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PLANE STRAIN DEFORMATION WITH REPEATED CHARACTERISTICS VALUES

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Abstract: The objective of the present study is to derive the analytical expression for deformation field in an orthotropic elastic medium by using repeated characteristics values as a result of inclined line-load . For the procedure the method of equal characteristics value and Fourier transformation is used. To represent graphically, the elastic constants for two distinct elastic materials have been considered. To see the effect of inclination, the variations in displacements and stresses for different values of inclination i.e. at $\delta = 0^0$, $30^0, 60^0, 90^0$ have been depicted graphically. It is found that normal and tangential loading influence the displacement and stresses significantly for distinct material.

Keywords and Phrases: Characteristics values, Inclined line-load, Orthotropic elastic medium, Tangential line-load, Vertical line-load.

2020 Mathematics Subject Classification: 74-10, 86A15.

1. Introduction

Although it is almost impossible to predict an Earthquake, but with the study of seismology and identifying rock properties, seismologists can find out which is the most affected zone of earthquakes. In theoretical aspects, mathematical modeling plays an important role in order to understand Seismology and related phenomena. With the help of an appropriate model one can analyze the effect of loading, irregularities etc on deformation in distinct media which can proved to be helpful in issuing a real time warning and excessive damage can be prevented. Due to continuous changes in the interior of the earth, scientists have been giving keen attention to the field of seismology and its related problems. Thus, plane strain deformation problem and anti-plane strain deformation problem have become of much concern to the researchers due to their applications in various fields such as seismological research, geophysics, engineering etc. To examine the rocks around mining tremors and earth crust's drilling, the study of line- loading causing deformation has proven to be very useful. This study can also be helpful for theoretical understanding of seismic sources.

Maruyama [14] and Love [10] derived the analytical expressions for deformation as a result of line source and strike-slip fault of infinite length respectively in an isotropic elastic medium respectively. Garg et al. [5] studied anti plane strain deformation as a representation of seismic source. Authors concluded that the displacements are affected by both source and dip of the faults.

To resolve the problem of plane strain deformation, eigen value technique has been proved much qualifying and used by many researchers. Singh et al. [17] premeditated the case of static deformation of a monoclinic elastic medium and derived the analytical expressions for deformation in a transformed domain for distinct eigen values. Madan and Gaba [12] obtained the analytical expressions considering the elastic medium with the properties of transversely isotropic half-space. Further Madan et al. [13] derived the analytical expressions for deformation field for imperfect interface.

Many researchers such as Kar et al. [7], Selim [15], Acharya and Roy [1], Kumar et al. [8] etc. studied the plane strain problem for distinct medium using eigen value approach. Kumari and Madan [9] also studied the plane strain problem and derived the analytical expressions for deformation field due to seismic sources with imperfect interface. Wenwang et al. [19] studied the elastic field considering an isotropic elastic medium due to dislocation loops for dislocation and force-like model. In the absence of initial compressive stress Garg et al. [6] solved the plane strain deformation problem by considering distinct eigen values and derive the deformation field due to line-loading and studied the effect graphically.Most of these studies considered the case of distinct eigen values. Selim and Ahmed [16] studied the effect of inclined line-load for an elastic medium stressed initially and derived the set of mathematical expressions for displacements and stresses. Particular case of the results obtained in the paper was contradicted by Chugh et al. [2] by proving that the eigen value does not remain distinct for a stress free elastic medium. With repeated eigen values, Madan et al. [11] studied the deformation for transversely isotropic elastic medium.

The upper layer of Earth is termed as Earth's crust which is acknowledged to have an orthorhombic symmetry. The combination of vertical cracks with horizontal axis of symmetry caused orthorhombic symmetry. If any plane of the orthorhombic symmetry is horizontal, then the resulting symmetry is defined as orthotropic symmetry. The Earth is considered to be orthotropic in nature, (Dziewonski and Anderson [4]), therefore for improved estimation, in this paper we have considered an orthotropic elastic medium and derived the deformation field for the case of repeated characteristics values. The calculation is made simpler by applying Fourier transformation technique and matrix method.

The boundary condition for normal and tangential loading is used to obtain the mathematical expressions for displacements and stresses and examined the effect of inclined line-load. To show the variation in displacements and stresses, graphical representation of deformation at different angles of inclination $\delta = 0^0$, $30^0, 60^0, 90^0$ has been done. It has been observed that the displacements and stresses are influenced by different loading and variation in displacements also depends on the properties of distinct rock forming materials. The results obtained here are theoretical but can be proved to be very helpful to experimental research of engineering problems, crystal physics, geophysics and geodynamics which is certainly the most interesting geophysical research. Our study is aimed at better understanding of the role of loading in the plane strain deformation problem.

2. Basic Theory and Equations

As most of the earthquakes are sufficiently large, thus two dimensional approximation is well justified in the study of earthquakes and related phenomena. Therefore in this case we have considered plane strain model, parallel to xy-plane, the non-zero stresses for an orthotropic elastic medium can be written as:

$$s_{11} = m_{11}\frac{\partial u}{\partial x} + m_{12}\frac{\partial v}{\partial y} \tag{1}$$

$$s_{22} = m_{12}\frac{\partial u}{\partial x} + m_{22}\frac{\partial v}{\partial y} \tag{2}$$

$$s_{33} = m_{13}\frac{\partial u}{\partial x} + m_{23}\frac{\partial v}{\partial y} \tag{3}$$

$$s_{12} = m_{66}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) \tag{4}$$

Garg et al. [16] gives the equilibrium equations for plane strain deformation problem considering an orthotropic elastic half-space:

$$m_{11}\frac{\partial^2 u}{\partial x^2} + m_{66}\frac{\partial^2 u}{\partial y^2} + (m_{66} + m_{12})\frac{\partial^2 v}{\partial x \partial y} = 0 \tag{5}$$

$$(m_{66}+m_{12})\frac{\partial^2 u}{\partial x\partial y}+m_{66}\frac{\partial^2 v}{\partial x^2}+m_{22}\frac{\partial^2 v}{\partial y^2}=0 \tag{6}$$

Using Fourier transformation on equilibrium equation (5) and (6) to avoid unnecessary mathematical complications (Debnath [3]), we find:

$$m_{11}\frac{d^2\bar{u}}{dx^2}-s^2m_{66}\bar{u}+(m_{66}+m_{12})(-is)\frac{d\bar{v}}{dx}=0 \tag{7}$$

$$(m_{66}+m_{12})(-is)\frac{d\bar{u}}{dx}+m_{66}\frac{d^2\bar{v}}{dx^2}+m_{22}(-s^2\bar{v})=0 \tag{8}$$

where s being the Fourier transform parameter.
The equations (7) and (8) are unified in the following vector-matrix equation.

$$T_1\frac{d^2W}{dx^2}-isT_2\frac{dW}{dx}-s^2T_3W=0 \tag{9}$$

where

$$T_1=\begin{pmatrix} m_{11} & 0\\ 0 & m_{66}\end{pmatrix}, T_2=\begin{pmatrix} 0 & m_{66}+m_{12}\\ m_{66}+m_{12} & 0\end{pmatrix}, T_3=\begin{pmatrix} m_{66} & 0\\ 0 & m_{22}\end{pmatrix}, W=\begin{pmatrix}\bar{u}\\ \bar{v}\end{pmatrix} \tag{10}$$

Here T_1, T_2 and T_3 represents the symmetric matrices. Let solution of equation (9) is

$$W(x,s)=C(s)e^{\Delta x} \tag{11}$$

where C(s)is a 2*1 type matrix and Δ being a parameter. Using equation (9) and equation (11), the characteristic equation is

$$(m_{11}m_{66})\Delta^4-(m_{11}m_{22}-2m_{66}m_{12}-m_{12}^2)s^2\Delta^2+(m_{66}m_{22})s^4=0 \tag{12}$$

As we have considered the case of repeated eigen values, where the eigen values has been obtained by solving characteristics equation (12) i.e.:
$\Delta=\pm\alpha|s|,\pm\alpha|s|$
with

$$\Delta_1=\Delta_2=\alpha|s|,\Delta_3=\Delta_4=-\alpha|s| \tag{13}$$

where $\alpha=\dfrac{m_{11}m_{22}-2m_{66}m_{12}-m_{12}^2}{2m_{11}m_{66}}$

The corresponding eigen vectors for Δ_1, Δ_2, Δ_3 and Δ_4 can be obtained by solving the characteristics equations

$(\Delta^2 T_1 - i\Delta s T_2 - s^2 T_3)V(s) = 0$
where T_1, T_2 and T_3 are given in equation (10). The corresponding eigen vectors are found to be

$$V_1 = \begin{bmatrix} M \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} \frac{A_4}{s^2} + xM \\ \frac{A_5}{k^2} + x \end{bmatrix} V_3 = \begin{bmatrix} -M \\ 1 \end{bmatrix}, V_4 = \begin{bmatrix} \frac{A_4}{s^2} - xM \\ \frac{A_5}{k^2} + x \end{bmatrix} \tag{14}$$

where

$$M = i\frac{\alpha(m_{66} + m_{12})}{\alpha^2 m_{11} - m_{66}}, A_1 = \alpha^2 m_{11} - m_{66}, A_2 = i\alpha(m_{66} + m_{12}), A_3 = \alpha^2 m_{66} - m_{22}$$

$$A_4 = \frac{MA_3 + A_2}{A_1 A_3 - A_2^2}, A_5 = \frac{MA_2 + A_1}{A_1 A_3 + A_2^2} \tag{15}$$

Therefore for the case of orthotropic elastic medium, we can write the solution of matrix equation (9) in the form:

$$U(x, s) = G_1 V_1 e^{\alpha|s|x} + G_2 V_2 e^{\alpha|s|x} + G_3 V_3 e^{-\alpha|s|x} + G_4 V_4 e^{-\alpha|s|x} \tag{16}$$

where G_1, G_2, G_3 and G_4 are constants to be determined using boundary conditions for normal and tangential loading. Using the values of eigen vectors from equation (14), the inversion of equation (16) gives the displacements and stresses in integral form as:

$$u(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Big[\left(MG_1 + \left(\frac{A_4}{s^2} + xM \right) G_2 \right) e^{\alpha|s|x} + \left(-MG_3 + G_4 \left(\frac{A_4}{s^2} - xM \right) \right) e^{-\alpha|s|x} \Big] e^{-isy} ds$$

$$v(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Big[\left(G_1 + G_2 \left(\frac{A_5}{s^2} + x \right) \right) e^{\alpha|s|x} + \left(G_3 + G_4 \left(\frac{A_5}{s^2} + x \right) \right) e^{-\alpha|s|x} \Big] e^{-isy} ds$$

$$s_{11}(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Big[G_1(M\alpha m_{11}|s| - ism_{12}) + G_2 \Big(\alpha m_{11} \frac{A_4}{|s|} - im_{12} \frac{A_5}{s} + xM\alpha m_{11}|s| - isxm_{12} + Mm_{11} \Big) e^{\alpha|s|x} + G_3(M\alpha m_{11}|s| - ism_{12}) + G_4 \Big(-\alpha m_{11} \frac{A_4}{|s|} - im_{12} \frac{A_5}{s} + xM\alpha m_{11}|s| - isxm_{12} - Mm_{11} \Big) e^{-\alpha|s|x} \Big] e^{-isy} ds$$

$$s_{12}(x, y) = \frac{m_{66}}{2\pi} \int_{-\infty}^{\infty} \Big[G_1 \left(-isM + \alpha|s| \right) + G_2 \left(-i\frac{A_4}{s} + \alpha \frac{A_5}{|s|} - isxM + \alpha|s|x + 1 \right) e^{\alpha|s|x} + G_3 \left(isM - \alpha|s| \right) + G_4 \left(-i\frac{A_4}{s} - \alpha \frac{A_5}{|s|} + isxM - \alpha|s|x + 1 \right) e^{-\alpha|s|x} \Big] e^{-isy} ds \tag{17}$$

3. Formulation and Solution of the Problem

In the present plane-strain deformation problem parallel to xy-plane, we have considered an infinite elastic half-space with orthotropic symmetry. The medium is assumed to be free from initial compressive stress. In the problem x-axis is acting vertically downwards. Let an inclined line-load (L_I) per unit length be acting on z-axis and the angle of inclination with x-axis is denoted by δ (figure 1). The resulting closed form analytical expressions for deformation field at any point of the medium due to loading shall be calculated using boundary conditions of normal and tangential loading. For the purpose, we have considered an the elastic medium consisting of Medium 1 ($x > 0$) and Medium 2 ($x < 0$).

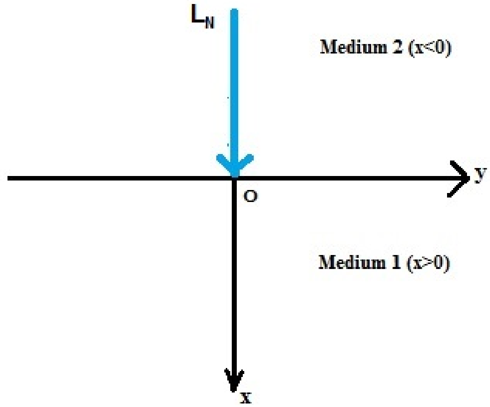

Figure 1: A vertical line load

The displacement and stress field due to loading are
For Medium 1:

$$u^1(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(-MG_3 + G_4\left(\frac{A_4}{s^2} - xM\right)\right)e^{-\alpha|s|x}e^{-isy}ds$$

$$v^1(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(G_3 + G_4\left(\frac{A_5}{s^2} + x\right)\right)e^{-\alpha|s|x}e^{-isy}ds$$

$$s^1_{11}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(G_3(M\alpha|s|m_{11} - ism_{12}) + G_4\left(-\alpha m_{11}\frac{A_4}{|s|} - im_{12}\frac{A_5}{s} + xM\alpha m_{11}|s|\right.\right.$$
$$\left.\left. - isxm_{12} - Mm_{11}\right)\right)e^{-\alpha|s|x}e^{-isy}ds$$

$$s_{12}^{1}(x,y) = \frac{m_{66}}{2\pi}\int_{-\infty}^{\infty} G_3\left(isM - \alpha|s|\right) + G_4\left(-i\frac{A_4}{s} - \alpha\frac{A_5}{|s|} + isxM - \alpha|s|x + 1\right) e^{-\alpha|s|x}e^{-isy}ds \tag{18}$$

For Medium 2 :

$$\begin{aligned}
u^2(x,y) =& \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(MG_1 + \left(\frac{A_4}{s^2} + xM\right)G_2\right)e^{\alpha|s|x}e^{-isy}ds \\
v^2(x,y) =& \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(G_1 + G_2\left(\frac{A_5}{s^2} + x\right)\right)e^{\alpha|s|x}e^{-isy}ds \\
s_{11}^2(x,y) =& \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(G_1(M\alpha|s|m_{11} - ism_{12}) + G_2\left(\alpha m_{11}\frac{A_4}{|s|} - im_{12}\frac{A_5}{s} + xM\alpha m_{11}|s|\right.\right. \\
& \left.\left. - isxm_{12} + Mm_{11}\right)\right)e^{\alpha|s|x}e^{-isy}ds \\
s_{12}^2(x,y) =& \frac{m_{66}}{2\pi}\int_{-\infty}^{\infty} G_1\left(-isM + \alpha|s|\right) + G_2\left(-i\frac{A_4}{s} + \alpha\frac{A_5}{|s|} - isxM + \alpha|s|x + 1\right) \\
& e^{\alpha|s|x}e^{-isy}ds
\end{aligned} \tag{19}$$

3.1. Line-Load acting Vertically

Applying a vertical line-load L_N, per unit length, acting on the interface x=0, along the z-axis (figure 1). Let at x=0, the boundary conditions be:

$$\begin{aligned}
u^1(x,y) - u^2(x,y) =& 0, \\
v^1(x,y) - v^2(x,y) =& 0, \\
s_{11}^1(x,y) - s_{11}^2(x,y) =& -L_N\delta(y), \\
s_{12}^1(x,y) - s_{12}^2(x,y) =& 0.
\end{aligned} \tag{20}$$

$\delta(y)$: Dirac delta function that satisfies the following properties

$$\int_{-\infty}^{\infty}\delta(y)dy = 1, \delta(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-isy}ds \tag{21}$$

Applying boundary conditions (20) into the equations (18) and (19) we obtain the following values of elastic constants.

$$\begin{aligned} G_1 &= L_N\delta(y)B_1\left[\frac{1}{|s|}-\frac{1}{s}\right], \\ G_2 &= L_N\delta(y)\left[B_1(|s|-s)-B_2(s^2-|s|)\right], \\ G_3 &= -L_N\delta(y)B_3\left[\frac{1}{|s|}-\frac{1}{s}\right], \\ G_4 &= -L_N\delta(y)\left[B_3(|s|-s)-B_2(s^2-|s|)\right], \end{aligned} \tag{22}$$

where
$B_1=\dfrac{A_5(MA_5+A_4)}{2\alpha m_{11}(M^2A_5^2-A_4^2)}$, $B_2=\dfrac{M}{2\alpha^2 m_{11}(M^2A_5^2-A_4^2)}$, $B_3=\dfrac{A_5(MA_5-A_4)}{2\alpha m_{11}(M^2A_5^2-A_4^2)}$.
Now using the values of elastic constants from equation (22) in to the equations (18)-(19) and using Wolfram Mathematica for integral calculations, we obtain the following analytical expressions of displacements and stresses for vertical line-load.

$$\begin{aligned} u^N(x,y) =& \frac{L_N\delta(y)}{2\pi}\Big[-B_1(MA_5\mp A_4)\Big(\log(\alpha^2x^2+y^2)\mp 2i\tan^{-1}\Big(\frac{y}{\alpha x}\Big)\Big) \\ &+B_2A_4\Big(\frac{-iy\mp\alpha x}{\alpha^2x^2+y^2}\Big)\mp 2xM\Big(-B_1\frac{(\alpha^2x^2-y^2)\pm 2i\alpha xy}{(\alpha^2x^2+y^2)^2} \\ &+B_2\frac{-2iy(3\alpha^2x^2-y^2)\mp 2\alpha x(\alpha^2x^2-3y^2)}{(\alpha^2x^2+y^2)^3}\Big)\Big] \end{aligned} \tag{23}$$

$$\begin{aligned} v^N(x,y) =& \frac{L_N\delta(y)}{2\pi}\Big[B_1(A_4\mp A_5)\Big(\log(\alpha^2x^2+y^2))\mp 2i\tan^{-1}\Big(\frac{y}{\alpha x}\Big)\Big) \\ &+B_2A_5\Big(\frac{-iy\mp\alpha x}{\alpha^2x^2+y^2}\Big)+2x\Big(\pm B_1\frac{(\alpha^2x^2-y^2)\pm 2i\alpha xy}{(\alpha^2x^2+y^2)^2} \\ &+B_2\frac{-2iy(3\alpha^2x^2-y^2)\mp 2\alpha x(\alpha^2x^2-3y^2)}{(\alpha^2x^2+y^2)^3}\Big)\Big] \end{aligned} \tag{24}$$

$$\begin{aligned} s_{11}^N(x,y) =& \frac{L_N\delta(y)}{2\pi}\Big[2M_1\frac{\pm\alpha x+iy}{\alpha^2x^2+y^2}+2M_2\frac{(\alpha^2x^2-y^2)\pm 2i\alpha xy}{(\alpha^2x^2+y^2)^2} \\ &+4M_3\frac{\alpha x(\alpha^2x^2-3y^2)+it(3\alpha^2x^2-y^2)}{(\alpha^2x^2+y^2)^3} \\ &-12M_4\frac{(\alpha^4x^4-6\alpha^2x^2y^2+y^4)\pm 4iy\alpha x(\alpha^2x^2-y^2)}{(\alpha^2x^2+y^2)^4}\Big] \end{aligned} \tag{25}$$

$$s_{12}^N(x,y) = \frac{L_N\delta(y)}{2\pi}\Big[2M_5\frac{\pm\alpha x+iy}{\alpha^2x^2+y^2}+2M_6\frac{(\alpha^2x^2-y^2)\pm 2i\alpha xy}{(\alpha^2x^2+y^2)^2}$$
$$+4M_7\frac{\alpha x(\alpha^2x^2-3y^2)+it(3\alpha^2x^2-y^2)}{(\alpha^2x^2+y^2)^3}$$
$$-12M_8\frac{(\alpha^4x^4-6\alpha^2x^2y^2+y^4)\pm 4iy\alpha x(\alpha^2x^2-y^2)}{(\alpha^2x^2+y^2)^4}\Big] \tag{26}$$

where
$M_1 = B_1(M\alpha m_{11} + im_{12} - \alpha m_{11}A_5 - m_{12})$,
$M_2 = -Mm_{11}B_1 \mp \alpha m_{11}A_4B_2 - im_{12}A_5B_2$,
$M_3 = xM\alpha m_{11}B_1 + ixm_{12}B_1 \pm Mm_{11}B_2$,
$M_4 = xM\alpha m_{11}B_2 - ixm_{12}B_2$,
$M_5 = -B_1(iM - \alpha \pm iA_4 + \alpha A_5), M_6 = \pm B_1 - iA_4B_2 \mp \alpha A_5B_2$,
$M_7 = -ixMB_1 - x\alpha B_1 - B_2$,
$M_8 = \mp(ixMB_2 + x\alpha B_2)$.

The superscript (N) is used to show the deformation due to vertical loading L_N and superscript (T) is used to show the deformation due to tangential loading L_T.

3.2. Line-Load acting Horizontally

Here we apply a line-load (L_T, per unit length),that is acting at the origin in the positive y-direction (figure 2). At x=0, let the required boundary conditions be:

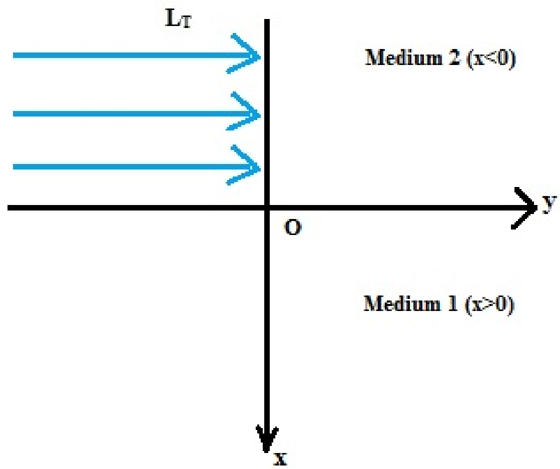

Figure 2: A horizontal line-load

$$\begin{aligned} u^1(x,y)-u^2(x,y) =&0,\\ v^1(x,y)-v^2(x,y) =&0,\\ s^1_{11}(x,y)-s^2_{11}(x,y) =&0,\\ s^1_{12}(x,y)-s^2_{12}(x,y) =&-L_T\delta(y). \end{aligned} \tag{27}$$

Applying boundary conditions (27) into the equations (18) and (19) we obtain the following values of elastic constants.

$$\begin{aligned} G_1 &= -L_T\delta(y)C_1\left[\alpha A_5\left[\frac{1}{|s|}-\frac{1}{s}\right]+M\left[1-\frac{s}{|s|}\right]\right],\\ G_2 &= 2L_T\delta(y)M\left[\alpha C_1[|s|-s]+C_2[s^2-s|s|]\right],\\ G_3 &= -L_T\delta(y)C_3\left[\alpha A_5\left[\frac{1}{|s|}-\frac{1}{s}\right]+M\left[1-\frac{s}{|s|}\right]\right],\\ G_4 &= -2L_T\delta(y)M\left[\alpha C_3[|s|-s]+C_2[s^2-s|s|]\right], \end{aligned} \tag{28}$$

where $C_1=\dfrac{MA_5+A_4}{2\alpha^2(M^2A_5^2-A_4^2)}$, $C_2=\dfrac{M}{2\alpha^2(M^2A_5^2-A_4^2)}$, $C_3=\dfrac{MA_5-A_4}{2\alpha^2(M^2A_5^2-A_4^2)}$. Now using the values of elastic constants from equation (28) in to the equations (18) - (19) and using Wolfram Mathematica for integral calculations, we obtain the following analytical expressions of displacements and stresses for horizontal line load.

$$\begin{aligned} u^T(x,y) =& -\frac{L_T\delta(y)}{2\pi}\Big[\pm MC_1(A_5-A_4)\Big(-\log(\alpha^2x^2+y^2))\pm 2i\tan^{-1}\left(\frac{y}{\alpha x}\right)\Big)\\ &+2M(C_2A_5-MC_1)\frac{(\alpha x+iy)}{(\alpha^2x^2+y^2)}-2xM^2\Big(C_1\frac{(\alpha^2x^2-y^2)\pm 2i\alpha xy}{(\alpha^2x^2+y^2)^2}\\ &+C_2\frac{2iy(3\alpha^2x^2-y^2)\pm 2\alpha x(\alpha^2x^2-3y^2)}{(\alpha^2x^2+y^2)^3}\Big)\Big] \end{aligned} \tag{29}$$

$$\begin{aligned} v^T(x,y) =& -\frac{L_T\delta(y)}{2\pi}\Big[C_1(A_5\pm A_4)\Big(-\log(\alpha^2x^2+y^2))\pm 2i\tan^{-1}\left(\frac{y}{\alpha x}\right)\Big)\\ &+2(MC_1-C_2A_5)\frac{(\alpha x+iy)}{(\alpha^2x^2+y^2)}\pm 2xM\Big(C_1\frac{(\alpha^2x^2-y^2)+2i\alpha xy}{(\alpha^2x^2+y^2)^2}\\ &+C_2\frac{2iy(3\alpha^2x^2-y^2)+2\alpha x(\alpha^2x^2-3y^2)}{(\alpha^2x^2+y^2)^3}\Big)\Big] \end{aligned} \tag{30}$$

$$\begin{aligned} s^T_{11}(x,y) =& \frac{L_T\delta(y)}{2\pi}\Big[2M_1^I\frac{\pm\alpha x+iy}{\alpha^2x^2+y^2}+2M_2^I\frac{(\alpha^2x^2-y^2)\pm 2i\alpha xy}{(\alpha^2x^2+y^2)^2}\\ &+4M_3^I\frac{\alpha x(\alpha^2x^2-3y^2)+it(3\alpha^2x^2-y^2)}{(\alpha^2x^2+y^2)^3} \end{aligned}$$

$$- 12M_4^I \frac{(\alpha^4x^4 - 6\alpha^2x^2y^2 + y^4) \pm 4iy\alpha x(\alpha^2x^2 - y^2)}{(\alpha^2x^2 + y^2)^4}\Big] \quad (31)$$

$$s_{12}^T(x,y) = \frac{L_T\delta(y)}{2\pi}\Big[2M_5^I \frac{\pm\alpha x + iy}{\alpha^2x^2 + y^2} + 2M_6^I \frac{(\alpha^2x^2 - y^2) \pm 2i\alpha xy}{(\alpha^2x^2 + y^2)^2} + 4M_7^I \frac{\alpha x(\alpha^2x^2 - 3y^2) + it(3\alpha^2x^2 - y^2)}{(\alpha^2x^2 + y^2)^3} - 12M_8^I \frac{(\alpha^4x^4 - 6\alpha^2x^2y^2 + y^4) \pm 4iy\alpha x(\alpha^2x^2 - y^2)}{(\alpha^2x^2 + y^2)^4}\Big] \quad (32)$$

where
$M_1^I = \alpha C_1A_4m_{11}M + iC_1m_{12}A_4 - \alpha C_1A_4m_{11} \pm iC_1m_{12}A_5$,
$M_2^I = -i\alpha C_1M^2m_{11} + \alpha m_{11}C_2A_4 - iMm_{12} \mp im_{12}A_5C2 + C_1Mm_{11}$,
$M_3^I = \pm x\alpha Mm_{11}C_1 \pm ixm_{12}C_1 - Mm_{11}C_2$,
$M_4^I = \mp x\alpha Mm_{11}C_2 \pm ixm_{12}C_2$,
$M_5^I = \mp iMC_1A_4 - \alpha C_1A_4 \mp iC_1A_4 + \alpha C_1A_5$,
$M_6^I = iC_1M^2 + \alpha MC_1 - iC_2A_4 \pm \alpha C_2A_5 \mp C_1$,
$M_7^I = \mp C_2 - ixMC_1 - x\alpha C_1$,
$M_8^I = ixMC_2 + x\alpha C_2$.

3. Inclined Line Load

When an inclined line load L_I is applied (figure 3), we have the following relations with normal and tangential loading: $L_N = L_I\cos\delta$,$L_T = L_I\sin\delta$.

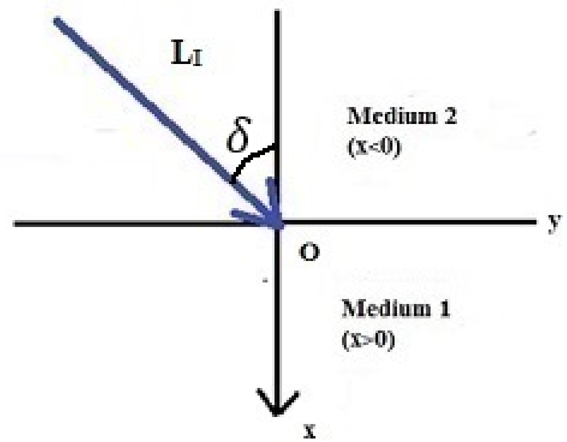

Figure 3: An inclined line load

The final formulated problem of deformation due to inclined line load is:

$$\begin{aligned} u^I(x,y) &= u^N(x,y) + u^T(x,y) \\ v^I(x,y) &= v^N(x,y) + v^T(x,y) \\ s^I_{11}(x,y) &= s^N_{11}(x,y) + s^T_{11}(x,y) \\ s^I_{12}(x,y) &= s^N_{12}(x,y) + s^T_{12}(x,y) \end{aligned} \tag{33}$$

The deformation due to vertical line load L_N has been obtained in equations (23)-(26) and the deformation due to tangential line load L_T has been obtained in equations (29)-(32).

4. Numerical Results and Discussion

In this section, we wish to examine the effect of inclined line-load L_I on displacement and stresses against the horizontal distance (y) in an infinite

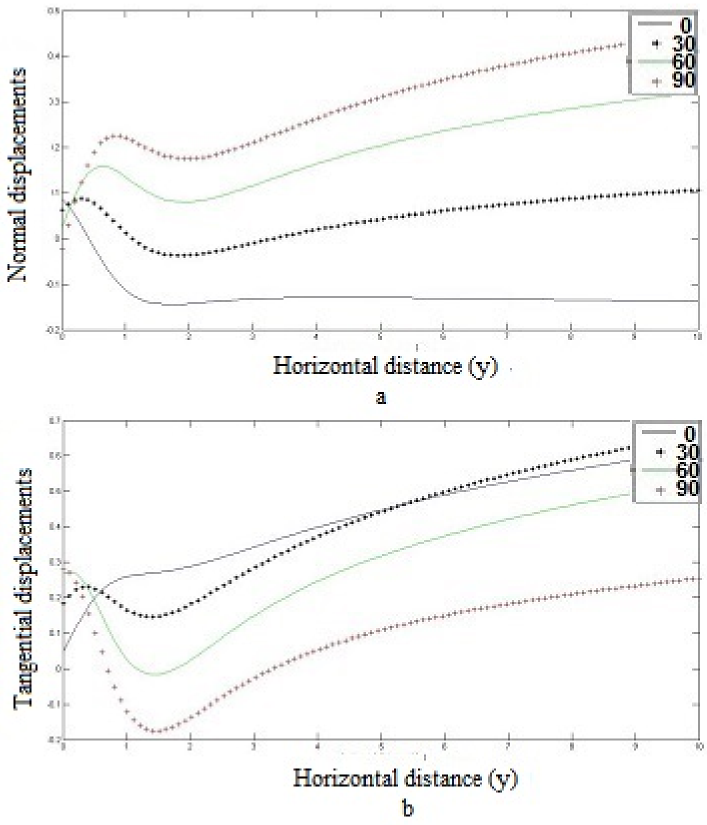

Figure 4: (a) Normal displacement (u) and (b) Tangential displacements (v) against the horizontal distance (y) at δ=0^0, 30^0, 60^0 and 90^0

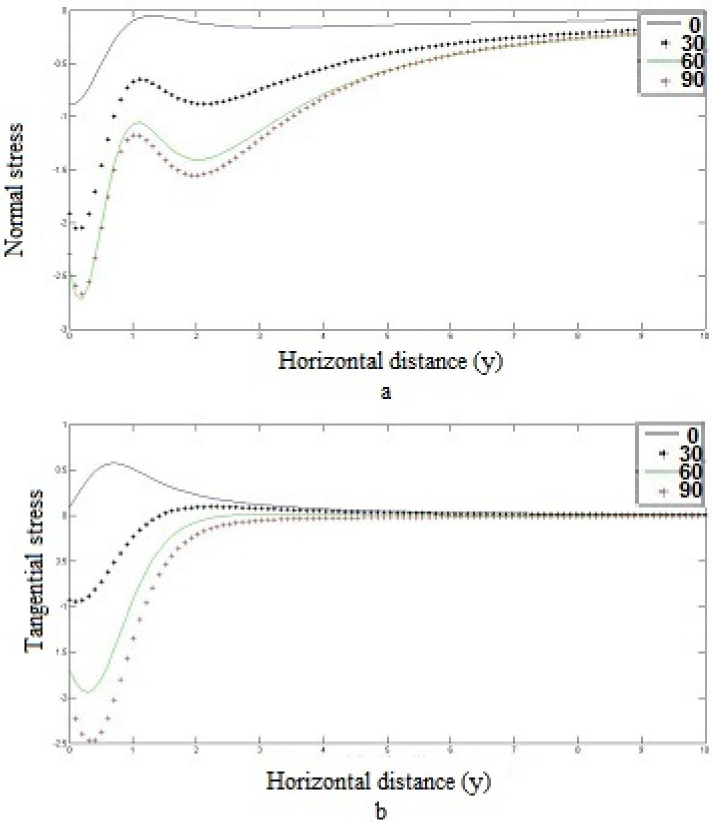

Figure 5: (a) Normal stress (s_{11}) and (b) Tangential stress (s_{12}) against the horizontal distance (y) at δ=0^0, 30^0, 60^0 and 90^0

orthotropic elastic medium. As different rock forming materials have different properties and influence of loading also depends on different rock materials, therefore for detailed observation of the variation in displacements and stresses due to loading, we have computed graphical results by considering elastic constants for two different materials Olivine [18] and Topaz [10].

As Olivine is most common mineral in earth's crust, thus we have used the value of elastic constants for olivine material in figure (4) and figure (5) i.e
$m_{11} = 324.0, m_{22} = 198.0, m_{33} = 249.0, m_{12} = 59.0, m_{23} = 78.0, m_{31} = 79.0$
$m_{66} = 79.3, m_{44} = 66.7, m_{55} = 81.0$ in terms of unit stress 10^{11}dynes cm^{-2}.

Figure 4(a) and 4 (b) depict comparison in variation of normal and tangential displacements against the horizontal distance y influenced by distinct angle of inclination ($\delta = 0^0, 30^0, 60^0, 90^0$) Comparison between stresses against the horizontal distance y due to various angle of inclination has been done in figure 5(a) and 5(b), considering the rock properties of olivine materiel.

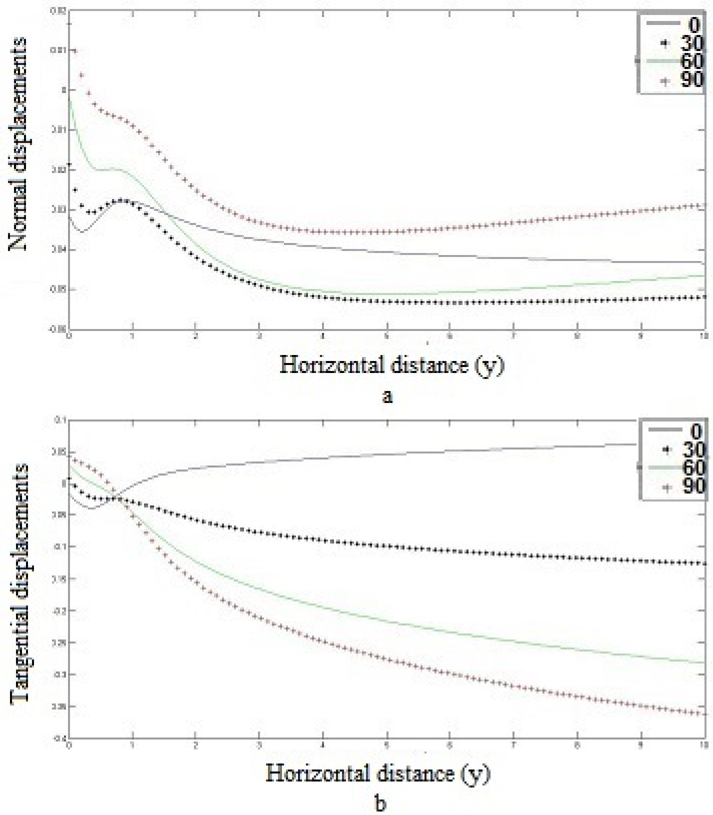

Figure 6: (a) Normal displacement (u) and (b) Tangential displacements (v) against the horizontal distance (y) at δ=0^0, 30^0, 60^0 and 90^0

Topaz in rare silicate material which usually forms fractures and cavities of igneous rocks, therefore in figure (6) and figure (7) we have used elastic constants for Topaz material i.e
$m_{11} = 2870, m_{22} = 3560, m_{33} = 3000, m_{12} = 1280, m_{23} = 900, m_{31} = 860$
$, m_{44} = 1100, m_{55} = 1350, m_{66} = 1330$ in terms of unit stress 10^6dynes cm^2
Figure (6) and figure (7), shows the variation in displacements and stresses at different values of inclined line-load against the horizontal distance y for Topaz material.
From these figures it is observed that the line-loading affect the displacement and stresses significantly but the variation of minerals present in the medium also influence the deformation field.

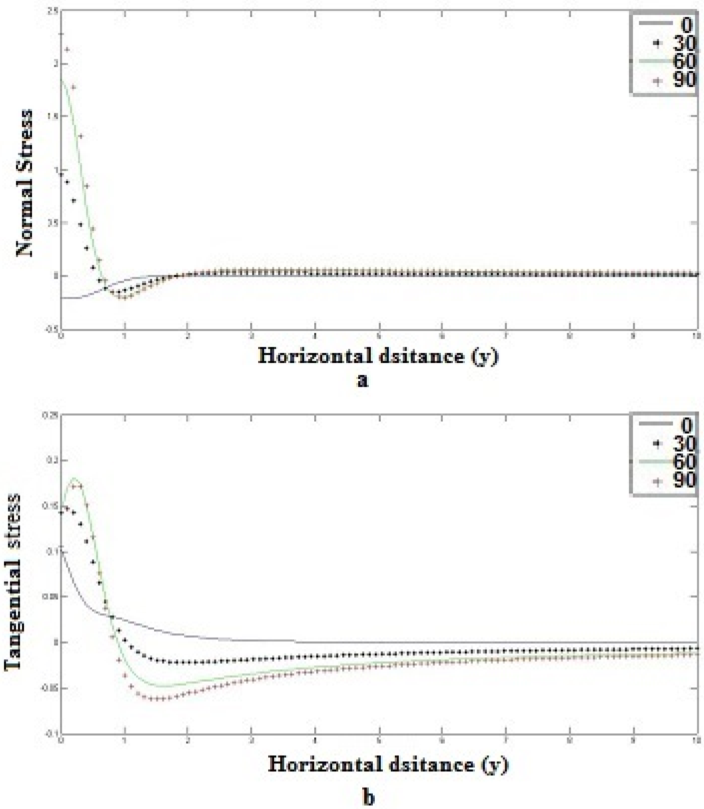

Figure 7: (a) Normal stress (s_{11}) and (b) Tangential stress (s_{12}) against the horizontal distance (y) at δ=0^0, 30^0, 60^0 and 90^0

5. Conclusion

Two dimensional approximation is useful to study Earthquakes as most of the Earthquakes are adequately large and shallow for eg. San Andreas Fault. Thus we have considered a plane strain deformation problem and examined the effect of loading on displacements and stresses for an orthotropic elastic medium that is free from initial compressive stress. To obtain the analytical expressions for deformation we used the technique of equal characteristics value and Fourier transformation. With minor substitution, the results for isotropic and transversely isotropic medium can also be derived.

The obtained graphical results clearly shows that the affect of distinct loading in the variation of displacements and stresses varies as the rock material properties changes. As the rock forming minerals of deep crust and upper mantle are different

therefore in the present study we have considered Olivine and Topaz materials for graphical computations.
The graphical representation can proved to be helpful in experimental study of analyzing field, mining tremors and also the study of loading is useful for the theoretical understanding of seismic and volcanic sources. The results are obtained by considering orthotropic elastic half-space therefore this study may find applications in geological and engineering problems.

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