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NON-EXISTENCE OF ENTIRE SOLUTIONS OF NON-LINEAR GENERAL DIFFERENCE EQUATIONS

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Abstract: The main objective of this article is to investigate the solvability of a non-linear difference equation generated by general difference polynomial of a transcendental entire function of finite order.

Keywords and Phrases: Transcendental entire function, meromorphic function, small function, non-linear difference polynomial of a function, finite order, Nevanlinna theory.

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1. Introduction, Notations, Definitions and Main Results

Many problems arising in a wide variety of application areas like physics, engineering, biology, ecology and economics give rise to mathematical models which includes complex difference equations. In studying difference equations in the complex plane $\mathbb{C}$, it is an interesting and quite difficult to prove the existence of transcendental entire solution of a given difference equation.

Recently, Nevanlinna's theory has been utilising by many researchers to study the properties of entire or meromorphic solutions of differential-difference equations in the complex plane.

In order to introduce our work, we assume that the reader is familiar with the fundamental results of Nevanlinna theory and its standard notations such as characteristic function $T(r, f)$, proximity function $m(r, f)$ and counting function for poles $N(r, f)$ of a non-constant meromorphic function in $\mathbb{C}$. (See [7], [23]). We denote by $\rho(f)$ the order of growth of a meromorphic function $f(z)$ which is defined as

$$\rho(f) = \limsup_{r\to\infty} \frac{\log T(r, f)}{\log r},$$

and $\rho_2(f)$ by the hyper order of growth of $f(z)$ which is defined by

$$\rho_2(f) = \limsup_{r\to\infty} \frac{\log\log T(r, f)}{\log r}.$$

Throughout the paper we denote by $S(r, f)$ any function satisfying $S(r, f) = o(T(r, f))$ as $r \to +\infty$, possibly outside of finite measure.
A meromorphic function $a(z)$ is said to be small function of $f(z)$ or a function with slow growth as measured by the Nevanlinna characteristic if it satisfies $T(r, a(z)) = S(r, f)$. We call

$$M_\lambda(z, f) = (f(z))^{i_{\lambda,0}}(f(z + c_1))^{i_{\lambda,1}}(f(z + c_2))^{i_{\lambda,2}} \cdots (f(z + c_k))^{i_{\lambda,k}}$$

is a monomial in $f(z)$ and its shifts $f(z+c_1), \cdots, f(z+c_k)$, where $c_1, c_2, c_3, \cdots, c_k$ are distinct non-zero complex constants, $i_{\lambda,0}$, $i_{\lambda,1}, \cdots, i_{\lambda,k}$ are non-negative integers and $d(\lambda) = i_{\lambda,0} + i_{\lambda,1} + \cdots + i_{\lambda,k}$ is its degree. Further

$$\begin{aligned} P(z, f) &= \textstyle\sum_{\lambda\in I} a_\lambda(z) M_\lambda(z, f) \\ &= \sum_{\lambda\in I} a_\lambda(z)(f(z))^{i_{\lambda,0}}(f(z + c_1))^{i_{\lambda,1}}(f(z + c_2))^{i_{\lambda,2}} \cdots (f(z + c_k))^{i_{\lambda,k}} \end{aligned} \tag{1.1}$$

is called a difference polynomial in $f(z)$ and its shifts, and $d(P) = \max_{\lambda\in I} d(\lambda)$ is its degree, where I is a finite set of the index $\lambda = \{i_{\lambda,0}, i_{\lambda,1}, \cdots, i_{\lambda,k}\}$ and $a_\lambda(z)$ are coefficients being small with respect to $f(z)$ in the sense that $T(r, a_\lambda) = S(r, f)$, $\lambda \in I$.

Recently, many researchers have been focusing on existence and solvability of entire and meromorphic solution of differential and difference equations (See [1] - [3], [5] - [6], [11] - [22], [24] - [26]). In particular, the following results are obtained for difference equations.
In the year 2014, X. Qi, J. Dou and L. Yang in [16] studied on existence of an entire solution of a non-linear difference equation and proved the following theorem.

Theorem 1.A. [16] *Consider non-linear difference equation of the form*

$$f^n(z) + p(z)(\Delta_c f)^m = r(z)e^{q(z)}, \tag{1.2}$$

where $p(z) \not\equiv 0$, $q(z)$, $r(z)$ be polynomials, n and m are positive integers, $\Delta_c f = f(z+c) - f(z) \not\equiv 0$ is a difference operator. Suppose that $f(z)$ is a transcendental entire function of finite order, not of period c. If $n > m$, then $f(z)$ cannot be a solution of (1.2).

Later, in the year 2016 in [5], Dyavanal and Mathai considered the above theorem for the difference equation (1.2) in which $(\Delta_c f)^m$ is replaced by general linear difference polynomial and proved the following theorem.

Theorem 1.B. [5] *Let $n > 1$ be an integer, $L(z, y) = a_0 y(z) + a_1 y(z + c_1) + \cdots + a_k y(z + c_k)$ be a non-zero linear difference polynomial of $y(z)$ and $p(z) \not\equiv 0$, $q(z)$ and $r(z)$ be polynomials. Consider a linear difference equation of the form*

$$f^n(z) + p(z)L(z, f) = r(z)e^{q(z)}. \tag{1.3}$$

Then a transcendental entire function $f(z)$ of finite order cannot be a solution of (1.3).

Our main purpose of this paper is to replace linear difference operator $L(z, f) = a_0 f(z)+a_1 f(z+c_1)+\cdots+a_k f(z+c_k)$ by difference polynomial $P(z, f)$ as defined in (1.1) in non-linear difference equation (1.3) and investigate for it's solution. Meanwhile, we proved the following theorem which says that any transcendental entire function of finite order such that all coefficients of $P(z, f)$ are small with respect to $f(z)$ cannot be a solution of difference equation (1.3) even if we replace linear difference operator $L(z, f)$ by general difference polynomial $P(z, f)$ as defined in (1.1).

Theorem 1.1. *Let $n > d(P)$ be an integer and $P(z, f)(\not\equiv 0)$ is difference polynomial as defined in (1.1). If $t(z)$ and $r(z)$ are polynomials, then any transcendental entire function $f(z)$ of finite order such that all coefficients of $P(z, f)$ are small with respect to $f(z)$ cannot be a solution of non-linear difference equation of the form*

$$f^n(z) + P(z, f) = r(z)e^{t(z)}. \tag{1.4}$$

Remark 1.1. *The following example shows that the condition $n > d(P)$ in the above theorem is sharp.*
It is easy to verify that transcendental entire function $f(z) = e^z$ of order 1 satisfies non-linear difference equation of the type (1.4) when $n = 3$ and $P(z, f) = zf(z)f^2(z+1) + 2z^2 f(z+1)f^2(z+2) + 3z^3 f^3(z)$.
Here note that $n = d(P) = 3$.

Remark 1.2. ***Theorem 1.A*** *and* ***Theorem 1.B*** *are particular cases of* ***Theorem 1.1*** *whenever $P(z, f)$ is $\Delta_c f = f(z+c)-f(z)$ and linear difference polynomial*

$L(z, f)$ as $d(P)$ is m and 1 respectively.

2. Some Preliminary Results

To prove our theorem, we require the following lemmas.

Lemma 2.1. [9] *Let $f(z)$ be a non-constant meromorphic function of finite order, $c \in \mathbb{C}$ and $\delta < 1$. Then*

$$m\left(r, \frac{f(z+c)}{f(z)}\right) = o\left(\frac{T(r,f)}{r^{\delta}}\right)$$

for all r outside of a possible exceptional set E with a finite logarithmic measure.

Lemma 2.2. [21] *Let $f(z)$ be a transcendental meromorphic solution of finite order $\rho(f)$ of a difference equation of the form*

$$H(z,f)P(z,f) = Q(z,f),$$

where $H(z,f)$, $P(z,f)$ and $Q(z,f)$ are difference polynomials in $f(z)$ such that the total degree of $H(z,f)$ in $f(z)$ and its shifts is n, and that the corresponding total degree of $Q(z,f)$ is $\leq n$. If $H(z,f)$ contains just one term of maximal total degree, then for any $\epsilon > 0$,

$$m(r, P(z,f)) = O(r^{\rho(f)-1+\epsilon}) + S(r,f),$$

possibly outside of an exceptional set of finite logarithmic measure.

Yang and Laine in ([21]) further pointed out the following.

Remark 2.1. *If in the above Lemma 2.2, $H(z,f) = f^n$, then a similar conclusion holds, if $P(z,f)$, $Q(z,f)$ are differential-difference polynomial in f.*

Lemma 2.3. [10] *Let $f(z)$ be a transcendental meromorphic function with $\delta(\infty, f) > 0$, and let $P(f(z))$ be an algebraic polynomial in $f(z)$ of the form*

$$P(f(z)) = a_n(z)f^n(z) + a_{n-1}(z)f^{n-1}(z) + \cdots + a_1(z)f(z) + a_0(z),$$

where $a_n(z) \not\equiv 0$, $a_j (j = 0, \cdots, n)$ satisfy $m(r, a_j) = S(r,f)$, then

$$m(r, P(f)) \leq nm(r,f) + S(r,f).$$

Lemma 2.4. [23] *Let $f_j(z)(j = 1, 2, 3)$ be meromorphic functions that satisfy*

$$\textstyle\sum_{j=1}^{3} f_j(z) \equiv 1.$$

If $f_1(z)$ is not a constant, and

$$\sum_{j=1}^{3} N\left(r, \frac{1}{f_j}\right) + 2\sum_{j=1}^{3} \overline{N}(r, f_j) \leq S(r, f) < (\alpha + o(1))T(r),$$

where $\alpha < 1$ *and* $T(r) = max_{1\leq j\leq 3}T(r, f_j)$, *then either* $f_2(z) \equiv 1$ *or* $f_3(z) \equiv 1$.

3. Proof of Theorem 1.1.

First we examine two cases for $t(z)$ and $r(z)$.

Case 1. If $t(z)$ is constant or $r(z) \equiv 0$ in (1.4), then it leads $f^n(z)+P(z, f) = K(z)$, where $K(z)$ is a polynomial. This implies

$$f^n(z) = K(z) - P(z, f).$$

Hence, $\quad nT(r, f(z)) \leq T(r, K(z)) + d(P)T(r, f) + S(r, f).$

Which leads to contraction for $n > d(P)$ and $f(z)$ is transcendental. Thus, a transcendental entire function $f(z)$ of finite order cannot be solution of (1.4) whenever $t(z)$ is constant or $r(z) \equiv 0$.

Case 2. Consider $t(z)$ is a non-constant polynomial and $r(z) \not\equiv 0$. Assume that $f(z)$ is finite order transcendental entire solution of (1.4).

Differentiating (1.4) and eliminating $e^{t(z)}$, we have

$$f^{n-1}(z)\left[nf^{'}(z) - \left(t^{'}(z) + \frac{r^{'}(z)}{r(z)}\right)f(z)\right] = \left(t^{'}(z) + \frac{r^{'}(z)}{r(z)}\right)P(z, f) - P^{'}(z, f). \tag{3.1}$$

If $nf^{'}(z) - \left(t^{'}(z) + \dfrac{r^{'}(z)}{r(z)}\right)f(z) \equiv 0$, then we have $f^n(z) = Cr(z)e^{t(z)}$, where C is non-zero constant. Hence

$$T(r, Cr(z)e^{t(z)}) = nT(r, f) + S(r, f). \tag{3.2}$$

Let $v^n(z) = Cr(z)$, then $f(z) = v(z)\,e^{\frac{t(z)}{n}}$.

Hence (1.4) becomes

$$(C - 1)r(z)e^{t(z)} + P(z, f) = 0. \tag{3.3}$$

Notice that, if $C = 1$, then $P(z, f) \equiv 0$, which contradicts to the assumption. Thus $C \neq 1$, and if $h(z) = e^{\frac{t(z)}{n}}$, then substituting $f(z) = v(z)h(z)$ in $P(z, f)$, we get

$P(z, f) = \sum_{\lambda\in I} a_\lambda(z)(v(z)h(z))^{i_{\lambda,0}}(v(z+c_1)h(z+c_1))^{i_{\lambda,1}} \cdots (v(z+c_k)h(z+c_k))^{i_{\lambda,k}}.$

Next, $P(z, f)$ can be arranged by collecting together all terms having the same total degree and writing $P(z, f)$ as

$$P(z, f) = \sum_{p=0}^{d(P)} B_p(z)h^p(z), \tag{3.4}$$

where the coefficients $B_p(z)$, $p = 0, 1, \cdots, d(P)$ is a finite sum of products of functions of the form

$$\left(\frac{v(z+c_i)h(z+c_i)}{h(z)}\right)^{i_\lambda, j}$$

with each such product being multiplied by the original coefficient $a_\lambda(z)$.
By Lemma 2.1 and hypothesis on the coefficients a_λ, we get

$$m(r, B_p(z)) = S(r,h), \ p = 0, 1, \cdots, degP(z,f). \tag{3.5}$$

Using (3.4), (3.5) and Lemma 2.3, we see that

$$m(r, P(z,f)) \leq d(P)m(r,h) + S(r,h), \tag{3.6}$$

which yields

$$T(r, P(z,f)) = m(r, P(z,f)) + N(r, P(z,f)) = m(r, P(z,f))$$

$$\leq d(P)T(r, h(z)) + S(r, h(z)). \tag{3.7}$$

Combining (3.7) with (3.3), we have

$$T\left(r, (1-C)r(z)e^{t(z)}\right) = T(r, (1-C)r(z)h^n(z)) = T(r, P(z,f))$$
$$\leqslant d(P)T(r, h(z)) + S(r,h). \tag{3.8}$$

Using $h(z) = e^{\frac{t(z)}{n}}$ and (3.8), we obtain that

$$(n - d(P))T(r, h(z)) \leqslant S(r,h),$$

which is contradiction to $n > d(P)$. Hence $nf^{'}(z) - \left(t^{'}(z) + \frac{r^{'}(z)}{r(z)}\right)f(z) \not\equiv 0$.
Now, we consider the following two subcases for n.
Subcase 1. Let $n > d(P) + 1$. We rewrite (3.1) as

$$f^n(z)\left[nf^{'}(z) - \left(t^{'}(z) + \frac{r^{'}(z)}{r(z)}\right)f(z)\right]$$
$$= \left(t^{'}(z) + \frac{r^{'}(z)}{r(z)}\right)P(z,f)f(z) - P^{'}(z,f)f(z) \tag{3.9}$$

and

$$f^{n}(z)\left[f(z)\left(nf^{'}(z)-\left(t^{'}(z)+\frac{r^{'}(z)}{r(z)}\right)f(z)\right)\right]$$
$$=\left(t^{'}(z)+\frac{r^{'}(z)}{r(z)}\right)P(z,f)f^{2}(z)-P^{'}(z,f)f^{2}(z). \tag{3.10}$$

Applying Lemma 2.2 and Remark 2.1 to (3.9) and (3.10), we get

$$m\left(r,nf^{'}(z)-\left(t^{'}(z)+\frac{r^{'}(z)}{r(z)}\right)f(z)\right)=S(r,f), \tag{3.11}$$

and

$$m\left(r,f(z)\left(nf^{'}(z)-\left(t^{'}(z)+\frac{r^{'}(z)}{r(z)}\right)f(z)\right)\right)=S(r,f). \tag{3.12}$$

Since $f(z)$ is an entire function and from (3.11) and (3.12), we get

$$T\left(r,nf^{'}(z)-\left(t^{'}(z)+\frac{r^{'}(z)}{r(z)}\right)f(z)\right)=S(r,f), \tag{3.13}$$

and

$$T\left(r,f(z)\left(nf^{'}(z)-\left(t^{'}(z)+\frac{r^{'}(z)}{r(z)}\right)f(z)\right)\right)=S(r,f). \tag{3.14}$$

It follows from (3.13) and (3.14) that $T(r,f(z))=S(r,f)$, a contradiction.
Subcase 2. If $n=d(P)+1$, then (1.4) yields

$$f^{d(P)+1}(z)+P(z,f)=r(z)e^{t(z)}. \tag{3.15}$$

This leads to

$$f^{d(P)}(z)\left[(d(P)+1)f^{'}(z)-\left(t^{'}(z)+\frac{r^{'}(z)}{r(z)}\right)f(z)\right]$$
$$=\left(t^{'}(z)+\frac{r^{'}(z)}{r(z)}\right)P(z,f)-P^{'}(z,f). \tag{3.16}$$

$$\text{Let } Q=(d(P)+1)f^{'}(z)-\left(t^{'}(z)+\frac{r^{'}(z)}{r(z)}\right)f(z).$$

Since $f(z)$ is entire function, we get $N(r,Q)=S(r,f)$. Applying Lemma 2.2 to (3.16), we obtain that $m(r,Q)=S(r,f)$ which in turn gives $T(r,Q)=S(r,f)$. Differentiating $Q(z)$, we get

$$(d(P)+1)f''(z)-\left(t'(z)+\frac{r'(z)}{r(z)}\right)'f(z)-\left(t'(z)+\frac{r'(z)}{r(z)}\right)f'(z)=\frac{Q'(z)}{Q(z)}Q(z).$$

This implies $(d(P)+1)f''(z)-\left(t'(z)+\frac{r'(z)}{r(z)}+(d(P)+1)\frac{Q'(z)}{Q(z)}\right)f'(z)-$

$$\left(t''(z)-\frac{Q'(z)}{Q(z)}t'(z)+\left(\frac{r'(z)}{r(z)}\right)'-\frac{Q'(z)r'(z)}{Q(z)r(z)}\right)f(z)=0.$$

The above equation can be rewritten in the following form

$$(d(P)+1)\left(\left(\frac{f'(z)}{f(z)}\right)'+\left(\frac{f'(z)}{f(z)}\right)^2\right)-\left(t'(z)+\frac{r'(z)}{r(z)}+(d(P)+1)\frac{Q'(z)}{Q(z)}\right)\frac{f'(z)}{f(z)}$$

$$-\left(t''(z)-t'(z)\frac{Q'(z)}{Q(z)}+\left(\frac{r'(z)}{r(z)}\right)'-\frac{Q'(z)r'(z)}{Q(z)r(z)}\right)=0. \tag{3.17}$$

By (3.17) it is clear that $N(r,\frac{1}{f})$ is $S(r,f)$. Thus, by Hadamard's factorization theorem $f(z)$ can be expressed as $f(z)=B(z)e^{d(z)}$, where $B(z)$ is an entire function satisfying $N\left(r,\frac{1}{B(z)}\right)=S(r,f)$ and $d(z)$ is non-constant polynomial. Substituting $f(z)=B(z)e^{d(z)}$ in (3.15), we have

$$B^{d(P)+1}(z)e^{(d(P)+1)d(z)}+a_0(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}$$

$$\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}+\sum_{\lambda=1}^{k}a_\lambda(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}$$

$$\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}=r(z)e^{t(z)}.$$

This implies

$$-\frac{B^{d(P)+1}(z)e^{(d(P)+1)d(z)}}{a_0(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}}$$

$$+\frac{r(z)e^{t(z)}}{a_0(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}}$$

$$-\frac{\sum_{\lambda=1}^{k} a_\lambda(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}}{a_0(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}} = 1.$$

Defining f_1, f_2 and f_3 by

$$f_1 = -\frac{B^{d(P)+1}(z)e^{(d(P)+1)d(z)}}{a_0(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}},$$

$$f_2 = \frac{r(z)e^{t(z)}}{a_0(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}},$$

$$f_3 = -\frac{\sum_{\lambda=1}^{k} a_\lambda(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}}{a_0(z)(B(z)e^{d(z)})^{i_{\lambda,0}}(B(z+c_1)e^{d(z+c_1)})^{i_{\lambda,1}}\cdots(B(z+c_k)e^{d(z+c_k)})^{i_{\lambda,k}}},$$

which immediately yield that

$$f_1 + f_2 + f_3 = 1.$$

Notice that f_1 is non-constant and we deduce that

$$\sum_{j=1}^{3} N(r, \frac{1}{f_j}) + 2\sum_{j=1}^{3} \overline{N}(r, f_j) \le S(r, f) < (\alpha + o(1))T(r)$$

Thus by Lemma 2.4, we get either $f_2(z) \equiv 1$ or $f_3(z) \equiv 1$. If $f_2(z) \equiv 1$, then by (3.15), we deduce $T(r, f) \le S(r, f)$, which is contradiction. If $f_3(z) \equiv 1$, then $P(z, f) = 0$. By hypothesis, we again get a contradiction. Hence, transcendental entire solution of finite order such that all coefficients of $P(z, f)$ are small with respect to $f(z)$ cannot be a solution of (1.4).

Conclusions

- The key tool to prove our result is the Lemma 2.1 which extends to the case of hyper order $\rho_2(f) < 1$. Hence our result for the non-existence of entire solutions of finite order may be extended to the case of hyper order less than one as well.
- The condition that $n > d(P)$ in Theorem 1.1 is sharp for non-existence of entire solutions of finite order.
- Difference equations of the type (1.4) that we have considered in Theorem 1.1 is larger class which includes difference equations of the type (1.2) as well as (1.3). Therefore we can claim that our result generalizes the earlier results of X. Qi, J. Dou and L. Yang in [16] and Dyavanal and Mathai in [5].
- We would like to pose the following problem for further research work.

Conjecture: There exists no entire function of infinite order that satisfies a non-linear difference equation of the type

$$f^n(z) + P(z, f) = r(z)e^{t(z)},$$

where $n > d(P)$ be an integer, $P(z, f)$ is difference polynomial as defined in (1.1) and $t(z)$, $r(z)$ are polynomials.

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