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COMMON FIXED POINT OF COMPATIBLE TYPE (K) MAPPINGS IN FUZZY METRIC SPACE

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Abstract: The purpose of this paper is to establish a common fixed point theorem for six self-mappings in complete fuzzy metric space, using the concept of compatibility of type (K) with another functional inequality and our result generalize the result of K. B. Manandhar and et al. [7] and other similar results in the literature.

Keywords and Phrases: Common fixed point, fuzzy metric space, compatible mappings of type (E), compatible mappings of type (K).

2020 Mathematics Subject Classification: 54H25, 47H10.

1. Introduction

The concept of fuzzy set was introduced by Zadeh (1965) [13] as a new way to represent vagueness in everyday life. A large number of renowned mathematicians worked with fuzzy sets in different branches of Mathematics, Fuzzy Metric Space is one of them. This paper uses the concept of fuzzy metric space introduced by Kramosil and Michalek [6] and modified by George and Veeramani [1] with the help of a t-norm. Grabiec [2] obtained the fuzzy version of the Banach contraction principle, which is a milestone in developing fixed point theory in fuzzy metric space. In the sequel, Vasuki [12] introduced the concept of R-weakly commuting in fuzzy metric space and proved the common fixed point theorem. Jungck [4] proposed the concept of compatibility in 1986. In 1993, G. Jungck, P. P. Murthy and Y. J. Cho [5] gave a generalization of compatible mappings called compatible mappings of type (A) which is equivalent to the concept of compatible mappings under some conditions. The concept of compatibility in fuzzy metric space was proposed by Mishra et al. [8]. In 1996, H. K. Pathak, Y. J. Cho, S. S. Chang and S. M. Kang [9] introduced the concept of compatible mappings of type (P) and compared with compatible mappings of type (A) and compatible mappings. K. B. Manandhar et al. [7] introduced the notion of compatible mappings of type (E) and obtained a common fixed point theorem for self mappings in complete fuzzy metric space in 2014. K. Jha, V. Popa and K. B. Manandhar [3] introduced the concept of compatible mappings of type (K) in metric space. Rao R. and Reddy B. [10] have obtained fixed point theorems for compatible mappings of type (K) in complete fuzzy metric space in 2016, where four mappings are needed to be continuous. In this paper, a theorem has been proved on a common fixed point for six self mappings in complete fuzzy metrics space, using compatible of type (K) and generalizing the result of K. B. Manandhar et. al. [7] and similar previous results.

2. Preliminaries

Definition 2.1. [13] *Let X be any set. A fuzzy set A in X is a membership function with a domain in X and values in $[0, 1]$.*

Definition 2.2. [11] *A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous t-norm if it satisfies the following conditions:*
(i) $*$ is associative and commutative,
(ii) $*$ is continuous,
(iii) $a * 1 = a$, for all $a \in [0, 1]$,
(iv) $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.
Examples of t-norms are
$a * b = \min\{a, b\}$ (minimum t-norm),
$a * b = ab$ (product t-norm).

Definition 2.3. [1] *The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions*
(FM-1) $M(x, y, t) > 0$,
(FM-2) $M(x, y, t) = 1$ if and only if $x = y$,

(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(FM-5) $M(x, y, .) : (0, \infty) \to [0, 1]$ *is continuous,*
for all $x, y, z \in X$ *and* $t, s > 0$.
Let (X, d) *be a metric space and let* $a*b = ab$ *or* $a*b = \min\{a, b\}$ *for all* $a, b \in [0, 1]$.
Let $M(x, y, t) = \dfrac{t}{t + d(x, y)}$; *for all* $x, y \in X$ *and* $t > 0$.
Then $(X, M, *)$ is a fuzzy metric space, and this fuzzy metric M induced by d is called the standard fuzzy metric [1].

Definition 2.4. [2] *A sequence* $\{x_n\}$ *of a fuzzy metric space* $(X, M, *)$ *is said to be convergent to a point* $x \in X$, *if* $\lim_{n\to\infty} M(x_n, x, t) = 1$ *for all* $t > 0$. *Further, the sequence* $\{x_n\}$ *is said to be Cauchy if* $\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$, *for all* $t > 0$ *and* $p > 0$.
The space $(X, M, *)$ is said to be complete if every Cauchy sequence in X is convergent in X.

Lemma 2.5. [2] *Let* $(X, M, *)$ *be a fuzzy metric space. Then* M *is non-decreasing for all* $x, y \in X$.

Remark 2.6. *Let* $(X, M, *)$ *be a fuzzy metric space. Then* M *is a continuous function on* $X^2 \times (0, \infty)$.
Throughout this paper $(X, M, *)$ will denote the fuzzy metric space with the following condition:
(FM-6) $\lim_{n\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$.

Lemma 2.7. [8] *If there exists* $k \in (0, 1)$ *such that* $M(x, y, kt) \geq M(x, y, t)$ *for all* $x, y \in X$ *and* $t > 0$, *then* $x = y$.

Definition 2.8. [8] *Let* f *and* g *be self-mappings on a fuzzy metric space* $(X, M, *)$. *The pair* (f, g) *is said to compatible if*
$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$, *for all* $t > 0$,
whenever $\{x_n\}$ *is a sequence in* X *such that* $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, *for some* $z \in X$.

Definition 2.9. [5] *Two self-mappings* f *and* g *of a fuzzy metric space* $(X, M, *)$ *are said to be compatible of type (A) if*
$\lim_{n\to\infty} M(fgx_n, ggx_n, t) = 1$ *and* $\lim_{n\to\infty} M(gfx_n, ffx_n, t) = 1$,
for all $t > 0$, *whenever* $\{x_n\}$ *is a sequence in* X *such that* $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, *for some* $z \in X$.

Definition 2.10. [9] *Two self-mappings f and g of a fuzzy metric space $(X, M, *)$ are said to be compatible of type (P) if*
$\lim_{n\to\infty} M(ffx_n, ggx_n, t) = 1,$
whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some $z \in X$.

Definition 2.11. [7] *The self-mappings f and g of a fuzzy metric space $(X, M, *)$ are said to be compatible of type (E) iff*
$\lim_{n\to\infty} M(ffx_n, fgx_n, t) = gz$ *and* $\lim_{n\to\infty} M(ggx_n, gfx_n, t) = fz,$
whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some $z \in X$.

Definition 2.12. [10] *Two self-mappings f and g of a fuzzy metric space $(X, M, *)$ are said to be compatible of type (K) iff*
$\lim_{n\to\infty} M(ffx_n, gz, t) = 1$ *and* $\lim_{n\to\infty} M(ggx_n, fz, t) = 1$ *all* $t > 0,$
whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some $z \in X$.
Obviously two compatible of type (E) mappings are also compatible of type (K) however the converse is not true.
The following examples show that the mappings compatible (compatible mappings of type (A), compatible mappings of type (P)) and compatible mappings of type (K) are independent

Example 2.1. Let $X = [0, 2]$ be a complete metric space, f and g are two self-maps on X defined by

$$f(x) = \begin{cases} 2, & \text{if } x \in [0,1] - \left\{\frac{1}{2}\right\} \\ 0, & \text{if } x = \frac{1}{2} \\ 1 - \frac{x}{2}, & (1,2] \end{cases} \quad \text{and} \quad g(x) = \begin{cases} x, & \text{if } x \in \left[0, \frac{1}{2}\right) \\ 2, & \text{if } x = \frac{1}{2} \\ 0, & x \in \left(\frac{1}{2}, 1\right] \\ \frac{x}{2}, & x \in (1,2] \end{cases}$$

choose a sequence $x_n = 1 + \frac{1}{n}$ for $n \in N$ then

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} f\left(1 + \frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{2n}\right) = \frac{1}{2},$$
$$\lim_{n\to\infty} gx_n = \lim_{n\to\infty} g\left(1 + \frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{1}{2} + \frac{1}{2n}\right) = \frac{1}{2},$$
$$\Rightarrow \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \frac{1}{2}.$$

Hence as $n \to \infty$ fx_n and gx_n both converges to 1/2.

$$f\left(\frac{1}{2}\right) = 0 \quad \text{and} \quad g\left(\frac{1}{2}\right) = 2.$$

$$\lim_{n\to\infty} ffx_n = \lim_{n\to\infty} ff\left(1+\frac{1}{n}\right) = \lim_{n\to\infty} f\left(\frac{1}{2}-\frac{1}{2n}\right) = 2 = g\left(\frac{1}{2}\right),$$

$$\lim_{n\to\infty} ggx_n = \lim_{n\to\infty} gg\left(1+\frac{1}{n}\right) = \lim_{n\to\infty} g\left(\frac{1}{2}+\frac{1}{2n}\right) = 0 = \left(\frac{1}{2}\right),$$

$$\lim_{n\to\infty} fgx_n = \lim_{n\to\infty} fg\left(1+\frac{1}{n}\right) = \lim_{n\to\infty} f\left(\frac{1+\frac{1}{n}}{2}\right) = \lim_{n\to\infty} f\left(\frac{1}{2}+\frac{1}{2n}\right) = 2.$$

$$\lim_{n\to\infty} gfx_n = \lim_{n\to\infty} gf\left(1+\frac{1}{n}\right) = \lim_{n\to\infty} g\left(1-\frac{1+\frac{1}{n}}{2}\right) = \lim_{n\to\infty} g\left(\frac{1}{2}-\frac{1}{2n}\right) = \frac{1}{2}.$$

Therefore, the mappings are not compatible, compatible of type (A) and compatible of type (P), compatible of type (E) but the mappings are compatible of type (K).

Example 2.2. Let $X = [0, 2]$ be a complete metric space, f and g are two self-maps on X defined by

$$f(x) = \begin{cases} x, & \text{if } x \in [0,1] - \left\{\frac{1}{2}\right\} \\ 1/4, & \text{if } x = \frac{1}{2} \\ 1-\frac{x}{2}, & (1,2] \end{cases} \quad \text{and} \quad g(x) = \begin{cases} \frac{1}{2}, & \text{if } x \in [0,1] \\ \frac{x}{2}, & x \in (1,2] \end{cases}$$

Now choose a sequence $x_n = 1 + \frac{1}{n}$ for $n \in N$. Then as $n \to \infty fx_n$ and gx_n both converges to $\frac{1}{2}$.
$\Rightarrow \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = \frac{1}{2}$, and value of $f\left(\frac{1}{2}\right) = \frac{1}{4}$ and $g\left(\frac{1}{2}\right) = \frac{1}{2}$.

$$\lim_{n\to\infty} ffx_n = \lim_{n\to\infty} ff\left(1+\frac{1}{n}\right) = \lim_{n\to\infty} f\left(\frac{1}{2}-\frac{1}{2n}\right) = \frac{1}{2} = g\left(\frac{1}{2}\right),$$

$$\lim_{n\to\infty} ggx_n = \lim_{n\to\infty} gg\left(1+\frac{1}{n}\right) = \lim_{n\to\infty} g\left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2} \neq f\left(\frac{1}{2}\right),$$

$$\lim_{n\to\infty} fgx_n = \lim_{n\to\infty} fg\left(1+\frac{1}{n}\right) = \lim_{n\to\infty} f\left(\frac{1}{2}+\frac{1}{2n}\right) = \lim_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2},$$

$$\lim_{n\to\infty} gfx_n = \lim_{n\to\infty} gf\left(1+\frac{1}{n}\right) = \lim_{n\to\infty} g\left(\frac{1}{2}-\frac{1}{2n}\right) = \frac{1}{2}.$$

$$\Rightarrow \lim_{n\to\infty} fgx_n = \lim_{n\to\infty} gfx_n = \lim_{n\to\infty} ffx_n = \lim_{n\to\infty} ggx_n.$$

Shows that the mappings are compatible, compatible of type (A) and compatible of type (P) but not compatible of type (K).
K. B. Manandhar et al. [7] proved the following result:
If A, B, S and T are self-mappings in fuzzy metric space $(X, M, *)$, we denote

$$M_\alpha \left\{ \begin{array}{l} M(Sx, Ax, t) * M(Ty, By, t) * M(Sx, Ty, t) \\ *M(Ty, Ax, \alpha t) * M(Sx, By, (2-\alpha)t) \end{array} \right\},$$

for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$.

Theorem 2.13. [7] *Let $(X, M, *)$ be a complete fuzzy metric space with $a * a \geq a$ all for $a \in [0, 1]$ and with the condition (FM-6). Let one of the mapping of self-mappings (A, S) and (B, T) of X be continuous such that,*
(2.13.1) $AX \subset TX$, $BX \subset SX$;
(2.13.2) There exists $k \in (0, 1)$ such that $M(Ax, By, kt) \geq M_\alpha(x, y, t)$ for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$.
If (A, S) and (B, T) compatible of type (E) then A, B, S and T have a unique common fixed point.

3. Main Result

Our result generalizes the results of [7] as we are using the concept of compatible of type (K) and proving the result for six self-maps in a complete fuzzy metric space using another inequality.

Theorem 3.1. *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, P, Q and T be mappings from X into itself such that the following conditions are satisfied:*
(3.1.1) $A(X) \subset TP(X)$, $B(X) \subset SQ(X)$;
(3.1.2) $TP = PT, BP = PB, AQ = QA, SQ = QS$;
(3.1.3) (A, SQ) and (B, TP) is compatible of type (K) where one of them is continuous;
(3.1.4) There exists $k \in (0, 1)$ such that

$$M(Ax, By, kt) \geq \min \left\{ \begin{array}{l} M(SQx, Ax, t), M(TPy, By, t), M(SQx, TPy, t), \\ M(TPy, Ax, \alpha t), M(SQx, By, (2-\alpha)t) \end{array} \right\},$$

for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$.
Then A, B, S, T, P and Q have a unique common fixed point in X.
Proof. Let x_0 be an arbitrary point in X. According to (3.1.1) $A(X) \subset TP(X)$, $B(X) \subset SQ(X)$ there exists some points $x_1, x_2 \in X$ such that
$Ax_0 = TPx_1 = y_0$ and $Bx_1 = SQx_2 = y_1$.

We can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$Ax_{2n} = TPx_{2n+1} = y_{2n} \quad \text{and} \quad Bx_{2n+1} = SQx_{2n+2} = y_{2n+1}. \tag{i}$$

For $n = 0, 1, 2, ...,$ $t > 0$ and $\alpha = 1 - q$ where $q \in (0, 1)$ Now, we first show that $\{y_n\}$ is a Cauchy sequence in X.
Using condition (3.1.4)

$$M(y_{2n+1}, y_{2n}, kt) = M(y_{2n}, y_{2n+1}, kt) = M(Ax_{2n}, Bx_{2n+1}, kt)$$

Using condition (3.1.4)

$$M(Ax_{2n}, Bx_{2n+1}, kt) \geq \min\left\{\begin{array}{l} M(SQx_{2n}, Ax_{2n}, t), M(TPx_{2n+1}, Bx_{2n+1}, t), \\ M(SQx_{2n}, TP_{2n+1}, t), M(TP_{2n+1}, Ax_{2n}, \alpha t), \\ M(SQx_{2n}, B_{2n+1}, (2-\alpha)t) \end{array}\right\},$$

$$M(y_{2n+1}, y_{2n}, kt) \geq \min\left\{\begin{array}{l} M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), \\ M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n}, (1-q)t), \\ M(y_{2n-1}, y_{2n+1}, (1+q)t) \end{array}\right\}, \quad \text{since (i)}$$

$$M(y_{2n+1}, y_{2n}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n-1}, y_{2n+1}, (1+q)t)\}$$

$$M(y_{2n+1}, y_{2n}, kt) \geq \min\left\{\begin{array}{l} M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n-1}, y_{2n}, t), \\ M(y_{2n}, y_{2n}, qt) \end{array}\right\},$$

letting $q \to 1$ and using definition (2.3) and lemma (2.5) we get

$$M(y_{2n+1}, y_{2n}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t)\} \tag{ii}$$

Replacing t with t/k in equation (ii), (since $\forall t > 0 \Leftrightarrow t/k > 0$)

$$M(y_{2n+1}, y_{2n}, t) \geq \min\{M(y_{2n-1}, y_{2n}, t/k), M(y_{2n}, y_{2n+1}, t/k)\}$$
$$M(y_{2n+1}, y_{2n}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n-1}, y_{2n}, t/k), M(y_{2n}, y_{2n+1}, t/k)\}$$
$$M(y_{2n+1}, y_{2n}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t/k)\}$$
$$M(y_{2n+1}, y_{2n}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n-1}, y_{2n}, t/k^2), M(y_{2n}, y_{2n+1}, t/k^2)$$
$$M(y_{2n+1}, y_{2n}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t/k^2)\}$$

Similarly

$$M(y_{2n+1}, y_{2n}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n+1}, t/k^m)\}$$

taking $m \to \infty$

$$M(y_{2n+1}, y_{2n}, kt) \geq \min\{M(y_{2n-1}, y_{2n}, t), 1\}$$
$$M(y_{2n+1}, y_{2n}, kt) \geq M(y_{2n-1}, y_{2n}, t), \quad \forall\ t > 0$$

Thus for all n and $t > 0$ we get,

$$M(y_{n+1}, y_n, kt) \geq M(y_n, y_{n-1}, t), \quad \forall\ t > 0.$$

Therefore

$$M(y_{n+1}, y_n, t) \geq M(y_n, y_{n-1}, t/k) > M(y_{n-1}, y_{n-2}, t/k^2) > ... > M(y_1, y_0, t/k^n)$$

$\lim_{n\to\infty} M(y_{n+1}, y_n, t) = 1, \quad \forall\ t > 0.$
Now for any integer p we have

$$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, t/k) * M(y_{n+1}, y_{n+2}, t/k) * ... * M(y_{n+p-1}, y_{n+p}, t/k)$$

$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1 * 1 * ... * 1 = 1.$
Above result show that $\{y_n\}$ is a Cauchy sequence in X which is complete. Therefore $\{y_n\}$ sequence converges to $z \in X$ and all subsequences $\{Ax_{2n}\}$, $\{TPx_{2n+1}\}$, $\{Bx_{2n+1}\}$ and $\{SQx_{2n+2}\}$ also converge to $z \in X$.

$$\lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} TPx_{2n+1} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} SQx_{2n+2} = z. \qquad (iii)$$

Case (I) (A, SQ) is compatible of type (K) and either A or SQ is continuous. where $\lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} SQx_{2n+2} = z$ then $\lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} SQx_{2n} = z$.
(A, SQ) is compatible of type (K) then $\lim_{n\to\infty} AAx_{2n} = SQz$ and $\lim_{n\to\infty} SQSQx_{2n} = Az$.
If A is continuous then $\lim_{n\to\infty} Ax_{2n} = z \Rightarrow \lim_{n\to\infty} AAx_{2n} = Az$. Therefore $Az = SQz$.
Similarly, If SQ is continuous then $\lim_{n\to\infty} SQx_{2n} = z \Rightarrow \lim_{n\to\infty} SQSQx_{2n} = SQz$.

$$Az = SQz. \qquad (iv)$$

Putting $x = z$ and $y = x_{2n+1}$ in (3.1.4)

$$M(Az, Bx_{2n+1}, kt) \geq \min\left\{\begin{array}{l} M(SQz, Az, t), M(TPx_{2n+1}, Bx_{2n+1}, t), \\ M(SQz, TPx_{2n+1}, t), M(TPx_{2n+1}, Az, \alpha t), \\ M(SQz, Bx_{2n+1}, (2-\alpha)t) \end{array}\right\}$$

since (iv),

$$M(Az, Bx_{2n+1}, kt) \geq \min \left\{ \begin{array}{l} M(Az, Az, t), M(TPx_{2n+1}, Bx_{2n+1}, t), \\ M(Az, TPx_{2n+1}, t), M(TPx_{2n+1}, Az, \alpha t), \\ M(Az, Bx_{2n+1}, (2-\alpha)t) \end{array} \right\},$$

$$M(Az, Bx_{2n+1}, kt) \geq \min \left\{ \begin{array}{l} 1, M(TPx_{2n+1}, Bx_{2n+1}, t), M(Az, TPx_{2n+1}, t), \\ M(TPx_{2n+1}, Az, \alpha t), M(Az, Bx_{2n+1}, (2-\alpha)t) \end{array} \right\}.$$

Taking $n \to \infty$

$$M(Az, z, kt) \geq \min \{1, M(z, z, t), M(Az, z, t), M(z, Az, \alpha t), M(Az, z, (2-\alpha)t)\},$$

since (iii).
When $\alpha \to 1$

$$M(Az, z, kt) \geq \min \{1, 1, M(Az, z, t), M(z, Az, t), M(Az, z, t)\},$$

$$M(Az, z, kt) \geq \min \{1, 1, M(Az, z, t)\},$$

$$M(Az, z, kt) \geq M(Az, z, t).$$

Using lemma 2.2 we have $Az = z$. Therefore

$$Az = SQz = z. \tag{v}$$

Case (II) (B, TP) is compatible of type (K) and either B or TP is continuous.
$\lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} TPx_{2n+1} = z$.
(B, TP) is compatible of type (K).
$\lim_{n\to\infty} BBx_{2n} = TPz$ and $\lim_{n\to\infty} TPTPx_{2n} = Bz$.
If B is continuous then $\lim_{n\to\infty} Bx_{2n} = z \Rightarrow \lim_{n\to\infty} BBx_{2n} = Bz$.
If TP is continuous then $\lim_{n\to\infty} TPx_{2n+1} = z \Rightarrow \lim_{n\to\infty} TPTPx_{2n+1} = TPz$.
Therefore,

$$Bz = TPz. \tag{vi}$$

Putting $x = z$ and $y = z$ in (3.1.4), we obtain

$$M(Az, Bz, kt) \geq \min \left\{ \begin{array}{l} M(SQz, Az, t), M(TPz, Bz, t), M(SQz, TPz, t), \\ M(TPz, Az, \alpha t), M(SQz, Bz, (2-\alpha)t) \end{array} \right\}$$

by using (v) and (vi),

$$M(z, Bz, kt) \geq \min \left\{ \begin{array}{l} M(z, Az, t), M(Bz, Bz, t), M(z, Bz, t), \\ M(Bz, z, \alpha t), M(z, Bz, (2-\alpha)t) \end{array} \right\},$$

Taking $\alpha \to 1$

$$M(z, Bz, kt) \geq \min\{1, 1, M(z, Bz, t), M(Bz, z, t), M(z, Bz, t)\}$$

$$M(z, Bz, kt) \geq M(z, Bz, t)$$

Using lemma 2.2 we have $Bz = z$. Therefore

$$Az = Bz = SQz = TPz = z. \tag{vii}$$

Now putting $x = z$ and $y = Pz$ in (3.1.4), we obtain

$$M(Az, BPz, kt) \geq \min\left\{\begin{array}{l} M(SQz, Az, t), M(TPPz, BPz, t), \\ M(SQz, TPPz, t), M(TPPz, Az, \alpha t), \\ M(SQz, BPz, (2-\alpha)t) \end{array}\right\},$$

since (3.1.2)

$$M(Az, PBz, kt) \geq \min\left\{\begin{array}{l} M(SQz, Az, t), M(PTPz, PBz, t), \\ M(SQz, PTPz, t), M(PTPz, Az, \alpha t), \\ M(SQz, PBz, (2-\alpha)t) \end{array}\right\},$$

since (vii)

$$M(z, Pz, kt) \geq \min\left\{\begin{array}{l} M(z, z, t), M(Pz, Pz, t), M(z, Pz, t), \\ M(Pz, z, \alpha t), M(z, Pz, (2-\alpha)t) \end{array}\right\},$$

Taking $\alpha \to 1$ $M(z, Pz, kt) \geq \min\{1, M(z, Pz, t)\}$
$M(z, Pz, kt) \geq M(z, Pz, t)$.
Using lemma 2.2 we have $Pz = z$ and $TPz = z \Rightarrow Tz = z$. Hence,

$$Pz = Tz = z. \tag{viii}$$

Again putting $x = Qz$ and $y = z$ in (3.1.4)

$$M(AQz, Bz, kt) \geq \min\left\{\begin{array}{l} M(SQQz, AQz, t), M(TPz, Bz, t), \\ M(SQQz, TPz, t), M(TPz, AQz, \alpha t), \\ M(SQQz, Bz, (2-\alpha)t) \end{array}\right\},$$

since (3.1.2)

$$M(QAz, Bz, kt) \geq \min\left\{\begin{array}{l} M(QSQz, QAz, t), M(TPz, Bz, t), \\ M(QSQz, TPz, t), M(TPz, QAz, \alpha t), \\ M(QSQz, Bz, (2-\alpha)t) \end{array}\right\},$$

since (vii)

$$M(Qz, z, kt) \geq \min \left\{ \begin{array}{l} M(Qz, Qz, t), M(Tz, z, t), M(Qz, z, t), \\ M(z, Qz, \alpha t), M(Qz, z, (2-\alpha)t) \end{array} \right\},$$

$$M(Qz, z, kt) \geq \min\{1, 1, M(Qz, z, t), M(z, Qz, \alpha t), M(Qz, z, (2-\alpha)t)\}$$

Taking $\alpha \to 1$ $M(Qz, z, kt) \geq \min\{1, M(Qz, z, t)\}$
$M(Qz, z, kt) \geq M(Qz, z, t)$.
Using lemma 2.2 we have $Qz = z$ and $SQz = z \Rightarrow Sz = z$. Hence,

$$Sz = Qz = z. \qquad (ix)$$

Using (vii), (viii) and (ix), We conclude the following:

$$Az = Bz = Pz = Qz = Sz = Tz = z.$$

Finally we get 'z' is a common fixed point of self-maps A, B, P, Q, S and T.

Uniqueness:
Let 'u' is another common fixed point of self-mappings A, B, P, Q, S and T.
Such that, $Az = Bz = Pz = Qz = Sz = Tz = z$,
$Au = Bu = Pu = Qu = Su = Tu = u$.
Putting $x = z$ and $y = u$ in (3.1.4) we have

$$M(Az, Bu, kt) \geq \min \left\{ \begin{array}{l} M(SQz, Az, t), M(TPu, Bu, t), M(SQz, TPu, t), \\ M(TPu, Az, \alpha t), M(SQz, Bu, (2-\alpha)t) \end{array} \right\}.$$

When $\alpha \to 1$

$$M(z, u, kt) \geq \min \left\{ \begin{array}{l} M(Sz, z, t), M(Tu, u, t), M(Sz, Tu, t), \\ M(Tu, z, t), M(Sz, u, t) \end{array} \right\},$$

$$M(z, u, kt) \geq \min \left\{ \begin{array}{l} M(z, z, t), M(u, u, t), M(z, u, t), \\ M(u, z, t), M(z, u, t) \end{array} \right\},$$

$M(z, u, kt) \geq \min\{1, M(z, u, t)\}$, $M(z, u, kt) \geq M(z, u, t) \Rightarrow z = u$.

Remark 3.2. *If we put $P = Q = I$ in theorem 3.1 then, condition (3.1.2) is satisfied trivially and we get the following theorem.*

Corollary 3.3. *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be mappings from X into itself such that the following conditions are satisfied:*
(3.3.1) $A(X) \subset T(X)$, $B(X) \subset S(X)$;

(3.3.2) (A, S) *and* (B, T) *is compatible of type* (K) *where one of them is continuous.*
(3.1.3) There exists $k \in (0, 1)$ *such that*

$$M(Ax, By, kt) \geq \min \left\{ \begin{array}{l} M(Sx, Ax, t), M(Ty, By, t), M(Sx, Ty, t), \\ M(Ty, Ax, \alpha t), M(Sx, By, (2-\alpha)t) \end{array} \right\}.$$

for all $x, y \in X$ *and* $t > 0$.
Then A, B, S *and* T *have a unique common fixed point in* X.
On putting $A = B$ in corollary (3.3), we get the following theorem.

Theorem 3.4. *Let* $(X, M, *)$ *be a complete fuzzy metric space and let* A, T *and* S *be mappings from* X *into itself such that the following conditions are satisfied:*
(3.4.1) $A(X) \subset T(X) \cap S(X)$;
(3.3.2) (A, T) *and* (A, S) *are compatible of type* (K) *where* A *is continuous.*
(3.1.3) There exists $k \in (0, 1)$ *such that*

$$M(Ax, Ay, kt) \geq \min \left\{ \begin{array}{l} M(Sx, Ax, t), M(Ty, Ay, t), M(Sx, Ty, t), \\ M(Ty, Ax, \alpha t), M(Sx, Ay, (2-\alpha)t) \end{array} \right\}.$$

for all $x, y \in X$ *and* $t > 0$.
Then A, S *and* T *have a unique common fixed point in* X.

4. Conclusion

This paper is a generalization of the result of K. B. Manandhar et al. [7] in the sense of replacing compatible of type (E) to compatible of type (K) to prove a theorem on common fixed point theorems for six self-mappings in complete fuzzy metric space.

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