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CONTRIBUTIONS TO P - π REGULAR IN NEAR-RING

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Abstract: In this paper, with the useful resource of defining P- π regular near-ring, we make a new method of π regular of order two in the near ring. Every P- π regular is a strongly P- π regular and additionally strongly P- π regular is a weakly P- π regular all are equivalent. And discussed some of the results. Every regular near ring is a π regular ring and π regular is a regular near-ring. Previously, we introduce the conception of strongly P-regular Near rings [9]. We have displayed that a Near ring N is strongly P-regular if and only if it is also regular. A Near-ring N is called left(right) strongly P-regular if for every 'a' there is an 'n' in N such that $a = na^2 + p$ $(a = a^2n + p)$ and $a = ana$, position P is an arbitrary ideal. We specify some new concepts and justify them with suitable examples. And also, we discuss some of the theorems related to it.

Keywords and Phrases: Near-ring[NR], π regular, P-π regular, π-regular of order 2.

2020 Mathematics Subject Classification: 16Y30, 12K05.

1. Introduction

In mathematics, a near-ring is an algebraic structure like a ring yet fulfilling less aphorism. Near-rings emerge naturally from functions on groups. Near-rings arise naturally from functions on the group. The antiquity of the concept of near-ring is eminent influenced by the knowledge of ring theory. A near-ring is a ring (not undoubtedly with unity) if and only if addition is commutative and multiplication

is also distributivity on both sides is ample, and commutative of addition follows unquestionably. The generalization of rings (Near-rings) plays a vital role in the development of mathematics.
Let R be an associative ring with unity. If R is π regular if for every $a \in R$ there exists a positive integer n such that $a^n \in a^n R a^n$. The very first point out of this concept dates back to 1939, when it was added by Mccoy in [10], as a generalization of von neumann regular rings. While Mccoys paper,most attention was given to regular π rings that are commutative, the study of the common case used to be persevered by using other authors. Azumaya [1] proved that each and every π regular ring with bounded index of nilpotence is strongly π regular. In this paper discussed about the p-π regular near ring.

Definition 1.1. *A NR N is P-π regular NR, if for each element $a \in N$, there exists a positive integer n and $x \in N$ such that $a^n = a^n x a^n + p$,where $p \in P$ be an arbitrary ideal of N.*

Definition 1.2. *A NR N is Strongly P-π regular NR, if for each element $a \in N$, there exists a positive integer n and $x \in N$ such that $a^n = xa^{2n} + p$,where $p \in P$ be an arbitrary ideal of N.*

Definition 1.3. *Let $N_c = \{a \in N/0.a = a\}$ is called the constant zero-symmetric part of Near-ring N.*

Example 1.1. Let $N = \{0, m, n, q\}$ with addition and multiplication be defined as below,

+	0	m	n	q
0	0	m	n	q
m	m	0	q	n
n	n	q	0	m
q	q	n	m	0

·	0	m	n	q
0	0	0	0	0
m	0	0	m	m
n	0	0	n	n
q	0	m	n	q

It can be seen that $(N, +, \cdot)$ is a P-π regular(where P is an arbitrary ideal) as well as it is π regular but N is not regular, as the element m is not a regular element.

Remark 1.1. *Every regular NR is π-regular but not conversely.*

Definition 1.4. *A NR N is Weakly P-π regular NR, if for each element $a \in N$, there exists a positive integer n and $x \in N$ such that $a^n = (a^n x)^3 x + p$,where $p \in P$ be an arbitrary ideal of N.*

Definition 1.5. *A NR N is π regular of order 2, if for each element $a \in N$, there exists a positive integer $n = 2$ and $x \in N$ such that $a^n = a^2 x a^2$.*

Example 1.2. *If Z_2 is NR. Then $Z_2 = \{0, 1\}$is a π regular of order 2.*

Remark 1.2. *Regular NR $\Rightarrow$ π regular*
π regular $\not\Rightarrow$ Regular NR.

Remark 1.3. *If N is a π regular NR then it is a π regular of order 2.*
Proof. We know that N is a π regular. (i.e) $a^n x a^n = a^n$ where $a \in N$and $x \in N$and n is positive integer.Now take $n = 2$ then we get $a^2 x a^2 = a^2$ π regular of order 2.

3. Main Results

Theorem 2.1. *Let N be a strongly P-π regular NR.Then it is a π regular NR, where $p \in P$ be an arbitrary ideal.*
Proof. Let N be a strongly P-π regular of N. By definition of strongly P-π regular of N. $a^n = a^{2n} x + p$.By zero-symmetric part of N.

$$\begin{aligned}
(a^n - (a^n x a^n + p)) &= 0 \\
a^n(a^n - (a^n x a^n + p)) &= a^n 0 \in N_c \\
a^n x a^n(a^n - (a^n x a^n + p)) &= a^n x a^n 0 \in N_c
\end{aligned}$$

Thus we have to prove $(a^n - (a^n x a^n + p))^3 = (a^n - (a^n x a^n + p))^2$ Then

$$\begin{aligned}
(a^n - (a^n x a^n + p))^2 &= (a^n - (a^n x a^n + p))(a^n - (a^n x a^n + p)) \\
&= a^n(a^n - (a^n x a^n + p)) - (a^n x a^n + p)(a^n - (a^n x a^n + p)) \\
&= a^n(a^n - (a^n x a^n + p)) - a^n x a^n(a^n - (a^n x a^n + p)) \\
&- p(a^n - (a^n x a^n + p)) \qquad \in N_c
\end{aligned}$$

Now

$$\begin{aligned}
(a^n - (a^n x a^n + p))^3 &= (a^n - (a^n x a^n + p))^2(a^n - (a^n x a^n + p)) \\
&\in N_c(a^n - (a^n x a^n + p)) \\
&\in N_c
\end{aligned}$$

Then $(a^n - (a^n x a^n + p)) = 0$,Hence by assumption N is P-π regular NR (i.e) $a^n = a^n x a^n + p$ where p is an arbitrary ideal.
Then we have,

$$\begin{aligned} a^n &= a^n x a^n + p \\ &= a^n x a^n + 0 \\ &= a^n x a^n \end{aligned}$$

Hence N is π regular NR.

Corollary 2.1. *Let N be a P-π regular NR. Then it is a π regular NR, where $p \in P$ be an arbitrary ideal.*

Theorem 2.2. *Let N be a NR. Then followings are equivalent,*

(i) P-π regular

(ii) Strongly P-π regular

(iii) weakly P-π regular

Proof. Let N be a P-π regular, by definition of p-π regular is $a^n = a^n x a^n + p$ for $a^n \in N$, n is a positive integer and $x \in N$ where p is an arbitrary ideal of N. Then $(i) \Rightarrow (ii)$ Assume N is P-π regular of NR.
We have to prove N is a strongly P-π regular NR. Now,

$$\begin{aligned} a^n &= a^n x a^n + p \\ &= a^n (a^n x) + p \\ &= (a^n a^n) x + p \\ &= a^{2n} x + p \end{aligned}$$

Hence N is a strongly P-π regular of N.
$(ii) \Rightarrow (iii)$ Assume that N is strongly P-π regular of NR.
Then we have to prove N is weakly P-π regular of NR.
Thus

$$\begin{aligned} a^n &= a^{2n} x + p \\ &= a^n a^n (xxx) + p \\ &= (a^n x a^n)(a^n x a^n)(xxx) + p \\ &= a^n (x a^n)(x a^n a^n)(xxx) + p \\ &= a^n (x a^n x)(a^n a^n)(xxx) + p \\ &= (a^n x a^n)(a^n a^n)(xxx) x + p \end{aligned}$$

$$
\begin{aligned}
&= a^n(a^na^n)(xxx)x + p \\
&= (a^na^na^n)(xxx)x + p \\
&= (a^nx)^3x + p
\end{aligned}
$$

Hence N is weakly P-π regular of NR.
$(iii) \Rightarrow (i)$ Assume that N is weakly P-π regular NR of N.
we have to prove N is P-π regular of NR.
Now

$$
\begin{aligned}
(a^nx)^3x + p &= (a^na^na^n)(xxx)x + p \\
&= a^na^n(a^nxx)xx + p \\
&= a^n(a^nxa^n)(xa^nx) + p \\
&= a^n(a^nx) + p \\
&= a^nxa^n + p
\end{aligned}
$$

Hence N is P-π regular of NR.

3. Conclusion

In mathematics, study on near-rings becomes an object of the exercise of several researchers. In this paper we made an attempt to study the concept of π regular near-ring were characterized and based a portion of the theorem were demonstrated. In a zero symmetric near-ring idea utilizing by the above theorems. In π regular notion and some simple outcomes on it are delivered via Azumaya [1], Badawi [2]. Moreover, in this paper, P- regular is a P- π regular but not conversely. Then P-π regular is a P- π regular of order two when $n = 2$.

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