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MHD FLOW PAST A VERTICAL PLATE OF CASSON FLUID WITH HEAT AND MASS TRANSFER EFFECTS

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Abstract: The mode of heat transfer will play an important role in the heat engineering applications. The present work is focused on analytical investigation of unsteady heat and mass transfer rate through porous medium in the presence of uniform transverse magnetic field along with radiation/absorption, heat generation/ absorption and homogeneous chemical reaction effects. The coupled nonlinear partial equations into ordinary differential equations by perturbation method. The effects of various parameters on flow characteristics are investigated .The results are presented through various graphs which are plotted for the effect of different parameters on fluid flow. Impact of Casson parameter leads to decrease the fluid velocity. The heavier species with low conductivity reduces the flow within the boundary layer.The Casson parameter is taken due to the significance of non-Newtonian fluids in real time applications in chemical industries and petroleum refineries.

Keywords and Phrases: Casson fluid, thermal radiation, Grashof Number, Porous Medium, MHD.

2020 Mathematics Subject Classification: 35Q30, 35Q35, 65L12, 76S05, 76N20, 80A21, 80M15.

1. Introduction

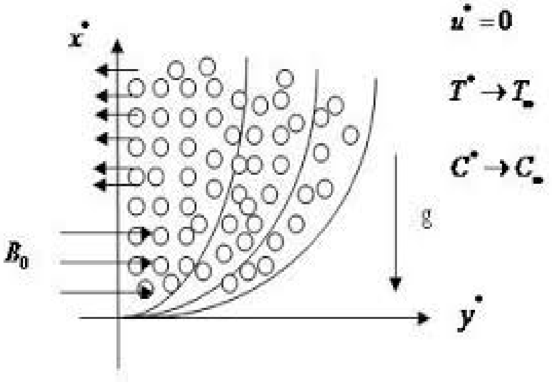
The present essential need for various chemical industries and manufacturing firms is how to reduce or completely eliminate the adverse effects or hazards happened in different chemical reactions takes place during the processing of products. The main cause of this is due to presence of any foreign elements or unexpected agents in the processing. The present investigation has been focused on how the influence of such foreign mater on the heat and mass transfer characteristics of the chemical fluid flows. Faruk Abdullahi et al. [1] have investigated the Casson fluid effects on magneto-hydrodynamics (MHD) unsteady heat and mass transfer free convective past an infinite vertical plate. Hayat et al. [2] presented several aspects by investigating oscillatory rotating flows of a fractional Jeffrey fluid filling a porous space. Saeed Islam et. al [3] have examined the impact of variable thickness and thermal conductivity characteristics in view of melting heat flow on Williamson nano fluid flow. Zeeshan et. al [4] have examined the impact of embedded parameters such as variable thickness, unsteadiness, Prandtl number, Schmidt number, Brownian-motion, and thermophoretic) on thin film nano fluids. Haroon ur Rasheed et.al [5-6] have done the computer analysis to investigate the effect of mathematical abstractions on velocity, energy, concentration and the influence of skin-friction and Nusselt number. Also the effects of Joule heating and viscous dissipation on magnetohydrodynamic boundary Layer Flow of Jeffrey nano fluid. Haroon Ur Rasheed et.al [7-8] have investigated the Joule heating and variable viscosity effect on reactive Casson fluids and also performed the numerical analysis with chemical reaction and Hall current on unsteady MHD flow of Casson fluid. Zeeshan Khan [9] have investigated the effect on physical appearance of material parameters on the flow field, temperature, concentration, drag force, and Nusselt number on convective heat transfer flow of Casson fluid. Abu Zeid et. al [10] have contributed their efforts to investigate the various effects of parameters in the MHD flow of Casson fluid flows.

Nomenclature :
k*=Thermal diffusion ratio;
n* = Frequency of the oscillations;
Ω= Rotation parameter;
k= Thermal conductivity [W.m-1.K-1];
Q=Heat absorption/generation parameter in non-dimensional form;
Q*=Heat absorption/generation parameter in dimensional form;
ρ =Density of the fluid [kg/m-3];

σ=Electrical conductivity of the fluid [ohm-1s-1];
K*=Non-dimensional porosity parameter;
K*p=Porosity parameter in dimensional form;
Cp= Specific heat of the fluid at constant pressure [J.kg-1 K-1];
ν=Kinematic viscosity [m2/s-1];
t*=Time in dimensional form;
g= Acceleration due to gravity [m.s-2];
Gr=Grashof number;
Gc= modified Grashof number;
R= radiation absorption parameter;
M= magnetic parameter;
Pr=Prandtl number;
Sc=Schmidt number,
H=Heat source parameter;
Kc=chemical reaction parameter;
Kp =Permeability of porous parameter;
Superscripts:
* dimensional;

2. Formulation

Let a vertical plate is considered in a fluid flow direction such that assume the –x^*- axis is along the plate and the-y^*- axis is normal to it.The physical model of the present investigation has shown in Fig. A.



Let us consider the magnetic Reynolds number is much less than unity so that induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. It is assumed that initially, at $t^* < 0$, the plate as well as fluids are at the same temperature and also concentration of the species is very low so that the Soret and Dofour effects are neglected. Whent^*, the temperature of the plate is instantaneously raised to ${T_W}^*$and the concentration of the species is to ${C_W}^*$. Let the permeability of the porous medium and the suction velocity be considered in the following forms respectively.

In the present investigation, the presence of radiation absorption and a transverse magnetic field is considered in an unsteady free convective flow of Casson fluid past an infinite vertical porous plate in a porous medium with time dependent oscillatory suction along with the permeability. This kind of environment is mostly present in various industries such as food processing firms, dairy industries, distilleries and beverage industries, polymer fabrication firms, glass manufacturing industries, pharmaceutical industries etc. The novelty of the present study is to analyse the effect of time defendant fluctuate suction and permeability of the medium on a Casson fluid flow in the presence of radiation, heat absorption, radiation absorption and chemical reaction. The Casson parameter is taken due to the significance of non-Newtonian fluids in real time applications in chemical industries and petroleum refineries.

$$K^*(t^*) = {K_p}^*(1+\varepsilon e^{n^*t^*}), \nu^*(t^*) = -\nu_0(1+\varepsilon e^{n^*t^*}) \tag{1}$$

Where $\nu_0 > 0$ and $\varepsilon \leq 1$ are positive constants. Under the above assumptions and with usual Boussinesq's approximation, the governing equations and boundary conditions are given by

$$\frac{\partial u^*}{\partial t^*} = (\frac{1}{1+\beta})\nu\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - {T_\infty}^*) + g\beta^*(C^* - {C_\infty}^*) - \sigma {B_0}^2\frac{u^*}{\rho} - (\frac{1}{1+\beta})\nu\frac{u^*}{k^*} \tag{2}$$

$$\frac{\partial T^*}{\partial t^*}\rho c_p = K\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q^*}{\partial y^*} - Q^*(T^* - {T_\infty}^*) + {Q_l}^*(C^* - {C_\infty}^*) \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_r(C^* - {C_\infty}^*) \tag{4}$$

$$u = f(t) = 1, T^* = T_\infty + \varepsilon(T_W - T_\infty)e^{n^*t^*}, C^* = C_\infty + \varepsilon(C_W - C_\infty)e^{n^*t^*} \text{at } y = 0\backslash n \tag{5}$$

Introducing the non-dimensional quantities,

$$y = \frac{\nu_0 t^*}{\nu}, t = \frac{{\nu_0}^2 t^*}{4\nu}, w = \frac{4\vartheta w^*}{{\nu_0}^2}, u = \frac{u^*}{\nu_0}, T = \frac{T^* - {T_\infty}^*}{T_W - T_\infty}, C = \frac{C^* - {C_\infty}^*}{C_W - C_\infty},$$

$$S = \frac{\vartheta S^*}{{\nu_0}^2}, K_p = \frac{\nu_0^2 K_p^2}{\nu^2}, M^2 = \sigma\frac{B_0^2\nu}{\nu_0^2\rho}, P_r = \frac{\nu}{K}, S_c = \frac{V}{D}, R_c = \frac{\nu_0^2 K_0}{\nu^2\rho},$$

$$G_c = \frac{\nu g\beta(C_W - C_\infty)}{\nu_0^3}, G_r = \frac{\nu g\beta(T_W - T_\infty)}{\nu_0^3}, F = \frac{4I_1\nu}{\nu_0^2\rho C_p}, s = \frac{Q\nu}{\nu_0^2\rho C_p},$$

$$R = \frac{Q_1\nu(C_W^* - C_\infty)}{\nu_0^2\rho(T_W^* - T_\infty)}, K_c = \frac{k_r\nu}{\nu_0^2}, H = F + S \tag{6}$$

The equations (2)-(5) reduce to following non-dimensional form

$$\frac{1}{4}\frac{\partial u}{\partial t} = (\frac{1}{1+\beta})\frac{\partial^2 u}{\partial y^2} + G_r T + G_c C - (M^2 + \frac{1}{K_p})u \tag{7}$$

$$\frac{1}{4}\frac{\partial T}{\partial t} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} - HT + RC \tag{8}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_c C \tag{9}$$

$$u = f(t) = 1, T = 1 + \varepsilon e^{\mathrm{nt}}, C = 1 + \varepsilon e^{\mathrm{nt}} \quad \mathrm{y} = 0\backslash n \tag{10}$$

3. Solution

In view of periodic suction, temperature and concentration at the plate let us assume the velocity, temperature, concentration the neighborhood of the plate be

$$u(y,t) = u_0(y) + \varepsilon e^{\mathrm{nt}}u_1(y), T(y,t) = T_0(y) + \varepsilon e^{\mathrm{nt}}T_1(y), \text{ and } C(y,t) = C_0(y) + \varepsilon e^{\mathrm{nt}}C_1(y) \tag{11}$$

Substituting equations (11) into (7-9) and comparing the no harmonic & harmonic terms we get

$$(\frac{1}{1+\beta}){u_0}^{11} - (M^2 + \frac{1}{K_p})u_0 = -G_r T_0 - G_c C_0 \tag{12}$$

$$(\frac{1}{1+\beta}){u_1}^{11} - (M^2 + \frac{1}{K_p} + \frac{n}{4})u_1 = -G_c C_1 - G_r T_1 \tag{13}$$

$${T_0}^{11} - P_r H T_0 = -RP_r C_0 \tag{14}$$

$${T_1}^{11} - (H + \frac{n}{4})P_r T_1 = -RP_r C_1 \tag{15}$$

$${C_0}^{11} - K_c S_c C_0 = 0 \tag{16}$$

$${C_1}^{11} - (K_c + \frac{n}{4})S_c C_1 = 0 \tag{17}$$

The boundary conditions now reduce to

$$u_0 = 1, u_1 = 0, T_0 = T_1 = 1, C_0 = C_1 = 1, \quad \text{at } \ y = 0\backslash n \tag{18}$$

Solving these differential equations (12)-(18) with the help of boundary conditions we get

$$u(y,t) = (1-b_3-b_4)e^{-\sqrt{a_5}y} + b_3e^{-\sqrt{a_3}y} + b_4e^{-\sqrt{a_1}y} + \varepsilon e^{\mathrm{nt}}\left\{(-b_5-b_6)e^{-\sqrt{a_8}y} + b_5e^{-\sqrt{a_4}y} + b_6e^{-\sqrt{a_2}y}\right\} \tag{19}$$

$$T(y,t) = (1-b_1)e^{-\sqrt{a_3}y} + b_1e^{-\sqrt{a_1}y} + \varepsilon e^{\mathrm{nt}}\left\{(1-b_2)e^{-\sqrt{a_4}y} + b_2e^{-\sqrt{a_2}y}\right\} \tag{20}$$

$$C(y,t) = e^{-\sqrt{a_1}y} + \varepsilon e^{\mathrm{nt}}\left\{e^{-\sqrt{a_2}y}\right\} \tag{21}$$

The skin friction at the plate in terms of amplitude and phase angle is given by $\tau = \frac{\partial u_0}{\partial y} + \varepsilon e^{\mathrm{nt}}\frac{\partial u_0}{\partial y}$, at $y = 0$

$$(\tau = [-(1-b_3-b_4)\sqrt{a_5} - b_3\sqrt{a_3} - b_4\sqrt{a_1}] + \varepsilon e^{\mathrm{nt}}[(b_5-b_6)\sqrt{a_8} - b_5\sqrt{a_4} - b_6\sqrt{a_2}] \tag{22}$$

The rate of heat transfer. That is heat flux at the N_u in terms of amplitude and phase is given by $N_u = -\left[\frac{\partial T_0}{\partial y} + \varepsilon e^{\mathrm{nt}}\frac{\partial T_1}{\partial y}\right]$ at $y = 0$

$$N_u = [(1-b_3)\sqrt{a_3} + b_1\sqrt{a_1}] + \varepsilon e^{\mathrm{nt}}[(1-b_2)\sqrt{a_4} + b_2\sqrt{a_2}] \tag{23}$$

The mass transfer coefficient, that is the Sherwood number S_h at the plate in terms of amplitude and phase is given by

$$S_h = -\left[\frac{\partial C_0}{\partial y} + \varepsilon e^{\mathrm{nt}}\frac{\partial C_1}{\partial y}\right] \text{ at } y = 0$$

$$S_h = [\sqrt{a_1}] + \varepsilon e^{\mathrm{nt}}[\sqrt{a_2}] \tag{24}$$

4. Results and Discussions

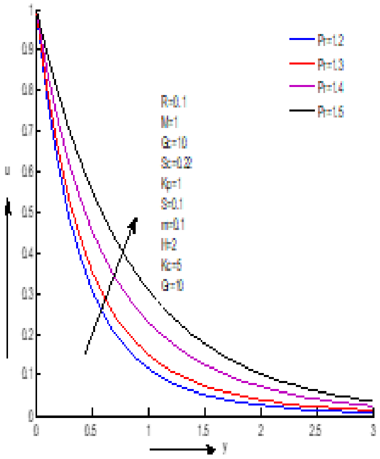


Fig. 1. Effect of Pr on Velocity

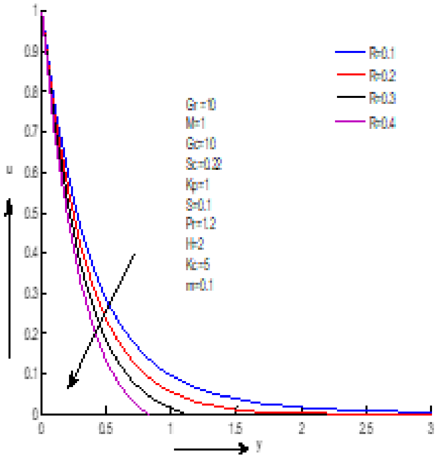


Fig. 2. Effect of Ron On Velocity

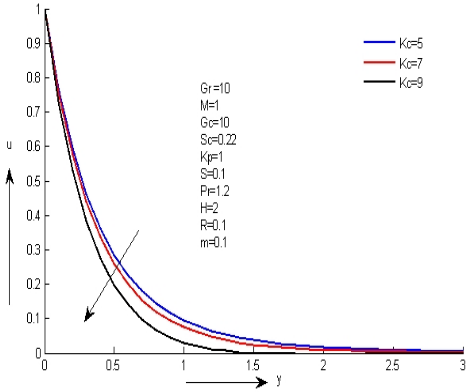


Fig. 3. Effect of Kc on Velocity

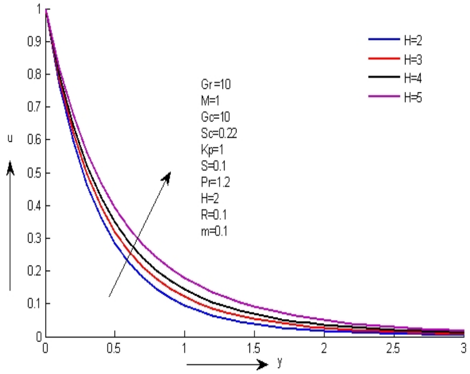


Fig. 4. Effect of H on Velocity

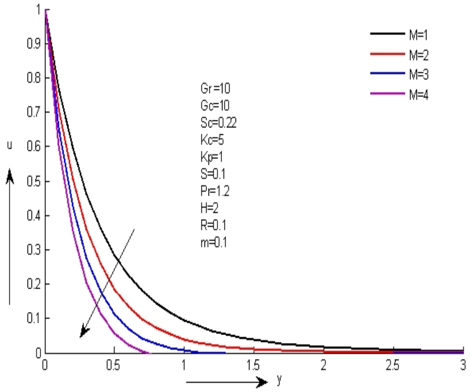


Fig. 5. Effect of M on Velocity

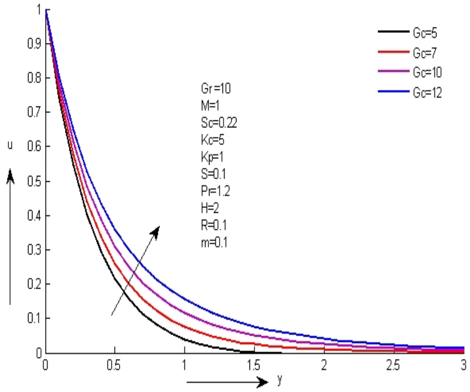


Fig. 6. Effect of Gc on Velocity

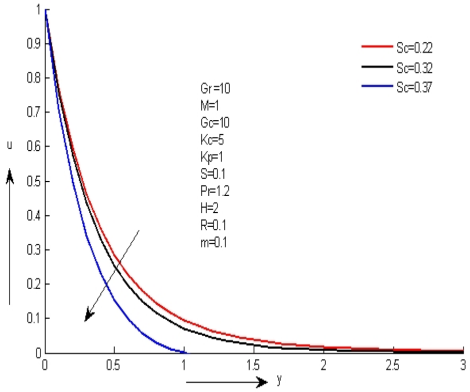


Fig. 7. Effect of Sc on Velocity

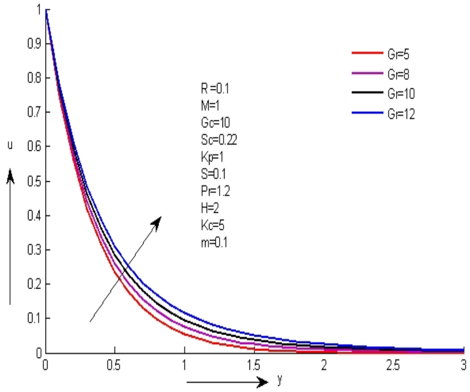


Fig. 8. Effect of Gr on Velocity

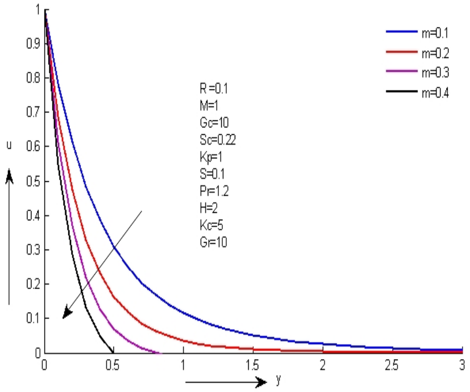


Fig. 9. Effect of β on Velocity

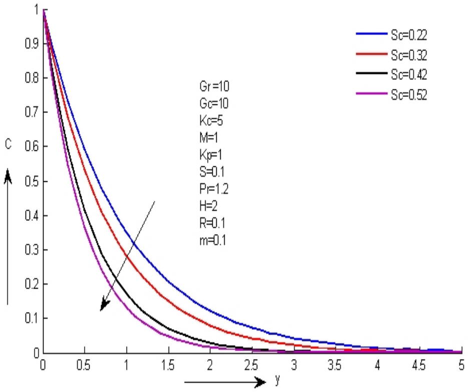


Fig. 10. Effect of Sc on Cnc

In order to assess the effects of the dimensionless thermo physical parameters on the regime calculations have been carried out on velocity field, temperature field and concentration field for varies physical parameters like magnetic parameter, Prandtl parameter, Grashof number, modified Grashof number, chemical reaction parameter etc. The results are represented through graphs in figures 1 to 10. From figure 1 shows that velocity increases for the increasing values of Pr number. From figure 2 it is observed that velocity decreases with the increasing values of R. From figure 3 shows that velocity decreases for increasing values of chemical reaction parameter. From figure 4 displays that velocity increases in chemical reaction. Figure 5, displays the velocity profiles for varies values of magnetic parameter M. It is observed that the velocity decreases with an increase in M. This is due to fact that the applied magnetic field which acts as retarding force that condenses the momentum boundary layer. From figure 6, it is displays that the velocity increases with an increases in Gc number. A similar effect is noticed from figure 7, in the presence of Schmidt number where velocity decreases. Figure 8, depicts the effects of Grashof number on velocity, from this figure it is observed that the velocity increases with an increase in Gr. From figure 9 it is noticed that the velocity decreases with an increase in m. Effect of Schmidt number parameter on concentration is presented in figure 10, which witnesses that concentration decreases as the values of Sc increase.

5. Conclusions

The present investigation has excellently helps to improve the heat and mass transfer effects in unsteady MHD free convection flow of Casson fluid past on a porous plate. It is mainly observed from the investigation in the presence of the heavier species with low conductivity in fluid flow results the reduction the flow rate with in the boundary layer. It is also observed that an increment in the elasticity of the fluid results the decrease the velocity of fluid flow. Hence it is finally concluded that the impact of Casson parameter leads to reduce the fluid velocity of flow.

Appendix:
$a_1 = S_c K_c`$, $a_2 = (K_c + \frac{n}{4}) S_c$ $a_3 = P_r H$, $a_4 = (H + \frac{n}{4}) P_r$, $a_5 = (M^2 + \frac{1}{K_P})/(\frac{1}{1+\beta})$,$a_6 = -Gr/(\frac{1}{1+\beta})$,$a_7 = -Gc/(\frac{1}{1+\beta})$,$a_8 = (M^2 + \frac{1}{K_P} + \frac{n}{4})/(1+\beta)$, ,$b_1 = \dfrac{-RP_r}{a_1 - a_3}$, $b_2 = \dfrac{-RP_r}{a_2 - a_4}$ $b_3 = \dfrac{a_6(1-b_1)}{a_3 - a_5}$,$b_4 = \dfrac{a_6 b_1 - a_7}{a_3 - a_5}$, $b_5 = \dfrac{a_6(1-b_2)}{a_4 - a_8}$, $b_6 = \dfrac{a_6 b_2 + a_7}{a_2 - a_8}$

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