South East Asian J. of Mathematics and Mathematical Sciences
Vol. 18, No. 2 (2022), pp. 379-388

DOI: 10.56827/SEAJMMS.2022.1802.31 **ISSN (Online): 2582-0850**

ISSN (Print): 0972-7752

BIANCHI TYPE VI_0 STRING COSMOLOGICAL MODEL IN LYRA'S MANIFOLD

S. R. Hadole, A. S. Nimkar and A. M. Pund*

Shri. Dr. R. G. Rathod Arts and Science College,
Murtizapur, Akola - 444107, Maharashtra, INDIA

E-mail : sangita.hadole2000@gmail.com, anilnimkar@gmail.com

*Department of Mathematics,
Shri Shivaji Education Society Amravati's Science College,
Congress Nagar, Nagpur - 440012, Maharashtra, INDIA

E-mail : ashokpund64@rediffmail.com

(**Received:** *Aug. 04, 2021* **Accepted:** *Aug. 12, 2022* **Published:** *Aug. 30, 2022)*

Abstract: A solution of field equations has been obtained for a Bianchi Type VI_0 space-time with cosmic string in Lyra's Manifold by using relation between metric coefficients and Reddy string. Also some physical and kinematic properties of the model have been examined.

Keywords and Phrases: Bianchi Type VI_0, Cosmic string, Lyra's Manifold.

2020 Mathematics Subject Classification: 83C50, 83F05.

1. Introduction

Lyra [9] proposed an alternation of Riemannian geometry by introducing a gauge function into the structure less manifold because of which the cosmological constant emerges naturally from the geometry. Sen [23] considered the cosmological model of the Universe based on Lyra's modified Riemannian geometry and showed that the model is just like the static Einstein model and has a finite density even without the creation of a cosmological constant. In curious investigation Sen and Dunn [22] expressed a new scalar tensor theory of gravitation and constructed comparison of the Einstein field equation based on Lyra's geometry. Cosmological model in the frame work of Lyra's geometry in the different contexts is investigated by several researchers such as Sethi, Nayak and Patra [24] studied string cosmological model with bulk viscosity in Lyra geometry. Yadav and Bhardwaj [27] studied Lyra's cosmology of hybrid universe in Bianchi V space time. Yadav, Goswami, Pradhan and Srivastava [26] studied Dark energy dominated universe in Lyra geometry. Reddy and Venkateswarlu [16] investigated a statics conformally flat cosmological model in lyra's manifold. Hegazy [6] studied Bulk viscous Bianchi type I cosmological model in Lyra geometry. Adhav [1] studied LRS Bianchi Type-I Universe with Anisotropic Dark Energy in Lyra Geometry. Naidu, Aditya, Ramesh and Reddy [12] studied Axially symmetric Bianchi type I cosmological model in presence of perfect fluid and attractive massive scalar field in Lyra manifold. Ram, Chandel and Verma [14] studied Kantowski-sachs universe with anisotropic dark energy in Lyra geometry. Katore and Kapse [8] studied Dynamical Behaviour of coupled magnetized dark energy in Lyra geometry. Agrawal, Pandey and Pradhan [2] studied Bianchi type II perfect fluid cosmological models in normal gauge for Lyra's manifold with constant deceleration parameter. Mete, Bansod and Murade [11] studied Accelerating Anisotropic Bianchi type–V model with Barotropic matter and viscous dark energy in Lyra geometry.Recently, Reddy, Anitla and Umadevi [18], Amirhashchi, Pradhan and Saha [3], Santhi, Rao and Aditya [20], Satish and Venkateswarlu [21], Bali, Pradhan and Amirhashchi [4] examined Bianchi Type VI_0 Cosmological Model. Pradhan and Chouhan [13], Chawala, Mishra and Pradhan [5], Reddy [17], Maurya Zia, and Pradhan [10], Sahoo and Mishra [19], Kanakavalli and Rao [7], Vinutha, Rao and Getaneh [25], Rao and Papa Rao [15] investigated cosmic string model. The motive of the nuclear work is to obtain Bianchi type VI_0 string cosmological model in Lyra's manifold.

This paper is arranged as follows.In section 2, we derive metric and field equation in presence of Lyra's manifolds. Section 3, deals with solutions of field equations. Section 4, contains physical and kinematical properties of the model. Section 5, contains some discussion. The last section contain conclusions.

2. Metric and Field Equations

We considered the Bianchi type VI_0 space time in the form,

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2e^{-2x}dz^2 \tag{1}$$

Where A, B, C are functions of 't' only.
Einstein's modified field equation in normal gauge for Lyra's Manifold obtained by Sen [23] is given by

$$R_i^j - \frac{1}{2}g_i^jR + \frac{3}{2}\phi_i\phi^j - \frac{3}{4}g_i^j\phi_k\phi^k = -T_i^j \tag{2}$$

Where ϕ_i is the displacement vector which is defined as $\phi_i = (0, 0, 0, \beta(t))$ and symbols have their usual meanings as in Riemannian geometry.
The energy-momentum tensor for Cosmic Strings is given by,

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{3}$$

Where $\rho = \rho_P + \lambda$ rest energy density of cloud of string with particle is attached to them, λ is the tension density of the string and ρ_P is the rest energy density of the particle.
The direction of the string will satisfy $u_i u^i = -1 = -x_i x^i$. From equation (3) we get,

$$T_1^1 = -\lambda, T_2^2 = T_3^3 = 0, T_4^4 = -\rho \tag{4}$$

The field equations in Lyra Manifold are given by,

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{1}{A^2} + \frac{3}{4}\beta^2 = \lambda \tag{5}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} + \frac{3}{4}\beta^2 = 0 \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} + \frac{3}{4}\beta^2 = 0 \tag{7}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} - \frac{3}{4}\beta^2 = \rho \tag{8}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{9}$$

Where suffix 4 denotes ordinary differentiation with respect to time t.

3. Solutions of Field Equations

The group of equations (5) to (9) is a system of five independent equations with six unknown A, B, C, β, λ and ρ. In order to get deterministic solution, the condition is used.

$$\lambda + \rho = 0 \quad \text{for Reddy [17]} \tag{10}$$

power law is given by

$$A = B^n, \quad \text{where n is positive integer.} \tag{11}$$

From equation (9) we get,

$$B = \mu C, \quad \text{where } \mu \text{ is integration constant.} \tag{12}$$

From equation (5)-(8) and using (10)-(12) we get,

$$\frac{B_{44}}{B}+k_1\frac{B_4^2}{B^2}=0 \quad \text{where } k_1=n+1 \tag{13}$$

By integrating equation (13) we get

$$B=k_4(k_2t+k_3)^{\frac{1}{k_1+1}} \tag{14}$$

Using (11)and (12) in (14) we get

$$A=k_4^n(k_2t+k_3)^{\frac{n}{k_1+1}} \tag{15}$$

$$C=\frac{k_4}{\mu}(k_2t+k_3)^{\frac{1}{k_1+1}} \tag{16}$$

Where k_1, k_2 and k_3 are integrating constant, $k_4=(k_1+1)^{\frac{1}{(k_1+1)}}$.
Using equation (14)-(16) in equation (6), (5) and (8) are obtained

$$\beta^2=\frac{4}{3}\frac{k_2^2}{(k_1+1)^2(k_2t+k_3)^2}\left[-n^2-nk_1+k_1+\frac{(k_1+1)^2}{k_2^2k_4^{2n}(k_2t+k_3)^{\frac{2(n-k_1-1)}{k_1+1}}}\right] \tag{17}$$

The string tension density and energy density is given by,

$$\rho=\frac{k_2^2}{(k_1+1)^2(k_2t+k_4)^2}\times\left[2n+1-\frac{k_5}{(k_2t+k_3)^{\frac{2(n-k_1-1)}{k_1+1}}}-\left(-n^2-k_1(n-1)+\frac{k_5}{(k_2t+k_3)^{\frac{2(n-k_1-1)}{k_1+1}}}\right)\right] \tag{18}$$

$$\lambda=\frac{k_2^2}{(k_1+1)^2(k_2t+k_4)^2}\times\left[-2k_1+1+\frac{k_5}{(k_2t+k_3)^{\frac{2(n-k_1-1)}{k_1+1}}}-\left(-n^2-k_1(n-1)+\frac{k_5}{(k_2t+k_3)^{\frac{2(n-k_1-1)}{k_1+1}}}\right)\right] \tag{19}$$

Using equations (14), (15) and (16) cosmological model in equation (1) takes the form,

$$ds^2=-dt^2+k_4^{2n}(k_2t+k_3)^{\frac{2n}{k_1+1}}dx^2+k_4^2(k_2t+k_3)^{\frac{2}{k_1+1}}e^{2x}dy^2+\frac{k_4^2}{\mu^2}(k_2t+k_3)^{\frac{2}{k_1+1}}e^{-2x}dz^2 \tag{20}$$

4. Physical and Kinematic Properties

Spatial Volume

$$\nu = \frac{k_4^{n+2}}{\mu}(k_2t + k_3)^{\frac{(n+2)}{(k_1+1)}} \tag{21}$$

Scalar Expansion

$$\theta = \frac{1}{3}\frac{n+2}{k_1+1}\frac{k_2}{(k_2t + k_3)} \tag{22}$$

Hubble Parameter

$$H = \frac{n+2}{k_1+1}\frac{k_2}{(k_2t + k_3)} \tag{23}$$

Shear Scalar

$$\sigma^2 = \frac{(n+2)^2}{54(k_1+1)^2(k_2t + k_3)^2} \tag{24}$$

Deceleration parameter

$$q = \frac{9(k_1+1)k_2 - (n+2)}{n+2} \tag{25}$$

Decomposition of time like tidal tensor

$$u_{a,b} = \frac{k_2k_4^2}{(k_1+1)(k_2t+k_3)}\Big\{nk_4^{2n-2}(k_2t+k_3)^{\frac{2n}{k_1+1}} + (k_2t+k_3)^{\frac{2}{k_1+1}}e^{2x} + (k_2t+k_3)^{\frac{2}{k_1+1}}e^{-2x}\Big\} \tag{26}$$

and Vorticity

$$\omega_{11} = \omega_{22} = \omega_{33} = \omega_{44} = 0 \tag{27}$$

Here Vorticity of model along X, Y, Z and t axes is zero. So, the obtained model is non rotating. Whereas when vorticity is nonzero, model is rotating.

$$\text{Red shift} \quad z = \left(\frac{1}{\frac{k_4^{\frac{n+2}{3}}}{\mu^{\frac{1}{3}}}(k_2t+k_3)^{\frac{n+2}{3(k_1+1)_1}}}\right) - 1 \tag{28}$$

5. Discussion

For the obtained cosmological model physical parameters are found in section 4. In this section, graphs are plotted for the particular value of physical parameters & integration constants $k_1 = k_2 = k_3 = 0.2$ and $\mu = 0.1$.

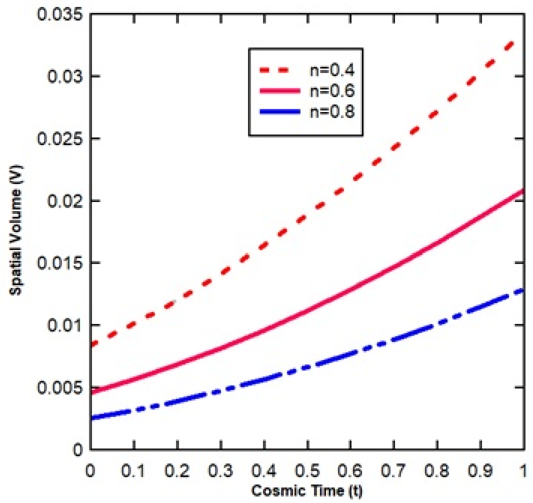

Figure 1: Plot of Spatial Volume Vs. Cosmic Time.

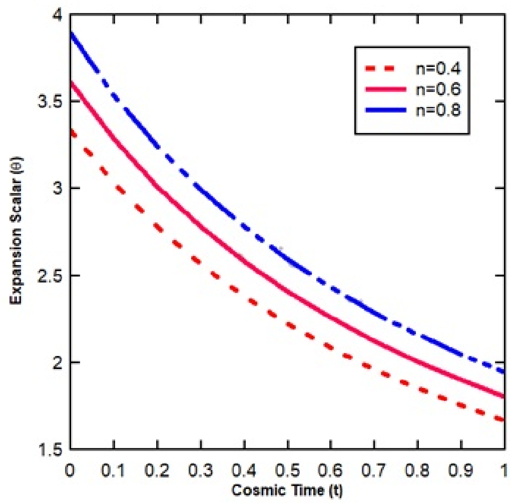

Figure 2: Plot of Expansion Scalar Vs. Cosmic Time

The behaviour of Spatial Volume and Expansion Scalar are represented in Fig. 1 and Fig. 2, as cosmic time increases spatial volume increases. But, expansion scalar decrease with increase in cosmic time and finally vanish at $t \to \infty$.
Spatial volume and Expansion scalar are finite at initial state i.e. $t = 0$.

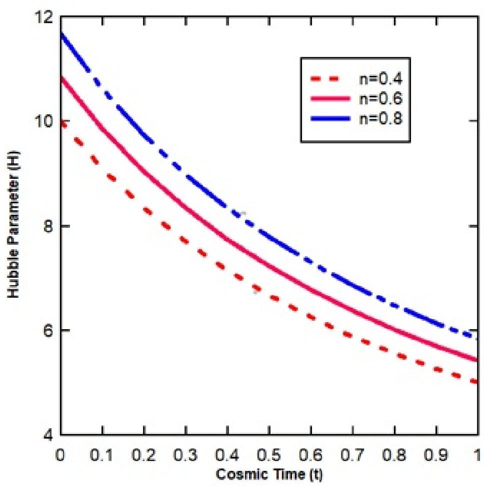

Figure 3: Plot of Hubble Parameter Vs. Cosmic Time.

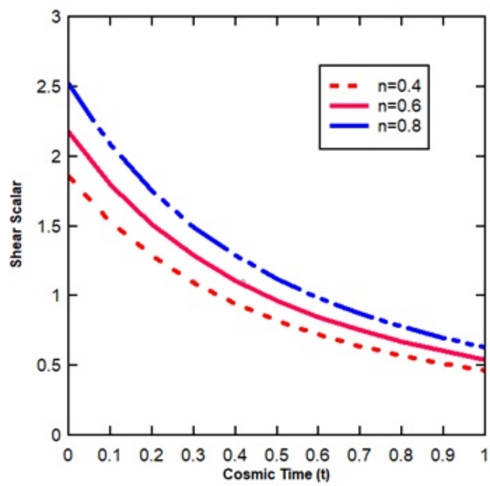

Figure 4: Plot of Shear Scalar Vs. Cosmic Time

The behavior of Hubble Parameter and Shear Scalar are represented in Fig. 3 and Fig. 4, as cosmic time increase Hubble Parameter and Shear scalar decreases with

increase in cosmic time and finally vanishes at $t \to \infty$.
It is observed that, as cosmic time t gradually increases the scalar expansion, Hubble parameter, Shear scalar decreases and vanishes when cosmic time is infinite. Also spatial volume is infinite for infinite cosmic time.

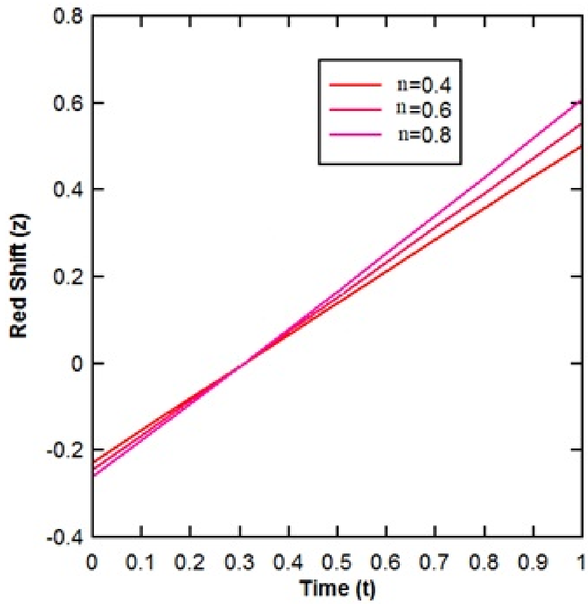

Figure 5: Plot of Red Shift Vs. Cosmic Time

From above Fig. 5, it is shown that how much light is emitted by red shift at particular time t. It is hoped that the formula will be useful for identifying objects at the early universe.

6. Conclusions

In this research work, it is investigated that Bianchi Type-VI_0 cosmological model in Lyra's manifold in presence of Cosmic String. Also, some Physical and Kinematical properties are studied. The Gauge function is calculated and found to the function of Cosmic Time. $\frac{\sigma}{\theta}$ is obtained to be constant and does not approach to the isotropy at any time and the obtained model is observed to be non-singular. Normal congruence to the homogeneous expansion also satisfies that $\frac{\sigma}{\theta}$ is constant similar to [13]. Also, the model is expanding, shearing and non-rotating in which the flow vector is geodesic. At Big Bang that is at $t = 0$ universe starts expanding and the expansion of the universe stops at $t = \infty$.

String energy density ρ is decreasing function of time but it is always positive and approaches to small positive value at present epoch. In early stages the values of Tension density and Hubble parameter is very large and with the expansion of the universe both decreases. Red Shift of the model is examined and is given in equation in (28).

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