P_{\Diamond A}(y)\}$.

(iii) Now $\mu^N_{\Box\Diamond A}(x) = \mu^N_{\Diamond A}(x) = -1 - \nu^N_A(x)$
$\leq \max\{-1 - \nu^N_A(x * y), -1 - \nu^N_A(y)\}$
$= \max\{\mu^N_{\Diamond A}(x * y), \mu^N_{\Diamond A}(y)\}$.
Therefore $\mu^N_{\Box\Diamond A}(x) \leq \max\{\mu^N_{\Diamond A}(x * y), \mu^N_{\Diamond A}(y)\}$.

(iv) Now $\nu^P_{\Box\Diamond A}(0) = 1 - \mu^P_{\Diamond A}(0) = 1 - [1 - \mu^P_A(0)] = \nu^P_A(0)$
$\leq \nu^P_A(x) = \nu^P_{\Diamond A}(x)$.
Therefore $\nu^P_{\Box\Diamond A}(0) \leq \nu^P_{\Diamond A}(x)$.
Now $\nu^N_{\Box\Diamond A}(0) = -1 - \mu^N_{\Diamond A}(0) = -1 - [-1 - \mu^N_A(0)] = \nu^N_A(0)$
$\geq \nu^N_A(x) = \nu^N_{\Diamond A}(x)$.
Therefore $\nu^N_{\Box\Diamond A}(0) \geq \nu^N_{\Diamond A}(x)$.

(v) Now $\nu^P_{\Box\Diamond A}(x) = 1 - \mu^P_{\Diamond A}(x) = 1 - [1 - \nu^P_A(x)] = \nu^P_A(x)$
$\leq \max\{\nu A^P(x * y), \nu^P_A(y)\} = \max\{\nu^P_{\Diamond A}(x * y), \nu^P_{\Diamond A}(y)\}$.
Therefore $\nu^P_{\Box\Diamond A}(x) \leq \max\{\nu^P_{\Diamond A}(x * y), \nu^P_{\Diamond A}(y)\}$.

(vi) Now $\nu^N_{\Box\Diamond A}(x) = -1 - \mu^N_{\Diamond A}(x) = -1 - [-1 - \nu^N_A(x)] = \nu^N_A(x)$
$\geq \min\{\nu A^N(x * y), \nu^N_A(y)\} = \min\{\nu^N_{\Diamond A}(x * y), \nu^N_{\Diamond A}(y)\}$.
Therefore $\nu^N_{\Box\Diamond A}(x) \geq \min\{\nu^N_{\Diamond A}(x * y), \nu^N_{\Diamond A}(y)\}$.

Therefore $\Box\Diamond A = \Diamond A$ is a bipolar intuitionistic fuzzy ideal of X.

Theorem 3.7. *If A is a bipolar intuitionistic fuzzy ideal of X, then $\Diamond\Box A = \Box A$ is a bipolar intuitionistic fuzzy ideal of X.*
Proof. Given A is a bipolar intuitionistic fuzzy ideal of X. Consider $0, x, y \in A$.

(i) Now $\mu^P_{\Diamond\Box A}(0) = 1 - \nu^P_{\Box A}(0) = 1 - [1 - \mu^P_A(0)] = \mu^P_A(0) \geq \mu^P_A(x) = \mu^P_{\Box A}(x)$.
Therefore $\mu^P_{\Box A}(0) \geq \mu^P_{\Box A}(x)$.
Now $\mu^N_{\Diamond\Box A}(0) = -1 - \nu^N_{\Box A}(0) = -1 - [-1 - \mu^N_A(0)] = \mu^N_A(0) \leq \mu^N_A(x) = \mu^N_{\Box A}(x)$.
Therefore $\mu^N_{\Box A}(0) \leq \mu^N_{\Box A}(x)$.

(ii) Now $\mu^P_{\Diamond\Box A}(x) = 1 - \nu^P_{\Box A}(x) = 1 - [1 - \mu^P_A(x)] = \mu^P_A(x)$
$\geq \min\{\mu^P_A(x * y), \mu^P_A(y)\} = \min\{\mu^P_{\Box A}(x * y), \mu^P_{\Box A}(y)\}$.
Therefore $\mu^P_{\Diamond\Box A}(x) \geq \min\{\mu^P_{\Box A}(x * y), \mu^P_{\Box A}(y)\}$.

(iii) Now $\mu^N_{\Diamond\Box A}(x) = -1 - \nu^N_{\Box A}(x) = -1 - [1 - \mu^N_A(x)] = \mu^N_A(x)$
$\leq \max\{\mu^N_A(x * y), \mu^N_A(y)\} = \max\{\mu^N_{\Box A}(x * y), \mu^N_{\Box A}(y)\}$.
Therefore $\mu^N_{\Diamond\Box A}(x) \leq \max\{\mu^N_{\Box A}(x * y), \mu^N_{\Box A}(y)\}$.

(iv) Now $\nu^P_{\Diamond\Box A}(0) = \nu^P_{\Box A}(0) = 1 - \mu^P_A(0) \leq 1 - \mu^P_A(x) = \nu^P_{\Box A}(x)$.
Therefore $\nu^P_{\Diamond\Box A}(0) \leq \nu^P_{\Box A}(x)$.
Now $\nu^N_{\Diamond\Box A}(0) = \nu^N_{\Box A}(0) = -1 - \mu^N_A(0) \geq -1 - \mu^N_A(x) = \nu^N_{\Box A}(x)$.
Therefore $\nu^N_{\Diamond\Box A}(0) \geq \nu^N_{\Box A}(x)$.

(v) Now $\nu^P_{\Diamond\Box A}(x) = \nu^P_{\Box A}(x) = 1 - \mu^P_A(x)$
$\leq \max\{1 - \mu^P_A(x * y), 1 - \mu^P_A(y)\} = \max\{\nu^P_{\Box A}(x * y), \nu^P_{\Box A}(y)\}$.
Therefore $\nu^P_{\Diamond\Box A}(x) \leq \max\{\nu^P_{\Box A}(x * y), \nu^P_{\Box A}(y)\}$.

(vi) Now $\nu^N_{\Diamond\Box A}(x) = \nu^N_A(x) = -1 - \mu^N_A(x)$
$\geq \min\{-1 - \mu^N_A(x * y), -1 - \mu^N_A(y)\}$
$= \min\{\nu^N_{\Box A}(x * y), \nu^N_{\Box A}(y)\}$.
Therefore $\nu^N_{\Diamond\Box A}(x) \geq \min\{\nu^N_{\Box A}(x * y), \nu^N_{\Box A}(y)\}$.

Therefore $\Diamond\Box A = \Box A$ is a bipolar intuitionistic fuzzy ideal of X.

Theorem 3.8. *If A is a bipolar intuitionistic fuzzy ideal of X, then $\Diamond\Diamond A = \Diamond A$ is a bipolar intuitionistic fuzzy ideal of X.*
Proof. Given A is a bipolar intuitionistic fuzzy ideal of X. Consider $0, x, y \in A$.

(i) Now $\mu^P_{\Diamond\Diamond A}(0) = 1 - \nu^P_{\Diamond A}(0) = 1 - \nu^P_A(0) \geq 1 - \nu^P_A(x) = \mu^P_{\Diamond A}(x)$.
Therefore $\mu^P_{\Diamond\Diamond A}(0) \geq \mu^P_{\Diamond A}(x)$.
Now $\mu^N_{\Diamond\Diamond A}(0) = -1 - \nu^N_{\Diamond A}(0) = -1 - \nu^N_A(0) \leq -1 - \nu^N_A(x) = \mu^N_{\Diamond A}(x)$.
Therefore $\mu^N_{\Diamond\Diamond A}(0) \leq \mu^N_{\Diamond A}(x)$.

(ii) Now $\mu^P_{\Diamond\Diamond A}(x) = 1 - \nu_{\Diamond A}A^P(x) = 1 - \nu^P_A(x)$
$\geq \min\{1 - \nu^P_A(x * y), 1 - \nu^P_A(y)\} = \min\{\mu^P_{\Diamond A}(x * y), \mu^P_{\Diamond A}(y)\}$.
Therefore $\mu^P_{\Diamond\Diamond A}(x) \geq \min\{\mu^P_{\Diamond A}(x * y), \mu^P_{\Diamond A}(y)\}$.

(iii) Now $\mu^N_{\Diamond\Diamond A}(x) = -1 - \nu_{\Diamond A}A^N(x) = -1 - \nu^N_A(x)$
$\leq \max\{-1 - \nu^N_A(x * y), -1 - \nu^N_A(y)\} = \max\{\mu^N_{\Diamond A}(x * y), \mu^N_{\Diamond A}(y)\}$.
Therefore $\mu^N_{\Diamond\Diamond A}(x) \leq \max\{\mu^N_{\Diamond A}(x * y), \mu^N_{\Diamond A}(y)\}$.

(iv) Now $\nu^P_{\Diamond\Diamond A}(0) = \nu^P_{\Diamond A}(0) = \nu^P_A(0) \leq \nu^P_A(x) = \nu^P_{\Diamond A}(x)$.
Therefore $\nu^P_{\Diamond\Diamond A}(0) \leq \nu^P_{\Diamond A}(x)$.
Now $\nu^N_{\Diamond\Diamond A}(0) = \nu^N_{\Diamond A}(0) = \nu^N_A(0) \geq \nu^N_A(x) = \nu^N_{\Diamond A}(x)$.
Therefore $\nu^N_{\Diamond\Diamond A}(0) \geq \nu^N_{\Diamond A}(x)$.

(v) Now $\nu^P_{\Diamond\Diamond A}(x) = \nu^P_{\Diamond A}(x) = \nu^P_A(x)$
$\leq \max\{\nu^P_A(x * y), \nu^P_A(y)\} = \max\{\nu^P_{\Diamond A}(x * y), \nu^P_{\Diamond A}(y)\}$.
Therefore $\nu^P_{\Diamond\Diamond A}(x) \leq \max\{\nu^P_{\Diamond A}(x * y), \nu^P_{\Diamond A}(y)\}$.

(vi) Now $\nu^N_{\Diamond\Diamond A}(x) = \nu^N_{\Diamond A}(x) = \nu^N_A(x)$
$\geq \min\{\nu^N_A(x * y), \nu^N_A(y)\} = \min\{\nu^N_{\Diamond A}(x * y), \nu^N_{\Diamond A}(y)\}$.
Therefore $\nu^N_{\Diamond\Diamond A}(x) \geq \min\{\nu^N_{\Diamond A}(x * y), \nu^N_{\Diamond A}(y)\}$.

Therefore $\Diamond\Diamond A = \Diamond A$ is a bipolar intuitionistic fuzzy ideal of X.

Theorem 3.9. *If A is a bipolar intuitionistic fuzzy ideal of X, then $\Box\overline{\overline{A}} = \Diamond A$ is a bipolar intuitionistic fuzzy ideal of X.*
Proof. Given A is a bipolar intuitionistic fuzzy ideal of X. Consider $0, x, y \in A$.

(i) Now $\mu^P_{\Box\overline{\overline{A}}}(0) = \nu^P_{\overline{A}}(0) = 1 - \nu^P_A(0) \geq 1 - \nu^P_A(x) = \mu^P_{\Diamond A}(x)$.
Therefore $\mu^P_{\Box\overline{\overline{A}}}(0) \geq \mu^P_{\Diamond A}(x)$.
Now $\mu^N_{\Box\overline{\overline{A}}}(0) = \nu^N_{\overline{A}}(0) = -1 - \nu^N_A(0) \leq -1 - \nu^N_A(x) = \mu^N_{\Diamond A}(x)$.
Therefore $\mu^N_{\Box\overline{\overline{A}}}(0) \leq \mu^N_{\Diamond A}(x)$.

(ii) Now $\mu^P_{\Box\overline{\overline{A}}}(x) = \nu^P_{\overline{A}}(x) = 1 - \nu^P_A(x)$
$\geq \min\{1 - \nu^P_A(x * y), 1 - \nu^P_A(y)\} = \min\{\mu^P_{\Diamond A}(x * y), \mu^P_{\Diamond A}(y)\}$
Therefore $\mu^P_{\Box\overline{\overline{A}}}(x) \geq \min\{\mu^P_{\Diamond A}(x * y), \mu^P_{\Diamond A}(y)\}$.

(iii) Now $\mu^N_{\Box\overline{\overline{A}}}(x) = \nu^N_{\overline{A}}(x) = -1 - \nu^N_A(x)$
$\leq \max\{-1 - \nu^N_A(x * y), -1 - \nu^N_A(y)\} = \max\{\mu^N_{\Diamond A}(x * y), \mu^N_{\Diamond A}(y)\}$
Therefore $\mu^N_{\Box\overline{\overline{A}}}(x) \leq \max\{\mu^N_{\Diamond A}(x * y), \mu^N_{\Diamond A}(y)\}$.

(iv) Now $\nu^P_{\Box\overline{\overline{A}}}(0) = \mu^P_{\overline{A}}(0) = \nu^P_A(0) \leq \nu^P_A(x) = \nu^P_{\Diamond A}(x)$
Therefore $\nu^P_{\Box\overline{\overline{A}}}(0) \leq \nu^P_{\Diamond A}(x)$.
Now $\nu^N_{\Box\overline{\overline{A}}}(0) = \mu^N_{\overline{A}}(0) = \nu^N_A(0) \geq \nu^N_A(x) = \nu^N_{\Diamond A}(x)$
Therefore $\nu^N_{\Box\overline{\overline{A}}}(0) \geq \nu^N_{\Diamond A}(x)$.

(v) Now $\nu^P_{\Box\overline{\overline{A}}}(x) = \mu^P_{\overline{A}}(x) = \nu^P_A(x)$
$\leq \max\{\nu^P_A(x*y), \nu^P_A(y)\} = \max\{\nu^P_{\Diamond A}(x*y), \nu^P_{\Diamond A}(y)\}$.
Therefore $\nu^P_{\Box\overline{\overline{A}}}(x) \leq \max\{\nu^P_{\Diamond A}(x*y), \nu^P_{\Diamond A}(y)\}$.

(vi) Now $\nu^N_{\Box\overline{\overline{A}}}(x) = \mu^N_{\overline{A}}(x) = \nu^N_A(x)$
$\geq \min\{\nu^N_A(x*y), \nu^N_A(y)\} = \min\{\nu^N_{\Diamond A}(x*y), \nu^N_{\Diamond A}(y)\}$.
Therefore $\nu^N_{\Box\overline{\overline{A}}}(x) \geq \min\{\nu^N_{\Diamond A}(x*y), \nu^N_{\Diamond A}(y)\}$.

Therefore $\Box\overline{\overline{A}} = \Diamond A$ is a bipolar intuitionistic fuzzy ideal of X.

Theorem 3.10. *If A is a bipolar intuitionistic fuzzy ideal of X, then $\Diamond\overline{\overline{A}} = \Box A$ is a bipolar intuitionistic fuzzy ideal of X.*
Proof. Given A is a bipolar intuitionistic fuzzy ideal of X. Consider $0, x, y \in A$.

(i) Now $\mu^P_{\Diamond\overline{\overline{A}}}(0) = \nu^P_{\Diamond\overline{A}}(0) = \mu^P_A(0) \geq \mu^P_A(x) = \mu^P_{\Box A}(x)$.
Therefore $\mu^P_{\Diamond\overline{\overline{A}}}(0) \geq \mu^P_{\Box A}(x)$.
Now $\mu^N_{\Diamond\overline{\overline{A}}}(0) = \nu^N_{\Diamond\overline{A}}(0) = \mu^N_A(0) \leq \mu^N_A(x) = \mu^N_{\Box A}(x)$.
Therefore $\mu^N_{\Diamond\overline{\overline{A}}}(0) \leq \mu^N_{\Box A}(x)$.

(ii) Now $\mu^P_{\Diamond\overline{\overline{A}}}(x) = \nu^P_{\Diamond\overline{A}}(x) = \mu^P_A(x)$
$\geq \min\{\mu^P_A(x*y), \mu^P_A(y)\} = \min\{\mu^P_{\Box A}(x*y), \mu^P_{\Box A}(y)\}$
Therefore $\mu^P_{\Diamond\overline{\overline{A}}}(x) \geq \min\{\mu^P_{\Box A}(x*y), \mu^P_{\Box A}(y)\}$.

(iii) Now $\mu^N_{\Diamond\overline{\overline{A}}}(x) = \nu^N_{\Diamond\overline{A}}(x) = \mu^N_A(x)$
$\leq \max\{\mu^N_A(x*y), \mu^N_A(y)\} = \max\{\mu^N_{\Box A}(x*y), \mu^N_{\Box A}(y)\}$
Therefore $\mu^N_{\Diamond\overline{\overline{A}}}(x) \leq \max\{\mu^N_{\Box A}(x*y), \mu^N_{\Box A}(y)\}$.

(iv) Now $\nu^P_{\Diamond\overline{\overline{A}}}(0) = \mu^P_{\Diamond\overline{A}}(0) = 1 - \mu^P_A(0) \leq 1 - \mu^P_A(x) = \nu^P_{\Box A}(x)$.
Therefore $\nu^P_{\Diamond\overline{\overline{A}}}(0) \leq \nu^P_{\Box A}(x)$.
Now $\nu^N_{\Diamond\overline{\overline{A}}}(0) = \mu^N_{\Diamond\overline{A}}(0) = -1 - \mu^N_A(0) \geq -1 - \mu^N_A(x) = \nu^N_{\Box A}(x)$.
Therefore $\nu^N_{\Diamond\overline{\overline{A}}}(0) \geq \nu^N_{\Box A}(x)$.

(v) Now $\nu^P_{\Diamond\overline{\overline{A}}}(x) = \mu^P_{\Diamond\overline{A}}(x) = 1 - \mu^P_A(x)$
$\leq \max\{1 - \mu^P_A(x * y), 1 - \mu^P_A(y)\} = \max\{\nu^P_{\Box A}(x * y), \nu^P_{\Box A}(y)\}$.
Therefore $\nu^P_{\Diamond\overline{\overline{A}}}(x) \leq \max\{\nu^P_{\Box A}(x * y), \nu^P_{\Box A}(y)\}$.

(vi) Now $\nu^N_{\Diamond\overline{\overline{A}}}(x) = \mu^N_{\Diamond\overline{A}}(x) = -1 - \mu^N_A(x)$
$\geq \min\{-1 - \mu^N_A(x * y), -1 - \mu^N_A(y)\} = \min\{\nu^N_{\Box A}(x * y), \nu^N_{\Box A}(y)\}$.
Therefore $\nu^N_{\Diamond\overline{\overline{A}}}(x) \geq \min\{\nu^N_{\Box A}(x * y), \nu^N_{\Box A}(y)\}$.

Therefore $\Diamond\overline{\overline{A}} = \Box A$ is a bipolar intuitionistic fuzzy ideal of X.

Here, we have given all the theorems which are proved about the operations on bipolar intuitionistic fuzzy ideal. Similarly, the above all theorems with proofs are applicable for the operations on bipolar intuitionistic anti fuzzy ideal also.

4. Conclusion

In this paper, the main idea of a bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are a new algebraic structure of BP-algebra and it is used through the possibility and necessity operators. The aim of this study is implemented. The relevant ideas between the operation on bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are discussed. We thought that our ideas can also be applied for other algebraic system.

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