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SUPER-EDGE MAGIC TOTAL LABELING IN CERTAIN CLASSES OF GRAPHS

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Abstract: Duplicate graph of a graph is constructed from a graph with vertex set V of p vertices and edge set E of q edges as a new graph with vertex set union of V, V' where V' is a set disjoint with V having p vertices such that uv is an edge in the graph G if and only if uv' and $u'v$ are the edges in its duplicate graph. Super-edge magic total labeling of a graph is a bijection which labels the vertices with integers 1 to p and edges with integers $p+1$ to $p+q$ such that the induced edge sum of edges defined as "sum of labels of end vertices and label of that edge" are all same. In this paper, we provide algorithms and prove existence of super-edge magic total labeling in extended duplicate graphs of twig, comb, star and bi-star graphs.

Keywords and Phrases: Extended duplicate graphs, Graph labeling, Edge magic total labeling.

2020 Mathematics Subject Classification: 05C78.

1. Introduction

Graph labeling is a branch of Graph theory having wide applications in the field of circuit design, networks, molecular biology, neural networks etc. In 1967, Rosa

[12] formally initiated labeling on graphs. Since then, over 2000 papers on labeling were published on variety of labeling. Intrigued and fascinated by magic squares in number theory, in 1963 Sedlacek introduced magic labeling in graph theory. In 1970, Kotzig and Rosa defined the concept of edge magic total labeling.

Definition 2.1. [13] *For a graph G with vertex set V construct another graph as follows: Let V' be the set such that $\cap V' = \phi$, $|V| = |V'|$, and $f : V \to V'$ be bijective. For $a \in V$ $f(a) \in V'$ we write as a' for convenience. Consider the graph DG on the vertex set $V \cup V'$, whose edges are given as follows: In the graph G, ab is an edge if and only if both ab' and $a'b$ are edges in DG. The graph DG is called Duplicate graph of G.*

Definition 1.2. *A path P_n is a sequence of arrangement of vertices $v_1, v_2, \ldots, v_n$ where the edges are in the form of v_iv_{i+1}, $i = 1, 2, \ldots, n-1$.*

Definition 1.3. *A comb CB_n is obtained by connecting a new pendant edge with each vertex of path P_n. Comb CB_n has $2n$ vertices and $2n-1$ edges and its duplicate graphs has $4n$ vertices and $4n-2$ edges. As this duplicate graph is disconnected, connecting any two vertices, preferably, one from v_i and one from v'_i makes it connected and called extended duplicate graph of CB_n. Here, the vertices v_{2n-1}, v'_{2n-1} are connected by an edge to get extended duplicate graph $EDG(CB_n)$.*

Definition 1.4. *A twig T_n is a graph obtained by connecting two new pendant edges with each of the internal vertices in P_{n+2}. Twig T_n has $3n+2$ vertices and $3n+1$ edges and its duplicate graph has $6n+4$ vertices and $6n+2$ edges. The vertices v_{3n-1}, v'_{3n-1} are connected by an edge to get extended duplicate graph $EDG(T_n)$.*

Definition 1.5. *A star graph $K_{1,n}$ is a graph having one apex vertex v_1 connected by an edge with each of n pendant vertices $v_2, v_3, \ldots, v_{n+1}$. Star $K_{1,n}$ has $n+1$ vertices and n edges and its duplicate graph has $2n+2$ vertices and $2n$ edges. The vertices v_1, v'_1 are connected by an edge to get extended duplicate graph $EDG(K_{1,n})$.*

Definition 1.6. *A bi-star graph $B_{m,n}$ is a graph having two apex vertices v_1, v_2 connected by an edge and each of these apex vertices are connected to n pendant vertices. Bi-star graph $B_{m,n}$ has $m+n+2$ vertices and $m+n+1$ edges and its duplicate graph $2m+2n+4$ vertices and $2m+2n+2$ edges. The vertices v'_1, v'_2 are connected by an edge to get extended duplicate graph $EDG(B_{m,n})$.*

Definition 1.7. *In a graph G with p vertices and q edges, a bijection $\phi : \{1, 2, 3, \ldots, p+q\} \to V \cup E$ is called an edge magic total labeling if the edge induced function ϕ^* defined by $\phi^*(uv) = \phi(u) + \phi(v) + \phi(uv)$ assigns the same value (constant) for all the edges.*

Illustration of above graphs and their duplicate graphs are given below:

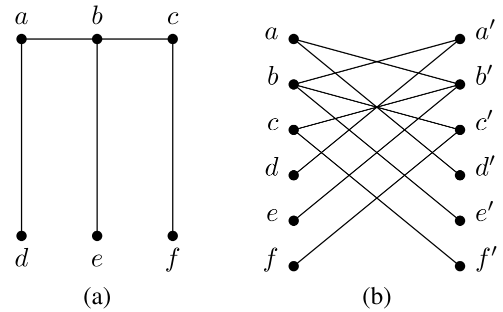


Figure 1: (a) Comb graph CB_3 (b) Duplicate graph of Comb $DG(CB_3)$

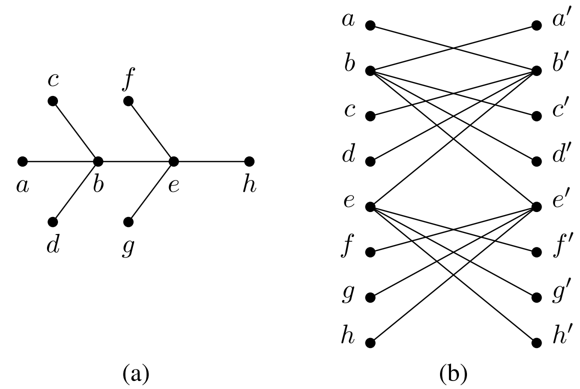


Figure 2: (a) Twig graph T_2 (b) Duplicate graph of Twig $DG(T_2)$

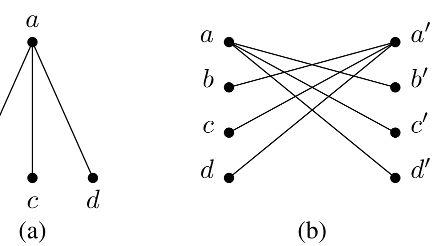


Figure 3: (a) Star graph $K_{1,3}$ (b) Duplicate graph $DG(K_{1,3})$

Definition 1.8. *An edge magic labeling is called a super-edge magic total labeling if vertices are labeled with integers from 1 to p and edges are labeled with integers*

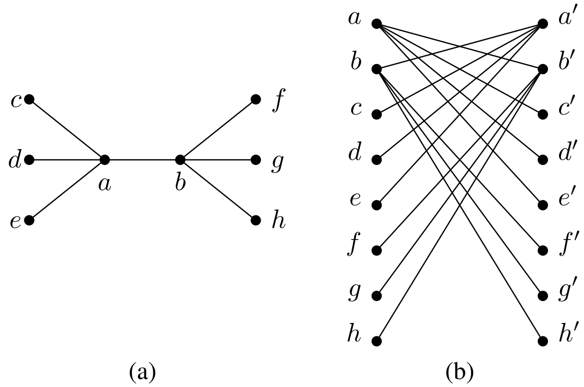


Figure 4: (a) Bi-star graph $B_{3,3}$ (b) Duplicate graph $DG(B_{3,3})$

from $p+1$ to $p+q$.

2. Main Results

Theorem 2.1. *The extended duplicate graph of twig $EDG(T_n)$, $n \geq 2$, admits super-edge magic total labeling.*

Proof. The vertices of $EDG(T_n)$ are labeled as below.

$f(v_1) = 6n+4 \quad f(v_1') = 1$

$f(v_{3n+2}) = \begin{cases} \frac{9n+6}{2}, & n \text{ is even} \\ \frac{9n+5}{2}, & n \text{ is odd} \end{cases} \qquad f(v_{3n+2}') = \begin{cases} \frac{3n+4}{2}, & n \text{ is even} \\ \frac{3n+5}{2}, & n \text{ is odd} \end{cases}$

For $i = 1, 3, 5, \ldots 2\left\lceil \frac{n}{2} \right\rceil - 1$

$f(v_{3i}) = 6n+3-\frac{3i-3}{2}, \qquad f(v_{3i+1}) = 6n+2-\frac{3i-3}{2},$
$f(v_{3i-1}) = 3n+3+\frac{3i-3}{2}, \qquad f(v_{3i}') = \frac{3i+1}{2},$
$f(v_{3i+1}') = 1+\frac{3i+1}{2}, \qquad f(v_{3i-1}') = 3n+2-\frac{3i-3}{2}.$

For $i = 2, 4, 6, \ldots 2\left\lfloor \frac{n}{2} \right\rfloor$

$f(v_{3i}) = 3n+4+\frac{3i-6}{2}, \qquad f(v_{3i+1}) = 3n+5+\frac{3i-6}{2},$
$f(v_{3i-1}) = 6n+4-\frac{3i}{2}, \qquad f(v_{3i}') = 3n+1-\frac{3i-6}{2},$
$f(v_{3i+1}') = 3n-\frac{3i-6}{2}, \qquad f(v_{3i-1}') = 1+\frac{3i}{2},$

The edges of $EDG(T_n)$ are labeled as follows:

$f(v_1v_2') = 6n+5 \quad f(v_1'v_2) = 12n+7$

For $i = 1, 3, 5, \ldots 2\left\lceil \frac{n}{2} \right\rceil - 1$

$f(v_{3i-1}v_{3i+2}') = 12n+7-3i, \qquad f(v_{3i}v_{3i-1}') = 6n+3+3i,$
$f(v_{3i+1}v_{3i-1}') = 6n+4+3i, \qquad f(v_{3i-1}'v_{3i+2}) = 6n+5+3i,$
$f(v_{3i}'v_{3i-1}) = 12n+9-3i, \qquad f(v_{3i+1}'v_{3i-1}) = 12n+8-3i.$

For $i = 2, 4, 6, \ldots 2\left\lfloor \frac{n}{2} \right\rfloor$

$f(v_{3i-1}v'_{3i+2}) = 6n+5+3i,$ $\quad f(v_{3i}v'_{3i-1}) = 12n+9-3i,$
$f(v_{3i+1}v'_{3i-1}) = 12n+8-3i,$ $\quad f(v'_{3i-1}v_{3i+2}) = 12n+7-3i,$
$f(v'_{3i}v_{3i-1}) = 6n+3+3i,$ $\quad f(v'_{3i+1}v_{3i-1}) = 6n+4+3i,$ and
$f(v_{3n+1}v'_{3n+1}) = 9n+6.$

The induced edge sum function f^* defined by $f^*(uv) = f(u)+f(v)+f(uv)$, gives the induced edge sums as below.

$f^*(v_1v'_2) = 6n+4+3n+2+6n+5 = 15n+11$

$f^*(v'_1v_2) = 1+3n+3+12n+7 = 15n+11$

For $i = 1,3,5,\ldots 2\left\lceil \frac{n}{2} \right\rceil - 1$

$f^*(v_{3i-1}v'_{3i+2}) = 3n+3+\frac{3i-3}{2}+1+\frac{3i+2+1}{2}+12n+7-3i = 15n+11,$

$f^*(v_{3i}v'_{3i-1}) = 6n+3-\frac{3i-3}{2}+3n+2-\frac{3i-3}{2}+6n+3+3i = 15n+11,$

$f^*(v_{3i+1}v'_{3i-1}) = 6n+2-\frac{3i-3}{2}+3n+2-\frac{3i-3}{2}+6n+4+3i = 15n+11,$

$f^*(v'_{3i-1}v_{3i+2}) = 3n+2-\frac{3i-3}{2}+6n+4-\frac{3i+2+1}{2}+6n+5+3i = 15n+11,$

$f^*(v'_{3i}v_{3i-1}) = \frac{1+3i}{2}+3n+3+\frac{3i-3}{2}+12n+9-3i = 15n+11,$

$f^*(v'_{3i+1}v_{3i-1}) = \frac{3i+1}{2}+1+3n+3+\frac{3i-3}{2}+12n+8-3i = 15n+11,$

For $i = 2,4,6,\ldots 2\left\lfloor \frac{n}{2} \right\rfloor$

$f^*(v_{3i-1}v'_{3i+2}) = 6n+4-\frac{3i}{2}+3n+2-\frac{3i+3-3}{2}+6n+5+3i = 15n+11,$

$f^*(v_{3i}v'_{3i-1}) = 3n+4+\frac{3i-6}{2}+1+\frac{3i}{2}+12n+9-3i = 15n+11,$

$f^*(v_{3i+1}v'_{3i-1}) = 3n+5+\frac{3i-6}{2}+1+\frac{3i}{2}+12n+8-3i = 15n+11,$

$f^*(v'_{3i-1}v_{3i+2}) = 1+\frac{3i}{2}+3n+3+\frac{3i+3-3}{2}+12n+7-3i = 15n+11,$

$f^*(v'_{3i}v_{3i-1}) = 3n+1-\frac{3i-6}{2}+6n+4-\frac{3i}{2}+6n+3+3i = 15n+11,$

$f^*(v'_{3i+1}v_{3i-1}) = 3n-\frac{3i-6}{2}+6n+4-\frac{3i}{2}+6n+4+3i = 15n+11,$

and for the edge $v_{3n-1}v'_{3n-1}$ the edge induced sum is

$$f^*(v_{3n-1}v'_{3n-1}) = \begin{cases} 3n+3+\frac{3i-3}{2}+3n+2-\frac{3i-3}{2}+9n+6 = 15n+11, & n \text{ is odd} \\ 6n+4-\frac{3i}{2}+1+\frac{3i}{2}+9n+6 = 15n+11, & n \text{ is even} \end{cases}$$

Hence the extended duplicate graph of twig $EDG(T_n)$, $n \geq 2$ is super-edge magic total graph.

An illustration is given in Figure 5 (a).

Theorem 2.2. *The extended duplicate graph of comb $EDG(CB_n)$, $n > 2$, admits super-edge magic total labeling.*

Proof. The vertices of $EDG(CB_n)$ are labeled as follows:

for $i = 1$ to $2n$

$$f(v_i) = \begin{cases} \frac{i}{2}, & i \equiv 2 \bmod 4 \\ \frac{i+2}{2}, & i \equiv 3 \bmod 4 \\ 2n - \left(\frac{i-1}{2}\right), & i \equiv 1 \bmod 4 \\ 2n - \left(\frac{i-2}{2}\right), & i \equiv 0 \bmod 4 \end{cases}$$

$$f(v_i') = \begin{cases} 4n - \left(\frac{i-2}{2}\right), & i \equiv 2 \bmod 4 \\ 4n - \left(\frac{i-1}{2}\right), & i \equiv 3 \bmod 4 \\ 2n + 1 + \left(\frac{i-1}{2}\right), & i \equiv 1 \bmod 4 \\ 2n + 1 + \left(\frac{i-2}{2}\right), & i \equiv 0 \bmod 4 \end{cases}$$

The edges of $EDG(CB_n)$ are labeled as follows:
For, $k = 1, 3, 5, \ldots 2\left\lfloor \frac{n}{2} \right\rfloor - 1$

$f(v_{2k-1}v'_{2k}) = 4n + 2k - 1,$ $\quad f(v'_{2k-1}v_{2k}) = 8n - 2k + 1,$

$f(v_{2k-1}v'_{2k+1}) = 4n + 2k,$ $\quad f(v'_{2k-1}v_{2k+1}) = 8n - 2k,$

For $k = 2, 4, 6, \ldots 2(\left\lceil \frac{n}{2} \right\rceil - 1)$

$f(v_{2k-1}v'_{2k}) = 8n - 2k + 1,$ $\quad f(v'_{2k-1}v_{2k}) = 4n + 2k - 1,$

$f(v_{2k-1}v'_{2k+1}) = 8n - 2k,$ $\quad f(v'_{2k-1}v_{2k+1}) = 4n + 2k,$

$f(v_{2n-1}v'_{2n-1}) = 6n$

$f(v_{2n-1}v'_{2n}) = \begin{cases} 6n - 1, & n \text{ is odd} \\ 6n + 1, & n \text{ is even} \end{cases}$ $\quad f(v'_{2n-1}v_{2n}) = \begin{cases} 6n + 1, & n \text{ is odd} \\ 6n - 1, & n \text{ is even} \end{cases}$

The induced edge-sums $f^*(uv) = f(u) + f(v) + f(uv)$ are calculated as follows:
For $k = 1, 3, 5, \ldots 2\left\lfloor \frac{n}{2} \right\rfloor - 1$

$f^*(v_{2k-1}v'_{2k}) = 2n - \left(\frac{2k-1-1}{2}\right) + 4n - \frac{2k-2}{2} + 4n + 2k - 1 = 10n + 1,$

$f^*(v'_{2k-1}v_{2k}) = 2n + 1 + \frac{2k-1-1}{2} + \frac{2k}{2} + 8n - 2k + 1 = 10n + 1,$

$f^*(v_{2k-1}v'_{2k+1}) = 2n - \frac{2k-1-1}{2} + 4n - \frac{2k+1-1}{2} + 4n + 2k = 10n + 1,$

$f^*(v'_{2k-1}v_{2k+1}) = 2n + 1 + \frac{2k-1-1}{2} + \frac{2k+1+1}{2} + 8n - 2k = 10n + 1,$

For $k = 2, 4, 6, \ldots 2(\left\lceil \frac{n}{2} \right\rceil - 1)$

$f^*(v_{2k-1}v'_{2k}) = \frac{2k-1+1}{2} + 2n + 1 + \frac{2k-2}{2} + 8n - 2k + 1 = 10n + 1,$

$f^*(v'_{2k-1}v_{2k}) = 4n - \frac{2k-1-1}{2} + 2n - \frac{2k-2}{2} + 4n + 2k - 1 = 10n + 1,$

$f^*(v_{2k-1}v'_{2k+1}) = \frac{2k-1+1}{2} + 2n + 1 + \frac{2k+1-1}{2} + 8n - 2k = 10n + 1,$

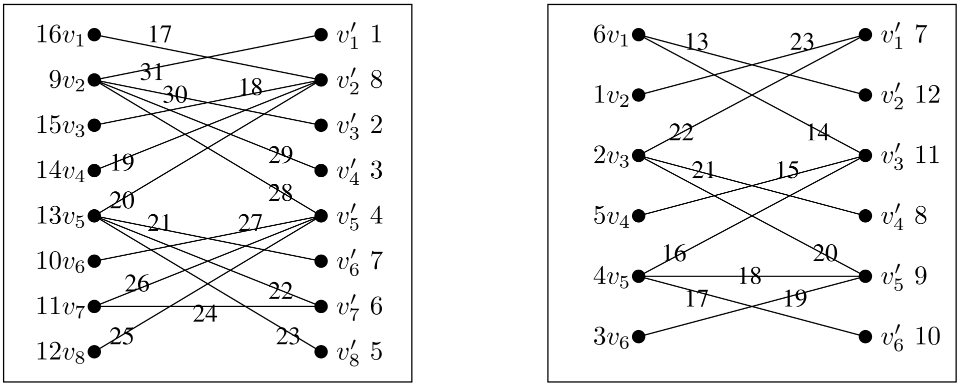
$f^*(v'_{2k-1}v_{2k+1}) = 4n - \left(\frac{2k-1-1}{2}\right) + 2n - \left(\frac{2k+1-1}{2}\right) + 4n + 2k = 10n + 1,$

and for the edge $v_{2n-1}v'_{2n-1}$ the induced edge sum is

$$f^*(v_{2n-1}v'_{2n-1}) = \begin{cases} \frac{2n-1+1}{2} + 4n - \frac{2n-1-1}{2} + 6n = 10n+1, & n \equiv 0 \bmod 4 \\ 2n - \frac{2n-1-1}{2} + 2n + 1 + \frac{2n-1-1}{2} + 6n = 10n+1, & n \equiv 1 \bmod 4 \\ \frac{2n-1+1}{2} + 4n - \frac{2n-1-1}{2} + 6n = 10n+1, & n \equiv 2 \bmod 4 \\ 2n - \frac{2n-1-1}{2} + 2n + 1 + \frac{2n-1-1}{2} + 6n = 10n+1, & n \equiv 3 \bmod 4 \end{cases}$$

Also, the edge induced sums for the edges $f^*(v_{2n-1}v'_{2n}) = 10n+1$ and $f^*(v_{2n}v'_{2n-1}) = 10n+1$ for all n which results into a super-edge magic total labeling of $EDG(CB_n)$, $n > 2$.

An illustration is given in Figure 5 (b).



(a) (b)

Figure 5: (a) Super-edge magic total labeling of $EDG(T_2)$, (b) Super-edge magic total labeling of $EDG(CB_3)$

Theorem 2.3. *The extended duplicate graph of star $EDG(K_{1,n})$, $n \geq 2$ admits super-edge magic total labeling.*

Proof. The vertices of $EDG(K_{1,n})$ are labeled as follows:

$f(v_i) = 2n + 3 - i, i = 1, 2, \ldots, n+1 \qquad f(v'_i) = i, i = 1, 2, \ldots, n+1$

The edges of $EDG(K_{1,n})$ are labeled as follows:

$f(v_1v'_i) = 3n + 4 - i, i = 1, 2, 3, \ldots, n+1$

$f(v'_1v_i) = 3n + 2 + i, i = 1, 2, 3, \ldots, n+1$

The induced edge sums $f^*(uv) = f(u) + f(v) + f(uv)$ are

$$f^*(v_1v_i') = 2n + 2 + i + 3n + 4 - i = 5n + 6$$
$$f^*(v_1'v_i) = 1 + 2n + 3 - i + 3n + 2 + i = 5n + 6 \quad \text{for } i = 1, 2, \ldots, n + 1$$

Thus labeling is super-edge magic total labeling for $n \geq 2$

An illustration is given in Figure 6 (a).

Theorem 2.4. *The extended duplicate graph of bi-star $EDG(B_{m,m})$, $m \geq 1$ admits super-edge magic total labeling.*

Proof. The vertices of $EDG(B_{m,m})$ are labeled as follows:

$$f(v_1) = 4m + 4 \quad f(v_2) = 1 \quad f(v_1') = 3m + 3 \quad f(v_2') = m + 2$$

$$f(v_i) = i - 1, 2 \leq i \leq m + 2$$
$$f(v_i) = 2m + 1 + i, m + 3 \leq i \leq 2m + 2$$
$$f(v_i') = m + i, 2 \leq i \leq 2m + 2$$

The edges of $EDG(B_{m,m})$ are labeled as follows:

$$f(v_1v_2') = 5m + 5 \quad f(v_1'v_2') = 6m + 6 \quad f(v_1'v_2) = 7m + 7$$

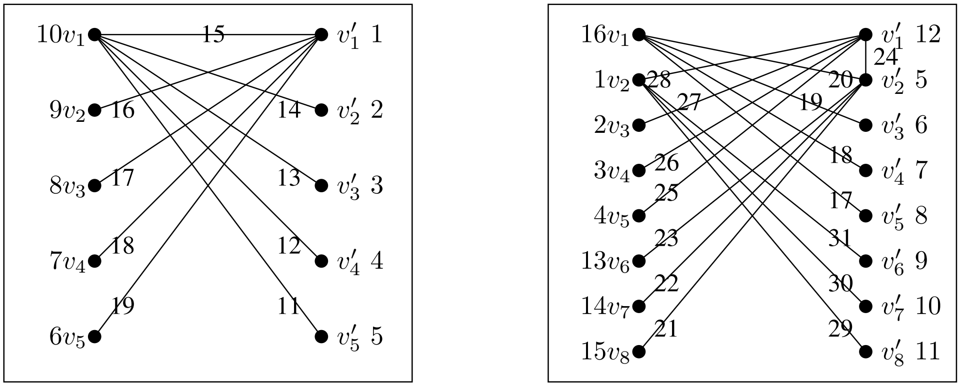
$$f(v_1v_i') = 5m + 7 - i, 3 \leq i \leq m + 2$$
$$f(v_1'v_i) = 7m + 9 - i, 3 \leq i \leq m + 2$$
$$f(v_2'v_i) = 7m + 8 - i, m + 3 \leq i \leq 2m + 2$$
$$f(v_2v_i') = 9m + 10 - i, m + 3 \leq i \leq 2m + 2$$

The induced edge-sums are calculated as $f^*(uv) = f(u) + f(v) + f(uv)$

$$f^*(v_1v_i') = 4m + 4 + m + i + 5m + 7 - i = 10m + 11, i = 2, 3, \ldots, m + 2$$
$$f^*(v_1'v_2') = 3m + 3 + m + 2 + 6m + 6 = 10m + 11,$$
$$f^*(v_1'v_2) = 3m + 3 + m + 1 + 7m + 7 = 10m + 11,$$
$$f^*(v_1'v_i) = 3m + 3 + i - 1 + 7m + 9 - i = 10m + 11, i = 3, 4, \ldots, m + 2$$
$$f^*(v_2'v_i) = m + 2 + 2m + 1 + i + 7m + 8 - i = 10m + 11, m + 3 \leq i \leq 2m + 2$$
$$f^*(v_2v_i') = 1 + m + i + 9m + 10 - i = 10m + 11, m + 3 \leq i \leq 2m + 2$$

and so the labeling is super-edge magic total labeling for $m \geq 1$.

An illustration is given in Figure 6 (b).



(a) (b)

Figure 6: (a) Super-edge magic total labeling of $EDG(K_{1,4})$, (b) Super-edge magic total labeling of $EDG(B_{3,3})$

3. Conclusion

We proved that extended duplicate graphs of Twig, Comb, Star and Bi-star admit Super-edge magic total labeling.

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