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ANALYTICAL AND APPROXIMATE SOLUTIONS FOR CONFORMABLE FRACTIONAL ORDER CORONA-VIRUS (COVID-19) EPIDEMIC MODEL

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Abstract: In this investigation, we discussed the SARS-CoV-2 virus into a system of equations and we apply the Conformable Fractional Differential Transformation Method (CFDTM) to COVID-19 mathematical model described by the system of non-linear conformable fractional order differential equations. The aspire of this study is to estimate the effectiveness of preventive measures, predicting future outbreaks and potential control strategies using the mathematical model. The impacts of various biological parameters on transmission dynamics of COVID-19 is examined. These results are based on different values of the fractional parameter and serve as a control parameter to identify the significant strategies for the control of the disease. In the end, the obtained results are demonstrated graphically to justify our theoretical findings.

Keywords and Phrases: Mathematical models, Epidemic model, Corona-virus (COVID - 19), $\alpha-$differentiable, Conformable Fractional Differential Transform Method.

2020 Mathematics Subject Classification: 34F05, 92D30, 65P20.

1. Introduction

The new Severe Acute Respiratory Syndrome Corona-virus (SARS-CoV-2), which causes the COVID-19 disease, was first reported in December 2019 at Wuhan city, China. This new pathogen has spread rapidly around over 200 countries. The disease was named as Corona-virus disease 2019 (COVID-19) by the World Health Organization (WHO) on 11 February 2020. Since then it killed over 9,391 on March 19 over the infected cases of 2,23,082 people in additional 180 countries [50, 51]. Moreover, while referring website of WHO it gives the data that globally, as of $2:00$ am CEST, 27 April 2020, there are $2,858,635$ confirmed cases of COVID-19, including $1,96,295$ deaths in additional 213 countries. As on date, there's no particular treatment, medicine or vaccine to cure infected patients completely and every day, there is always exponential increase in the death of the deceased people. Initially, every infected person has high fever, cough and shortness within the breath. The virus transmitted by touching the body of diseased patients to the uninfected person to his/her eyes, nose, mouth and a few other parts. So as to manage the spread of the virus, each country has taken most initializations and public health concerns are being paid globally on what percentage people are infected and suspected to forestall or avoid its effects on humankind. Since the beginning start of the outbreak in Wuhan, several modeling groups round the world have reported estimations and predictions for the COVID-19 (formerly called 2019-nCov) epidemic in journal publications or on websites, one can refer [12, 16, 17, 20, 34, 35, 38, 42]. With the efforts to explore clinical nature of the virus, a considerable amount of time and energy is being dedicated to understanding the mathematical nature of the transmission of the disease, the studies based on an extended SEIR model with fractional order are discussed in [5, 11, 15, 31, 37].

Mathematical models play an increasingly important role in our understanding of the transmission and control of infectious diseases [28]. Epidemiological models are studied by Mathematics are constructive in comprising, proposing, planning, implementing, testing theories, prevention, evaluating a range of detection, therapy and control programs. On the other hand, to study, examine, analyze, predict and capture the behaviour of viruses, diseases, threads et al., the mathematics is that the only tool that may help us in systematics, effective and accurate manner without much expense. To detect and cure those diseases properly, we want a good method to resolve these models. For the answer of the system of linear and nonlinear differential equations, there are many methods like exact, approximate and purely numerical, stochastic models are available. Most of those are computationally intensive or need complicated symbolic computations. Generally, the exact solutions of those models are unavailable and usually are very tough.

In 1986, Zhou [53] introduced differential transform method to solve linear and non-linear initial value problems in electric circuit analysis. Following the work of Zhou [53], there were several works in literature [1, 9, 19, 25, 36, 39-41, 43, 48, 52] and references therein for history and properties of DTM. Especially, this method got much attention to solve epidemic models [4, 6, 7, 13, 14, 18, 23, 26, 29, 30, 41, 45] also one can refer the references therein for more details. To speak about the advantages and generic nature of the DTM, it is worthwhile to mention that this method is directly applied to either linear or nonlinear ODEs and PDEs. We obtain a closed form series solution or approximate solution by using DTM method. This method obtains an analytical solution in the form of a polynomial and it is possible to obtain highly accurate results or exact solutions for linear and non-linear differential equations. Another important advantage is that, this method is capable of reducing the size of computational work and still accurately provides the series solution with fast convergence rate.

In 2007, Arikoglu [8] developed a new analytical technique namely, Fractional Differential Transform Method (FDTM) to solve fractional differential equations (see also [22]). Motivated in this line Ünal and Gökdoğan [47] developed a new technique conformable fractional derivative [32]. It is called as conformable fractional differential transform method (CFDTM), formulizes conformable fractional power series likewise FDTM formulizes fractional power series and DTM formulizes Taylor series. One can refer the recent studies and its application in [2, 21, 24, 27, 44-46] and the references therein.

2. On the Conformable Fractional Derivative

Employing the fundamental limit definition of the classical derivative, the newfangled conformable fractional derivative was introduced by Khalil et al. [32]. Further properties were discussed thoroughly by Abdeljawad [3]. From the time when the pioneering work was done, numerous works were discussed, and various problems were solved and that could be found in [2, 10, 21, 27, 33, 44-47] and the references there in for recent results on conformable fractional derivative. Now we provide the definition from [32]:

Given a function $f : [0,\ \infty) \rightarrow \mathbb{R}$, then for all $t > 0$, $\alpha \in (0,\ 1]$, let

$$\mathcal{T}_\alpha\ (f)\,(t) := \lim_{\varepsilon \to 0} \frac{f\left(t + \varepsilon\, t^{1-\alpha}\right) - f(t)}{\varepsilon}$$

$\mathcal{T}_\alpha$ is called the conformable fractional derivative of f order α. If f is $\alpha-$ differentiable in some $(0,\ a)$, $a > 0$, and $\lim_{t \to 0^+} \mathcal{T}_\alpha(f)(t)$ exists, then define

$$\mathcal{T}_\alpha(f)(0) = \lim_{t \to 0^+}\ \mathcal{T}_\alpha(f)(t).$$

The required fundamental properties are given below due to [32].

(1) $\mathcal{T}_\alpha(m\,f \pm n\,g)(t) = m\,\mathcal{T}_\alpha(f)(t) \pm n\,\mathcal{T}_\alpha(g)(t)$, $\forall\, m,\ n \in \mathbb{R}$.

(2) $\mathcal{T}_\alpha(t^p) = pt^{p-\alpha}$, $\forall p \in \mathbb{R}$.

(3) $\mathcal{T}_\alpha(f\,g)(t) = f(t)\,\mathcal{T}_\alpha(g)(t) + g(t)\,\mathcal{T}_\alpha(f)(t)$.

(4) $\mathcal{T}_\alpha\left(\frac{f}{g}\right)(t) = \dfrac{g(t)\,\mathcal{T}_\alpha(f)(t) - f(t)\,\mathcal{T}_\alpha(g)(t)}{g^2(t)}$.

(5) If $f(t)$ is constant function, then $\mathcal{T}_\alpha(f)(t) = 0$.

(6) $\mathcal{T}_\alpha(f)(t) = t^{\alpha-1}\dfrac{df(t)}{dt}$, if $f(t)$ is differentiable.

3. Conformable Fractional Differential Transform Method

In order to expand the analytical and continuous function $f(t)$ in terms of a factional power series, we consider the basic definition and its properties of the conformable fractional one-dimensional differential transform method introduced and studied by Ünal and Gökdoğan [47].

Definition 1. [47] *If we suppose that the function $f(t)$ is infinitely $\alpha-$differentiable function for some $\alpha \in (0,\ 1]$, then the conformable fractional differential transform of $f(t)$ is defined as follows:*

$$F_\alpha(k) = \frac{1}{\alpha^k k!}\left[\left(T_\alpha^{t_0}\, f\right)^{(k)}(t)\right]_{t=t_0}, \tag{3.1}$$

where $\left(T_\alpha^{t_0}\, f\right)^{(k)}(t)$ denotes the kth iterate of the fractional derivative of $\left(T_\alpha^{t_0}\, f\right)(t)$ for a function $f\ :\ [t_0,\ \infty) \to \mathbb{R}$ given by

$$\left(T_\alpha^{t_0}\, f\right)(t) := \lim_{\varepsilon\to 0}\left\{\frac{f\left(t + \varepsilon\,(t-t_0)^{1-\alpha}\right) - f(t)}{\varepsilon}\right\}, \quad t > t_0 \geq 0; \quad 0 < \alpha \leq 1. \tag{3.2}$$

Definition 2. [47] *If $F_\alpha(k)$ denotes the conformable fractional differential transform of the function $f(t)$ given by Definition 1, then the inverse fractional differential transform of $F_\alpha(k)$ is defined by*

$$f(t) = \sum_{k=0}^{\infty} F_\alpha(k)(t-t_0)^{\alpha\,k} = \sum_{k=0}^{\infty}\frac{1}{\alpha^k k!}\left[\left(T_\alpha^{t_0}\, f\right)^{(k)}(t)\right]_{t=t_0}(t-t_0)^{\alpha\,k}. \tag{3.3}$$

By applying Definitions 1 and 2, we are led to Definition 3 below:

Definition 3. [47] *The conformable fractional differential transform (CFDT) of the initial conditions for integer-order derivatives are defined as follows:*

$$F_\alpha(k) = \begin{cases} \frac{1}{(\alpha\, k)!} \left[\frac{d^{\alpha k}\{f(t)\}}{dt^{\alpha k}}\right]_{t=t_0}; & \alpha k \in \mathbb{N} \\ 0; & \alpha k \notin \mathbb{N}, \end{cases} \quad \textit{for} \quad k = 0,\ 1, \cdots,\ \left[\frac{n}{\alpha}\right] - 1, \tag{3.4}$$

where $\mathbb{N}$ *denotes the set of positive integers and* n *is the order of the corresponding fractional differential equation.*

From (3.1) and (3.3), we present following basic properties due to [47].

(1) If $z(t) = m\, f(t) \pm n\, g(t)$, then $Z_\alpha(k) = m\, F_\alpha(k) \pm n\, G_\alpha(k)$, where m and n are constants

(2) If $z(t) = m\, f(t)$, then $Z_\alpha(k) = m\, F_\alpha(k)$. where m is constant.

(3) If $z(t) = T_\alpha^{t_0}\,(f(t))$, then $Z_\alpha(k) = \alpha\,(k+1)\;\; F_\alpha(k+1)$.

(4) If $z(t) = T_\gamma^{t_0}\,(f(t))$ for $m < \gamma \le m+1$ then

$$Z_\alpha(k) = \frac{\Gamma(k\,\alpha + \gamma + 1)}{\Gamma(k\,\alpha + \gamma - m)}\, F_\alpha\left(k + \frac{\gamma}{\alpha}\right).$$

(5) If $z(x) = f(t)\; g(t)$, then $Z_\alpha(k) = \sum_{l=0}^{k} F_\alpha(l)\; G_\alpha(k-l)$.

(6) If $z(t) = (t - t_0)^{m\,\alpha}$, then $Z_\alpha(k) = \delta(k-m)$, where $\delta(k-m) := \begin{cases} 1 & : \; k = m; \\ 0 & : \; k \neq m. \end{cases}$

Motivated by the above useful applications of the conformable fractional differential transformation method, in this article, we discuss the solutions of dynamics of novel corona-virus model suggested by Khan et al. [34] in the form of the system of the fractional order nonlinear differential equations and approximating the solutions in a sequence of time intervals.

4. Mathematical Formulation

In the present investigation, we consider the compartmental mathematical model (epidemic model) developed by Khan and Atangana [34] for understanding the transmission of virus and presented and derived some interesting results for the projected model by comparison with some practical values (see also [16, 35, 42, 49] as a system of nonlinear ordinary differential equations :

$$\frac{d\,s}{dt} = b_r - \gamma\, s(t) - \frac{\lambda\, s(t)\, (i(t) + \xi\, a(t))}{N} - \zeta\, s(t)\, m(t) \tag{4.1}$$

$$\frac{d\,e}{dt} = \frac{\lambda s(t)\, (i(t) + \xi a(t))}{N} + \zeta s(t)\, m(t) - (1-\upsilon)\, \varphi e(t) - \upsilon\phi e(t) - \gamma e(t) \tag{4.2}$$

$$\frac{d\,i}{dt} = (1-\upsilon)\, \varphi\, e(t) - (\varrho + \gamma)\, i(t) \tag{4.3}$$

$$\frac{d\,a}{dt} = \upsilon\, \phi\, e(t) - (\varsigma + \gamma)\, a(t) \tag{4.4}$$

$$\frac{d\,r}{dt} = \varrho\, i(t) + \varsigma a(t) - \gamma r(t) \tag{4.5}$$

$$\frac{d\,m}{dt} = \chi\, i(t) + \psi\, a(t) - \varpi\, m(t), \tag{4.6}$$

where $N = S(0)+E(0)+I(0)+A(0)+R(0)+M(0)$ is a total number of populations at a time t, is divided into the following six compartments: $s(t)$ the susceptible people; $e(t)$ the exposed people; $i(t)$ the infected strength; $a(t)$ the asymptotically infected people; $r(t)$ the recovered people; $m(t)$ the reservoir; and the parameters b_r is the rate of birth; γ is rate of death of infected population; λ is the transmission coefficient; ξ is transmissibility multiple; ϕ is the transmission rate becomes infected; φ is the incubation period; υ is the amount of asymptomatic infection; ζ is the disease transmission coefficient; ϱ is recovery rate; ς is asymptotically infected population; χ, ψ is the influence of virus to m by i and a; ϖ is the rate of virus removing from m. Parameterized in equations (4.1) - (4.6) and their corresponding values are $b_r = \gamma * N$ $\gamma = \frac{1}{76.79 \times 365}$; $\lambda = 0.05$; $\xi = 0.02$; $\zeta = 0.000001231$; $\upsilon = 0.1243$; $\varphi = 0.00047876$; $\phi = 0.005$; $\varrho = 0.09871$; $\varsigma = 0.854302$; $\chi = 0.000398$; $\psi = 0.001$; and $\varpi = 0.01$, where the total population may vary with time (t) (see previous study [34]).

In this article we base our investigation upon the system of equations given in (4.1) - (4.6) [34]. The following system of generalized epidemic model is introduced by applying the conformal fractional derivative ${}_t\mathcal{T}_\alpha$ of order α $(0 < \alpha \leq 1)$ to the system of equations in (4.1) - (4.6) [32, 45]:

$${}_t\mathcal{T}_\alpha s(t) = b_r - \gamma\, s(t) - \frac{\lambda\, s(t)\, (i(t) + \xi\, a(t))}{N} - \zeta\, s(t)\, m(t) \tag{4.7}$$

$${}_t\mathcal{T}_\alpha e(t) = \frac{\lambda s(t)\, (i(t) + \xi a(t))}{N} + \zeta s(t) m(t) - (1-\upsilon)\, \varphi e(t) - \upsilon\phi e(t) - \gamma e(t) \tag{4.8}$$

$$ {}_t\mathcal{T}_\alpha i(t) = (1-\upsilon)\,\varphi\, e(t) \; - \; (\varrho+\gamma)\, i(t) \tag{4.9} $$
$$ {}_t\mathcal{T}_\alpha a(t) = \upsilon\,\phi\, e(t) \; - \; (\varsigma+\gamma)\, a(t) \tag{4.10} $$
$$ {}_t\mathcal{T}_\alpha r(t) = \varrho\, i(t) \; + \; \varsigma a(t) \; - \; \gamma r(t) \tag{4.11} $$
$$ {}_t\mathcal{T}_\alpha m(t) = \chi\, i(t) \; + \; \psi\, a(t) \; - \; \varpi\, m(t), \tag{4.12} $$

where the parameters are b_r, γ, λ, ξ, ζ, υ, φ, ϕ, ϱ, ς, χ, ψ and ϖ are positive and the initial conditions are considered from [34] and as follows:

$$ \begin{cases} S(0) = 8065518, & E(0) = 200000, \quad I(0) = 282 \\ A(0) = 200, & R(0) = 0, \qquad\quad M(0) = 50000. \end{cases} \tag{4.13} $$

Let $S_\alpha(k)$, $E_\alpha(k)$, $I_\alpha(k)$, $A_\alpha(k)$, $R_\alpha(k)$ and $M_\alpha(k)$ denote the conformal fractional differential transformations of $s(t)$, $e(t)$, $i(t)$, $a(t)$, $r(t)$ and $m(t)$ respectively. Then applying the fundamental operations of conformable fractional differential transformation method, discussed in Section 3 to each equations of the system (4.7) - (4.12), we have

$$ \begin{aligned} S_\alpha(k+1) =& \frac{1}{\alpha\,(k+1)} \Bigg[b_r\delta(k) - \gamma S_\alpha(k) - \frac{\lambda}{N}\sum_{l=0}^{k} S_\alpha(l) I_\alpha(k-l) \\ & - \frac{\lambda}{N}\xi \sum_{l=0}^{k} S_\alpha(l) A_\alpha(k-l) - \zeta \sum_{l=0}^{k} S_\alpha(l) M_\alpha(k-l) \Bigg] \end{aligned} \tag{4.14} $$

$$ \begin{aligned} E_\alpha(k+1) =& \frac{1}{\alpha(k+1)} \Bigg[\frac{\lambda}{N}\sum_{l=0}^{k} S_\alpha(l) I_\alpha(k-l) + \frac{\lambda}{N}\xi \sum_{l=0}^{k} S_\alpha(l) A_\alpha(k-l) \\ & + \zeta \sum_{l=0}^{k} S_\alpha(l) M_\alpha(k-l) - (1-\upsilon)\varphi E_\alpha(k) - \upsilon\phi E_\alpha(k) - \gamma E_\alpha(k) \Bigg] \end{aligned} \tag{4.15} $$

$$ I_\alpha(k+1) = \frac{1}{\alpha(k+1)} \left[(1-\upsilon)\varphi E_\alpha(k) - (\varrho+\gamma) I_\alpha(k) \right] \tag{4.16} $$

$$ A_\alpha(k+1) = \frac{1}{\alpha(k+1)} \left[\upsilon\phi E_\alpha(k) - (\varsigma+\gamma) A_\alpha(k) \right] \tag{4.17} $$

$$ R_\alpha(k+1) = \frac{1}{\alpha(k+1)} \left[\varrho I_\alpha(k) + \varsigma A_\alpha(k) - \gamma R_\alpha(k) \right] \tag{4.18} $$

$$ M_\alpha(k+1) = \frac{1}{\alpha(k+1)} \left[\chi I_\alpha(k) + \psi A_\alpha(k) - \varpi M_\alpha(k) \right]. \tag{4.19} $$

Now, we consider the initial conditions from [34] and which reduces to $S(0) = 8065518$, $E(0) = 200000$, $I(0) = 282$, $A(0) = 200$, $R(0) = 0$ and $M(0) = 50000$. Further, we take the parameter values from [34] to solve $S_\alpha(k+1)$, $E_\alpha(k+1)$, $I_\alpha(k+1)$, $A_\alpha(k+1)$, $R_\alpha(k+1)$ and $M_\alpha(k+1)$ in (4.14) - (4.19) up to certain order, we get, $S_\alpha(k)$, $E_\alpha(k)$, $I_\alpha(k)$, $A_\alpha(k)$, $R_\alpha(k)$ and $M_\alpha(k)$ respectively.

Then the series form of the solution, can be written as

$$s(t) = \sum_{k=0}^{\infty} S_\alpha(k) t^{\alpha k} = 8065518 - 5151695.006\,\frac{t^\alpha}{\alpha} + 1647850.696\,\frac{t^{2\alpha}}{\alpha^2} + \cdots \tag{4.20}$$

$$e(t) = \sum_{k=0}^{\infty} E_\alpha(k)\, t^{\alpha k} = 200000 + 380795.3031\,\frac{t^\alpha}{\alpha} - 271129.8298\,\frac{t^{2\alpha}}{\alpha^2} + \cdots \tag{4.21}$$

$$i(t) = \sum_{k=0}^{\infty} I_\alpha(k)\, t^{\alpha k} = 282 - 106.7610111\,\frac{t^\alpha}{\alpha} + 115.9054943\,\frac{t^{2\alpha}}{\alpha^2} + \cdots \tag{4.22}$$

$$a(t) = \sum_{k=0}^{\infty} A_\alpha(k)\, t^{\alpha k} = 200 - 162.0035330\,\frac{t^\alpha}{\alpha} + 234.2876001\,\frac{t^{2\alpha}}{\alpha^2} + \cdots \tag{4.23}$$

$$r(t) = \sum_{k=0}^{\infty} R_\alpha(k)\, t^{\alpha k} = 198.6966200\,\frac{t^\alpha}{\alpha} - 131.8145616\,\frac{t^{2\alpha}}{\alpha^2} + \cdots \tag{4.24}$$

$$m(t) = \sum_{k=0}^{\infty} M_\alpha(k)\, t^{\alpha k} = 50000 - 499.6877640\,\frac{t^\alpha}{\alpha} + 2.396191612\,\frac{t^{2\alpha}}{\alpha^2} + \cdots . \tag{4.25}$$

Taking $\alpha = 1$ in the obtained solutions (4.20) - (4.25), we have the following forms:

$$s(t) = 8065518 - 5151695.006\,t + 1647850.696\,t^2 + \cdots \tag{4.26}$$
$$e(t) = 200000 + 380795.3031\,t - 271129.8298\,t^2 + 87137.53826\,t^3 + \cdots \tag{4.27}$$
$$i(t) = 282 - 106.7610111\,t + 115.9054943\,t^2 + \cdots \tag{4.28}$$
$$a(t) = 200 - 162.0035330\,t + 234.2876001\,t^2 \tag{4.29}$$
$$r(t) = 198.6966200\,t - 131.8145616\,t^2 + 95.89294214\,t^3 + \cdots \tag{4.30}$$
$$m(t) = 50000 - 499.6877640\,t + 2.396191612\,t^2 + 0.08548535690\,t^3 + \cdots . \tag{4.31}$$

5. Results and Discussion

In this section, we evaluated the tabular values and figures obtained by CFDTM. Tables 1, 2, 3, 4, 5 and 6 shows the solution for $s(t)$, $e(t)$, $i(t)$, $a(t)$, $r(t)$ and $m(t)$ respectively obtained by DTM and we depict the solution by for $s(t)$, $e(t)$, $i(t)$, $a(t)$, $r(t)$ and $m(t)$ in Figures 1, 2, 3, 4, 5 and 6 respectively.

t	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
0	8.065518×10^6	8.065518×10^6	8.065518×10^6	8.065518×10^6
0.2	6.033369096×10^6	6.484589990×10^6	6.833065410×10^6	7.101093027×10^6
0.4	5.122725733×10^6	5.565943137×10^6	5.947133419×10^6	6.268496109×10^6
0.6	4.563350224×10^6	4.923185848×10^6	5.262212518×10^6	5.567727247×10^6
0.8	4.230875570×10^6	4.480399392×10^6	4.744329597×10^6	4.998786440×10^6
1	4.068914309×10^6	4.200665954×10^6	4.375796013×10^6	4.561673690×10^6

Table 1: Table of susceptible population $s(t)$ at different values of α.

t	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
0	2.00000×10^5	2.00000×10^5	2.00000×10^5	2.00000×10^5
0.2	3.268433550×10^5	3.026661389×10^5	2.824740027×10^5	2.660109677×10^5
0.4	3.701154772×10^5	3.497761938×10^5	3.312247590×10^5	3.145141508×10^5
0.6	3.967127917×10^5	3.791776166×10^5	3.637987762×10^5	3.496921515×10^5
0.8	4.194640877×10^5	4.013526485×10^5	3.875560639×10^5	3.757275710×10^5
1	4.447124079×10^5	4.225442742×10^5	4.079079382×10^5	3.968030116×10^5

Table 2: Table of exposed population $e(t)$ at different values of α.

t	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
0	282	282	282	282
0.2	257.4162224	258.9649135	262.0297516	265.2840176
0.4	267.2755013	259.6875819	257.4971462	257.8404747
0.6	291.0317094	273.2934962	264.1504561	259.6693712
0.8	324.6147581	297.0931357	280.7192215	270.7707074
1	366.0260950	329.6510709	306.4698571	291.1444832

Table 3: Table of infected population $i(t)$ at different values of α.

t	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
0	200	200	200	200
0.2	175.2188217	171.9948872	173.6759342	176.9707974
0.4	210.7055056	187.2084738	176.6761630	172.6846028
0.6	272.0075510	227.0910139	201.6657364	187.1414162
0.8	351.8823809	286.7638747	246.3121481	220.3412377
1	446.7043409	363.5699590	309.2400252	272.2840671

Table 4: Table of asymptotically infected population $a(t)$ at different values of α.

t	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
0	0	0	0	0
0.2	73.26341490	56.78922133	44.58936555	35.23388508
0.4	115.6931661	92.55862443	76.59150701	64.52546644
0.6	162.5740800	129.0616812	107.6391594	92.47760532
0.8	220.9500804	173.2748196	143.7130733	123.6931630
1	294.4142649	229.7014251	189.5803826	162.7750005

Table 5: Table of recovered population $r(t)$ at different values of α.

t	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
0	50000	50000	50000	50000
0.2	49769.14447	49827.93009	49869.73307	49900.15898
0.4	49625.51790	49700.78918	49757.18422	49800.51375
0.6	49503.24037	49586.62441	49650.62486	49701.06843
0.8	49393.12443	49480.22435	49547.85429	49601.82712
1	49291.29976	49379.30131	49447.86689	49502.79392

Table 6: Table of reservoir population $m(t)$ at different values of α.

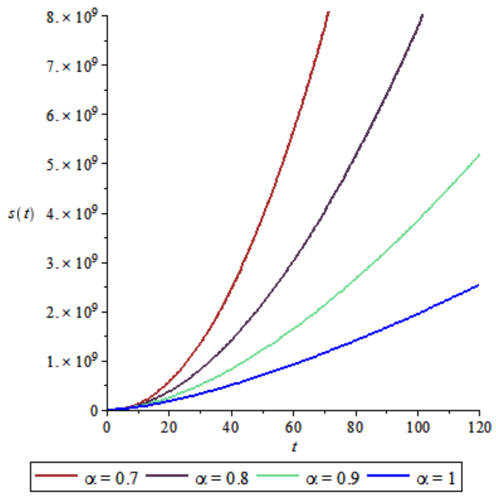

Figure 1: Numerical solutions for susceptible $s(t)$ population in a time t at different values of α.

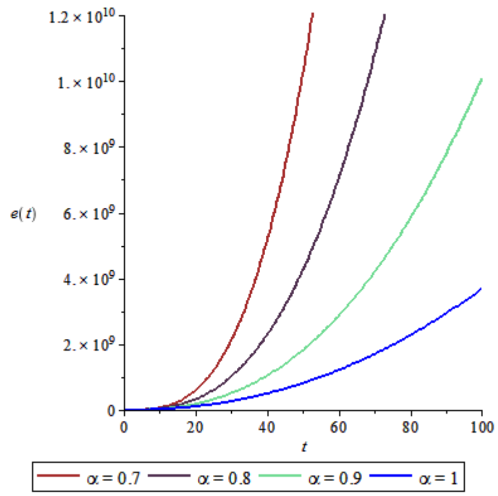

Figure 2: Numerical solutions for exposed $e(t)$ population in a time t at different values of α.

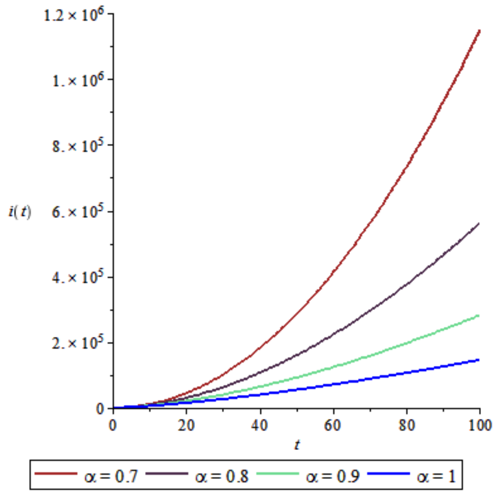

Figure 3: Numerical solutions for infected $i(t)$ population in a time t at different values of α.

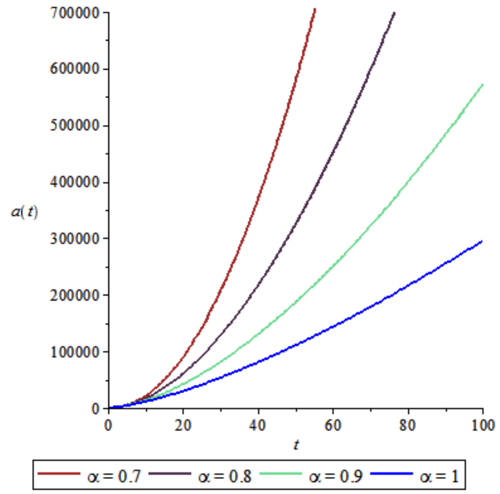

Figure 4: Numerical solutions for asymptotically infected $a(t)$ population in a time t at different values of α.

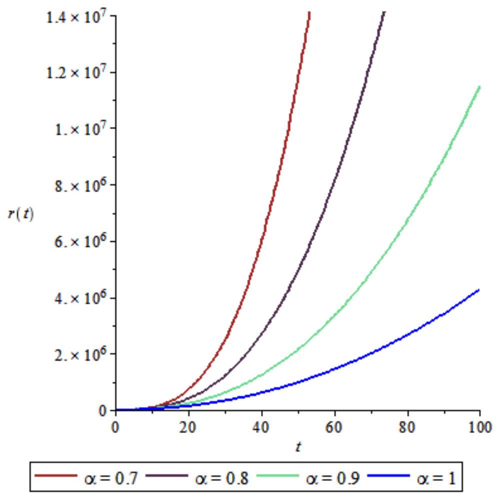

Figure 5: Numerical solutions for recovered $r(t)$ population in a time t at different values of α.

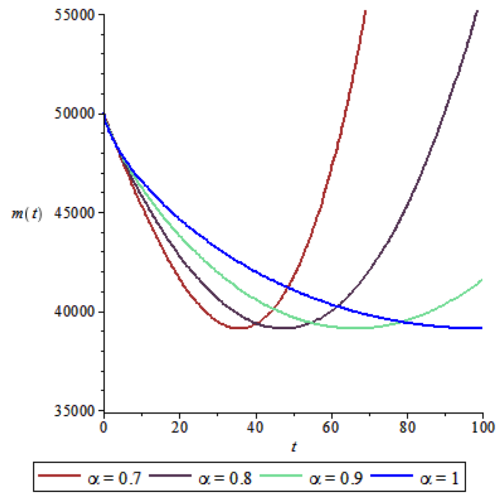

Figure 6: Numerical solutions for reservoir $m(t)$ population in a time t at different values of α.

6. Conclusion

In this work, we established the conformable fractional calculus in a non-linear system with the conformable fractional differential transform method is exploited to determine the solution of non-linear conformable differential system, analysed and examined transmission dynamics of COVID-19 infection formulated in terms of mathematical model based on fractional differential system which described by COVID-19 epidemic model. The outcomes show that these results agreed well for the case $\alpha = 1$. The study confirms that the conformable fractional differential transform method is an successful and appropriate procedure to solve the fractional-order COVID-19 epidemic model which we have examined here. The rest of the COVID-19 control measures are to be studied and their significance will be analyzed.

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