

Stellar Structures in Two-Dimensions

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Abstract: We have presented stellar structure in two-dimensions, Friedmann cosmology in two-dimensions and Schwarzschild-like solutions in two-dimensions.

1. Introduction

We have presented and obtained the field equations of a completely geometric Lagrangian based dynamical theory of gravitation in two dimension by using algebraically extended Hilbert theory of extension was investigated so as to give a framework where nontrivial geometric Lagrangian based extension of general theory of relativity could be studied.

Let us consider a self-gravitating body of mass M and radius R , so it will have gravitational potential energy

$$U = (GM^2/R) \quad (1.1)$$

If it is in equilibrium, then temperature T reads

$$Nk_B T \approx (GM^2/R) \quad (1.2)$$

or

$$T \approx (GMm_p/k_B R). \quad (1.3)$$

Let us consider a spherically symmetric star in a steady state where all physical variables depend on only radial coordinate r . The equation for hydrostatic equilibrium of a such a star is given as

$$\frac{dp}{dr} = p' = -G \frac{G(r)\rho(r)}{r^2}, \quad (1.4)$$

where $p(r)$ and $\rho(r)$ be the pressure and density respectively at radius r and $M(r)$ be the mass within the sphere of radius r , with

$$\frac{p(r)}{dr} = 4\pi r^2 \rho. \quad (1.5)$$

We have two equations connecting three variables $p(r)$, $\rho(r)$ and $M(r)$, so one needs a barotropic equation of the form

$$p = p(\rho), \quad (1.6)$$

to obtain

$$p \propto \rho^{5/3} \quad (1.7)$$

in nonrelativistic limit.

Therefore, we obtain the general equation for hydrostatic equilibrium in two-dimensions.

$$-p'' = 4G\pi(\rho^2 - p^2) - \frac{2\rho'^2 + p'\rho'}{p + \rho} \quad (1.8)$$

For given equation of state

$$p = p(\rho) \quad (1.9)$$

one obtains

$$\rho = \rho(r). \quad (1.10)$$

Let us compare it with Newtonian equation

$$p'' = 4\pi G\rho^2 - \frac{p'\rho'}{\rho} \quad (1.11)$$

2. Two Dimensional Friedmann Cosmology

In order to obtain the simplest model of the universe, let us assume that the geometrical features of three-dimensional space are the same at all spatial points and these geometrical features do not single out any special direction in space.

Let us consider now in two-dimensional case, the metric reads

$$ds^2 = -dt^2 + \frac{a^2(t)}{1 - kx^2} dx^2. \quad (2.1)$$

one may absorb the denominator $(1 - kx^2)$ into the definition of the spatial coordinate, and we get the metric

$$ds^2 = -dt^2 + a^2(t) dx^2, \quad (2.2)$$

where a be the cosmic scale factor. Let us now observe that in two-dimensions one does not have three different cosmological models corresponding to $k = 0 \pm 1$, in eq. (2.2).

It describes to a large $\pm r$ or two-dimensional spacetime is not asymptotically Riemannian at large distances from the origin. It is an important to note that the transformation to flat space breaks down at this point.

3. Conclusion:

We have presented and obtained the field equations of a completely geometric Lagrangian based dynamical theory of gravitation in two dimensions by using algebraically extended Hilbert theory of gravitation in two dimensions. The method of algebraic extension was investigated so as to give a framework where nontrivial geometric Lagrangian based extensions of general theory of relativity could be studied. We have presented stellar structure in two dimensions, Friedmann cosmology in two dimensions. However, it should be pointed out that the same method occurs in four-dimensional algebraically extended Hilbert theory of gravitation. There by this theory is presented as a viable for both pedagogical purposes as well as a laboratory for the testing of new theoretical views.

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