*South East Asian J. of Mathematics and Mathematical Sciences*
*Vol. 18, No. 1 (2022), pp. 397-408*

**ISSN (Online): 2582-0850**

**ISSN (Print): 0972-7752**

# THE CHROMATIC DETOUR NUMBER OF A GRAPH

**R. Suganya and V. Sujin Flower**

Department of Mathematics,
Holy Cross College (Autonomous),
Nagercoil - 629004, Tamil Nadu, INDIA

E-mail : suganya.maths34@gmail.com, sujinflower@gmail.com

(**Received:** *May 18, 2021* **Accepted:** *Apr. 14, 2022* **Published:** *Apr. 30, 2022)*

**Abstract:** A set $S \subseteq V(G)$ is called a chromatic detour set of $G$ if $S$ is both a chromatic set and a detour set of $G$. The minimum cardinality of a chromatic detour set of $G$ is called a chromatic detour number of $G$ and is denoted by $\chi_{dn}(G)$. Some of its general properties are studied. Connected graphs of order $n \geq 2$ with chromatic detour number $n$ or $n-1$ are characterized. It is shown that for every positive integer $a$ and $b$ with $2 \leq a < b$, there exists a connected graph $G$ such that $dn(G) = a$ and $\chi_{dn}(G) = b$. It is also shown that for every positive integers $a$ and $b$ with $2 \leq a \leq b$, there exists a connected graph $G$ such that $\chi(G) = a$ and $\chi_{dn}(G) = b$.

**Keywords and Phrases:** Chromatic detour number, chromatic number, detour number.

**2020 Mathematics Subject Classification:** 05C12, 05C15.

## 1. Introduction

Throughout this paper all graphs are simple. Let $G = (V, E)$ be a graph with $V(G)$ is the vertex set of $G$ and $E(G)$ is the edge set of $G$. For basic graph theoretic terminology, we refer to [2]. In a connected graph $G$, for any two vertices $u, v \in V(G)$, let $d_G(u, v)$ denote the length of the shortest path between $u$ and $v$ in $G$. The *diameter* of graph is the maximum distance between the pair of vertices of $G$. The *subgraph induced* by a set $S$ of vertices of a graph $G$ is denoted by $G[S]$ with $V(G[S]) = S$ and $E(G[S]) = \{uv \in E(G) : u, v \in S\}$. A set $S \subset V$ is called a *clique* if $\langle S \rangle$ is complete. The *clique number* of $G$ is the number of vertices in a maximum clique and is denoted by $\omega(G)$. A *split graph* is a graph in which the vertices can be partitioned into a clique and an independent set. A *semi split graph* is a graph in which the vertices can be partitioned into a clique and a set of end vertices. The *Helm* $H_n$ is the graph obtained from a wheel by attaching a pendant edge at each vertex of the $n$-cycle. A *flower* is the graph obtained from a helm by joining each pendant vertex to the central vertex of the helm. A *Fan graph* $F_n$ can be constructed by joined $n$ copies of the cycle graph $C_3$ with a common vertex. $F_n$ is a planar undirected graph with $2n+1$ vertices and $3n$ edges.

A *k-coloring* of $G$ is a function $c : V(G) \to \{1, 2, ..., k\}$ such that $c(u) \neq c(v)$ for every adjacent vertices $u, v \in V(G)$. The *chromatic number* of $G$ denoted by $\chi(G)$, is the smallest $k$ for which $G$ has a $k$-coloring. For simplicity we denote a $\chi(G)$-coloring of $G$ by $\chi$-coloring. A graph having chromatic number $k$ is called a $k$-chromatic graph. Let $G$ be a $k$-chromatic graph. A set $S \subseteq V(G)$ is called *chromatic set* if $S$ contains all $k$ vertices of different colors in $G$. The chromatic number of a graph was studied in [1, 6, 7].

The *detour distance D(u,v)* between two vertices $u$ and $v$ in a connected graph $G$ from $u$ to $v$ is defined as the length of a longest $u-v$ path in $G$. A $u-v$ path of length $D(u,v)$ is called a $u-v$ *detour*. The detour eccentricity $e_D(v)$ of a vertex $v$ in $G$ is the maximum detour distance from $v$ to a vertex of $G$. The detour radius, $rad_D G$ of $G$, is the minimum detour eccentricity among the vertices of $G$, while the detour diameter, $diam_D G$ of $G$, is the maximum detour eccentricity among the vertices of $G$. Denote the detour radius and detour diameter by $R$ and $D$ respectively. A vertex $x$ is said to lie on a $u-v$ detour $P$ if $x$ is a vertex of $P$ including the vertices $u$ and $v$. For two vertices $u$ and $v$, the closed interval $I_D[u, v]$ consists of all vertices lying in a $u$-$v$ detour. For a set $S$ of vertices, let $I_D[S] = \cup_{u,v \in S} I_D[u, v]$. Then certainly $S \subseteq I_D[S]$. A set $S \subseteq V(G)$ is called a *detour set* of $G$ if $I_D[S] = V(G)$. The detour number $dn(G)$ of $G$ is the minimum order of its detour sets and any detour set of order $dn(G)$ is a called a $dn-set$ of $G$. The detour number of a graph was studied in [3, 4, 8-12]. The geochromatic number of a graph was studied in [1]. The monophonic chromatic number of a graph was studied in [6]. This motivated us to define a new parameter chromatic detour number of a graph.

The chromatic number has application in Time Table Scheduling, Map coloring, channel assignment problem in radio technology, town planning, GSM mobile phone networks etc [5]. When we apply detour concept, there is a effective in Scheduling. Throughout the following $G$ denotes a connected graph with at least two vertices. The following theorems are used in sequel.

**Theorem 1.1.** [4] *Each end vertex of a connected graph $G$ belongs to every detour set of $G$.*

**Theorem 1.2.** [7] *For the complete graph $G = K_n$ $(n \geq 2)$, $\chi(G) = n$.*

## 2. The Chromatic Detour Number of a Graph

It is easily seen that a chromatic set of $G$ need not be a detour set of $G$. Also the converse is not valid in general. This has motivated us to define a new chromatic conception of chromatic detour number. We investigate those subset of vertices of a graph that are both chromatic set and a detour set. We call these sets as chromatic detour sets. Although the chromatic detour number is greater than or equal to the chromatic number for an arbitrary graph, the properties of the chromatic detour number are quite different from that of chromatic concept.

**Definition 2.1.** *A set $S \subseteq V(G)$ is called a chromatic detour set of $G$ if $S$ is both a chromatic set and a detour set of $G$. The minimum cardinality of a chromatic detour set of $G$ is called a chromatic detour number of $G$ and is denoted by $\chi_{dn}(G)$. Any chromatic detour set of cardinality of $\chi_{dn}(G)$ is called a $\chi_{dn}$-set of $G$.*

**Example 2.2.** For the graph $G$ given in Figure 2.1, $S_1 = \{v_1, v_2, v_3\}$ is a $\chi$-set of $G$ so that $\chi(G) = 3$ and $S_2 = \{v_1, v_8\}$ is a $dn$-set of $G$ so that $dn(G) = 2$. Also $S_3 = \{v_1, v_2, v_3, v_8\}$ is a $\chi_{dn}$-set of $G$ so that $\chi_{dn}(G) = 4$. For the star $G = K_{1,n-1}$ $(n \geq 4)$, $\chi(G) = 2$, $dn(G) = n - 1$ and $\chi_{dn}(G) = n$.

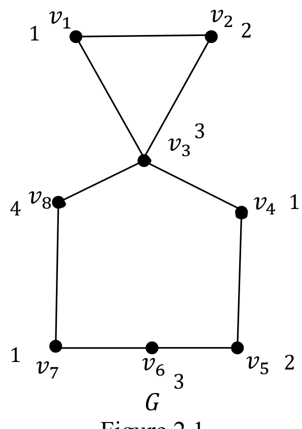


Figure 2.1

**Observation 2.3.** *If $v$ is either an end vertex or a universe vertex of $G$, then $v$ belongs every chromatic detour set of $G$.*

In the following, we determine the chromatic detour number of some standard graphs.

**Theorem 2.4.** *For the complete graph* $G = K_n$ $(n \geq 2)$, $\chi_{dn}(G) = n$.
**Proof.** This follows from Observation 2.3.

**Theorem 2.5.** *For the star* $G = K_{1,n}$ $(n \geq 3)$, $\chi_{dn}(G) = n$.
**Proof.** This follows from Observation 2.3.

**Theorem 2.6.** *For the complete bipartite* $G = K_{r,s}$ $(1 \leq r \leq s)$, $\chi_{dn}(G) = 2$.
**Proof.** If $r = s = 1$, then the result follows from Theorem 2.4. If $r = 1$, $s \geq 2$, then the result follows from Theorem 2.5. So let $2 \leq r \leq s$, $X = \{x_1, x_2, ..., x_r\}$ and $Y = \{y_1, y_2, ..., y_s\}$ be the bipartite sets of $G$. Then $S_{ij} = \{x_i, y_j\}$ $(2 \leq i, j \leq r \leq s)$ is a $\chi_{dn}$-set of $G$ so that $\chi_{dn}(G) = 2$.

**Theorem 2.7.** *For the path* $G = P_n$ $(n \geq 3)$,

$$\chi_{dn}(G) = \begin{cases} 3 & \text{if } n \text{ is odd} \\ 2 & \text{if } n \text{ is even} \end{cases}$$

**Proof.** Let $P_n$ be $v_1, v_2, ..., v_n$.
**Case(i)** $n$ is odd.
It can be easily seen that no two element subsets of $G$ is a chromatic detour set of $G$ and so $\chi_{dn}(G) \geq 3$. Let $S = \{v_1, v_2, v_n\}$. Then $S$ is a $\chi_{dn}$-set of $G$ so that $\chi_{dn}(G) = 3$.
**Case(ii)** $n$ is even.
Let $S = \{v_1, v_n\}$. Then $S$ is a $\chi_{dn}$-set of $G$ so that $\chi_{dn}(G) = 2$.

**Theorem 2.8.** *For the wheel graph* $G = K_1 + C_{n-1}$ $(n \geq 4)$,

$$\chi_{dn}(G) = \begin{cases} 3 & \text{if } n-1 \text{ is even} \\ 4 & \text{if } n-1 \text{ is odd.} \end{cases}$$

**Proof.** Let $x$ be the central vertex of $G$ and $V(C_{n-1}) = \{v_1, v_2, ..., v_{n-1}, v_1\}$. We consider the following two cases.
**Case(i)**: $n-1$ is even.
Let $S_i = \{x, v_i, v_{i+1}\}$ $(1 \leq i \leq n-2)$. We assign three different colours for $x, v_i, v_{i+1}$ $(1 \leq i \leq n-2)$ and so $\chi_{dn}(G) \geq 3$. It is clear that $S_1 = \{x, v_1, v_2\}$ is a detour chromatic set and so $\chi_{dn}(G) = 3$.
**Case(ii)**: $n-1$ is odd.
Let $S_j = \{x, v_j, v_{j+1}\}$ $(1 \leq j \leq n-3)$. We assign three different colours for $x, v_j, v_{j+1}$ $(1 \leq j \leq n-3)$. Since $n-1$ is odd, the vertex $v_{n-1}$ is not included in $S_j$ for any $j$ $(1 \leq j \leq n-3)$. There we assign a colour 4 to $v_{n-1}$ and so $\chi_{dn}(G) \geq 4$. Now $\{x, v_j, v_{j+1}, v_{n-1}\}$ is a chromatic detour set of $G$. Hence $\chi_{dn}(G) = 4$.

**Theorem 2.9.** *For the fan graph* $G = K_1 + P_{n-1}$ $(n \geq 3)$, $\chi_{dn}(G) = 3$.
**Proof.** Let $x$ be the central vertex of $G$ and $V(P_{n-1}) = \{v_1, v_2, ..., v_{n-1}\}$. Since

$S_i = \{x, v_i, v_{i+1}\}$ is a clique for $1 \leq i \leq n-2$. We assign three different colours for $x, v_i, v_{i+1}$ $(1 \leq i \leq n-2)$ and so $\chi_{dn}(G) \geq 3$. Since $S_i$ $(1 \leq i \leq n-2)$ is a chromatic detour set of $G$, $\chi_{dn}(G) = 3$.

**Theorem 2.10.** *For the graph* $G = K_n - \{e\}$ $(n \geq 4)$, $\chi_{dn}(G) = n-1$.
**Proof.** Let $e = uv$. Since $G[V - \{u\}]$ is a clique, the vertex set of $G[V - \{u\}]$ is assigned by distinct colours $c_1, c_2, ..., c_{n-1}$. Therefore $\chi_{dn}(G) \geq n-1$. Let $c_1 = c(u)$. Since $uv \notin E(G)$, we assign $c(v) = c_1$. Therefore $V - \{u\}$ is a chromatic set of $G$ as well as detour set of $G$ so that $\chi_{dn}(G) = n-1$.

**Theorem 2.11.** *For the graph* $G = K_n - \{e_1, e_2\}$ $(n \geq 5)$, $\chi_{dn}(G) = n-1$, *where* $e_1$ *and* $e_2$ *are adjacent edges of* $K_n$.
**Proof.** Let $V(K_n) = \{v_1, v_2, ..., v_n\}$. Without loss of generality, let $e_1 = v_2v_n$ and $e_2 = v_3v_n$. Since $n \geq 5$ $G[v_1, v_2, ..., v_{n-1}] = K_{n-1}$, $\chi_{dn}(G) \geq n-1$. Let $c(v_i) = c_i$ $(1 \leq i \leq n-1)$ and $c(v_n) = c_2$. Then $S = \{v_1, v_2, ..., v_{n-1}\}$ is a $\chi_{dn}$-set of $G$ so that $\chi_{dn}(G) = n-1$.

**Theorem 2.12.** *Let* $G$ *be a semi split graph of order* $n$. *Then* $\chi_{dn}(G) = n$.
**Proof.** For a semi split graph $G$, $S = V(G)$ is the unique chromatic detour set of $G$ so that $\chi_{dn}(G) = n$.

## 3. Some Results on the Chromatic Detour Number of a Graph

In this section, we look at some relationships between the chromatic detour number and other parameters. Further some improved upper bounds for the chromatic detour number of a graph are given. Also we characterized connected graphs of order $n \geq 2$ with chromatic detour number $n$ or $n-1$.

**Observation 3.1.** *Let* $G$ *be a connected graph of order* $n \geq 2$. *Then* $2 \leq max\{\chi(G), dn(G)\} \leq n$.

**Theorem 3.2.** *Let* $G$ *be a connected non-complete of order* $n \geq 4$ *with detour diameter* $D \geq 2$. *Let* $P_D$ *be a detour diametral path in* $G$ *such that* $G[P_D]$ *is neither* $P_3$ *nor* $K_{D+1}$ *nor* $K_{D+1} - \{e\}$ *nor* $K_{D+1} - \{e_1, e_2\}$ *nor a semi split graph nor the graphs given in Figures 3.2, 3.3 and 3.4, Then* $\chi_{dn}(G) \leq n-2$.
**Proof.** Let $P_D : u_0, u_1, u_2, ..., u_D$ be a detour diametral path of $G$. Since $G[P_D]$ is neither $P_3$ nor $K_{D+1}$ nor $K_{D+1} - \{e\}$ nor $K_{D+1} - \{e_1, e_2\}$ where $e_1$ and $e_2$ are adjacent edges of $K_{D+1}$ nor a semi split graph nor the graphs given in Figures 3.2, 3.3 and 3.4 there exists at least one chordless subpath of $P_D$, say $Q$ such that $|Q| \geq 2$. Let $Q : x_0, x_1, x_2, ..., x_k$, where $k \geq 2$. Let us assign $c(x_0) = c_1$, $c(x_1) = c_2$, $c(x_2) = c_1$. Then $S = V(G) - \{x_1, x_2\}$ is a detour chromatic set of $G$ so that $\chi_{dn}(G) \leq n-2$.

**Remark 3.3.** *The bound in Theorem 3.2 can be sharp. For the graph $G = P_4$, $\chi_{dn}(G) = 2 = n - 2$. Also the bound in Theorem 3.2 can be strict. For the graph given in Figure 3.1, $S = \{v_3, v_5\}$ is a $\chi_{dn}$-set of $G$ so that $\chi_{dn}(G) = 2 < n - 2$.*

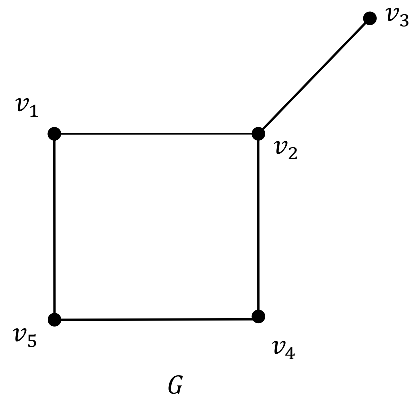


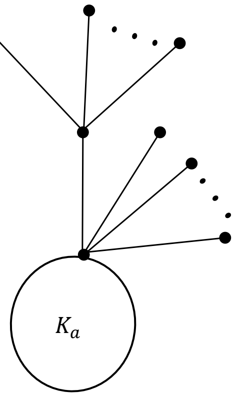
Figure 3.1

**Theorem 3.4.** *Let $G$ be a connected graph of order $n \geq 2$. Then $\chi_{dn}(G) = n$ if and only if $G$ is either $K_n$ or $K_{1,n-1}$ or $G$ is a semi split graph.*

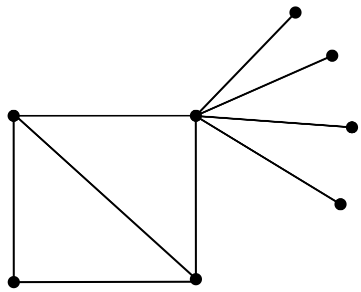
**Proof.** Let $\chi_{dn}(G) = n$. If $n = 2$, then $G = K_2$, which satisfies the requirements of this theorem. If $n = 3$, then $G$ is either $K_3$ or $P_3$, which satisfies the requirements of this theorem. So, let $n \geq 4$. Let $P_D : u_0, u_1, u_2, ..., u_D$ be a detour diametral path of $G$. If $G[P_D]$ is neither $P_3$ nor $K_{D+1}$ nor $K_{D+1} - \{e\}$ nor $K_{D+1} - \{e_1, e_2\}$ where $e_1$ and $e_2$ are adjacent edges of $K_{D+1}$ nor a semi split graph nor the graphs given in Figures 3.2, 3.3 and 3.4, then by Theorem 3.2, $\chi_{dn}(G) \leq n - 2$, which is a contradiction. Therefore $G[P_D]$ is either $P_3$ or $K_{D+1}$ or $K_{D+1} - \{e\}$ or $K_{D+1} - \{e_1, e_2\}$ or the graphs given in Figures 3.2, 3.3 and 3.4. If $G[P_D]$ is $P_3$, then $G = K_{1,n-1}$, which satisfies the requirements of this theorem. If $G[P_D]$ is $K_{D+1}$ then $G$ is $K_n$. which satisfies the requirements of this theorem. If $G[P_D]$ is $K_{D+1} - \{e\}$ then let $e = xy$. Let us assign $c(x) = c(y) = c$. Then $S = V(G) - \{x\}$ is a chromatic detour set of $G$ so that $\chi_{dn}(G) \leq n - 1$, which is a contradiction. If $G[P_D]$ is $K_{D+1} - \{e_1, e_2\}$, then by the similar way we can prove that $\chi_{dn}(G) \leq n - 1$, which is a contradiction. If $G$ is a semi split graph, then by Theorem 2.12, $\chi_{dn}(G) = n$, which satisfies the requirements of this theorem. If $G$ is the graph given in Figures 3.3, 3.4. It can be easily verified that $\chi_{dn}(G) = n - 1$, which is a contradiction. The converse is clear.

**Theorem 3.5.** *Let $G$ be a connected graph of order $n \geq 3$. Then $\chi_{dn}(G) = n - 1$ if and only if $G$ is either $G \neq K_n - \{e\}$ or $G \neq K_n - \{e_1, e_2\}$, where $e_1$ and $e_2$ are adjacent edges of $K_n$ or the graph given in Figures 3.2, 3.3 and 3.4.*

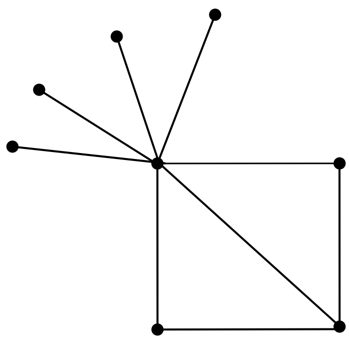
**Proof.** Let $\chi_{dn}(G) = n - 1$. Let $P_D : u_0, u_1, u_2, ..., u_D$ be a detour diametral path of $G$.




$G$
Figure 3.2



$G$
Figure 3.3

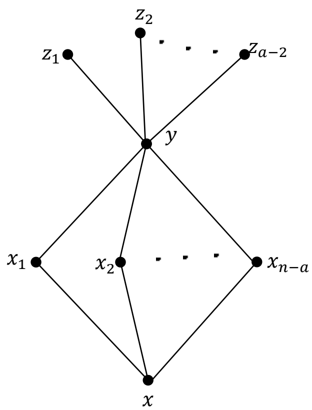


$G$
Figure 3.4

If $G[P_D]$ is neither $P_3$ nor $K_{D+1}$ nor $K_{D+1} - \{e\}$ nor $K_{D+1} - \{e_1, e_2\}$ where $e_1$ and $e_2$ are adjacent edges of $K_{D+1}$ nor a semi split graph nor the graphs given in Figures 3.2, 3.3 and 3.4. Then $\chi_{dn}(G) \leq n-2$, which is a contradiction. Therefore $G[P_D]$ is either $P_3$ or $K_{D+1}$ or $K_{D+1} - \{e\}$ or $K_{D+1} - \{e_1, e_2\}$ or a semi split graph or the graph given in Figures 3.2, 3.3 and 3.4. If $G[P_D]$ is either $P_3$ or $G = K_{D+1}$, then by Theorem 3.4,$\chi_{dn}(G) = n$, which is a contradiction. If $G$ is a semi split graph, then by Theorem 2.12, $\chi_{dn}(G) = n$, which is a contradiction. If $G$ is the graph given in Figures 3.2, 3.3 and 3.4, then $\chi_{dn}(G) = n - 1$, which satisfies the requirements of this theorem. If $G[P_D]$ is $K_{D+1} - \{e\}$ then $G$ is $K_n - \{e\}$, which satisfies the requirements of this theorem. If $G[P_D]$ is $K_{D+1} - \{e_1, e_2\}$ then $G$ is $K_n - \{e_1, e_2\}$, which satisfies the requirements of this theorem.

**Theorem 3.6.** *For every pair of integers $a$ and $n$ with $2 \leq a \leq n$, there exists a connected graph $G$ of order $n$ such that $\chi_{dn}(G) = a$.*

**Proof.** For $a = n$, let $G = K_a$. Then by Theorem 2.4, $\chi_{dn}(G) = a$. So, let $2 \leq a \leq n-1$. Let $V(\overline{K}_2) = \{x, y\}$ and $V(\overline{K}_{n-a}) = \{x_1, x_2, ..., x_{n-a}\}$. Let $H$ be the graph obtained from $\overline{K}_2$ and $\overline{K}_{n-a}$ by joining $x$ and $y$ with each $x_i$ $(1 \leq i \leq n-a)$. Let $G$ be the graph obtained from $H$ by adding new vertices $z_1, z_2, ..., z_{a-2}$ and joining $y$ with each $z_i$ $(1 \leq i \leq a-2)$. The graph $G$ is given in Figure 3.5. We prove that $\chi_{dn}(G) = a$. Let $Z = \{z_1, z_2, ..., z_{a-2}\}$ be a set of end vertices of $G$. Then by Observation 2.3, $Z$ is a subset of every chromatic detour set of $G$. Therefore we assign different colours for $z_i$ for each $i$ $(1 \leq i \leq a-2)$ and $y$. Let $c(z_i) = c_i$ $(1 \leq i \leq a-2)$. Since $yz_i \in E(G)$ for all $i$ $(1 \leq i \leq a-2)$, we assign different colour for $y$. Let $c(y) = c_{a-1}$. Therefore $\chi_{dn}(G) \geq a-1$. Let $Z_1 = Z \cup \{y\}$. Since $I_D[Z_1] \neq V(G)$, $Z_1$ is not a chromatic detour set of $G$ and so $\chi_{dn}(G) \geq a$. Let us assign the vertex $x$ with new colour and assign each vertex $x_i$ $(1 \leq i \leq n-a)$ with same colours. Let $c(x) = c_a$ and $c(x_i) = c_1$ $(1 \leq i \leq n-a)$. Then $S_1 = S \cup \{x\}$ is a detour chromatic set of $G$ so that $\chi_{dn}(G) = a$.




$G$
Figure 3.5

In view of Observation 3.1, we have the following realization result.

**Theorem 3.7.** *For every positive integers $a$ and $b$ with $2 \leq a \leq b$, there exists a connected graph $G$ such that $dn(G) = a$ and $\chi_{dn}(G) = b$.*

**Proof.** Let $V(K_{b-a+1}) = \{v_1, v_2, ..., v_{b-a+1}\}$. Let $G$ be the graph obtained from $K_{b-a+1}$ by adding new vertices $u_1, u_2, ...., u_{a-1}$ and the edges $v_1u_i$ $(1 \leq i \leq a-1)$. The graph $G$ is shown in Figure 3.6.

First we show that $dn(G) = a$. Let $Z = \{u_1, u_2, ...., u_{a-1}\}$ be the set of end vertices of graph $G$. By Theorem 1.1, $Z$ is a subset of every detour set of $G$. Since $I_D[Z] \neq V(G)$, $Z$ is not a detour set of $G$ and so $dn(G) \geq a$. Let $S = Z \cup \{v_2\}$. Then $S$ is a detour set of $G$ so that $dn(G) = a$.

Next we show that $\chi_{dn}(G) = b$. Since $G$ is a semi split graph, by Theorem 3.4, $\chi_{dn}(G) = n = b - a + 1 + 1 = b$.

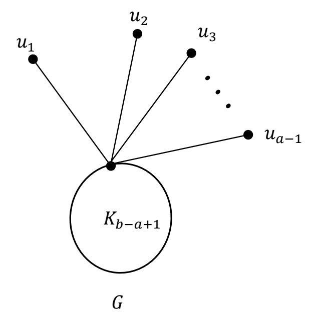


Figure 3.6

**Theorem 3.8.** *For every positive integers $a$ and $b$ with $2 \leq a \leq b$, there exists a connected graph $G$ such that $\chi(G) = a$ and $\chi_{dn}(G) = b$.*

**Proof.** For $a = b$, let $G = K_a$. Then by Theorems 1.2 and 2.4, $\chi(G) = a$ and $\chi_{dn}(G) = a$. So, let $2 \leq a < b$. Let $V(K_a) = \{v_1, v_2, ..., v_a\}$. Let $G$ be the graph obtained from $K_a$ by adding new vertices $x, z_1, z_2, ...., z_{b-a}$ and join each $z_i$ $(1 \leq i \leq b - a)$ with $x$ and join $x$ with $v_1$. The graph $G$ is shown in Figure 3.7. It is easily seen that $V(K_a)$ is a $\chi$-set of $G$ so that $\chi(G) = a$. By Theorem 3.5, $\chi_{dn}(G) = n - 1 = b - a + a + 1 - 1 = b$.

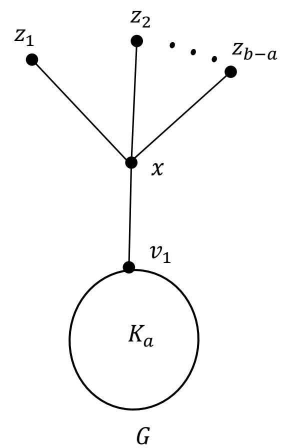


Figure 3.7

In the following, we present the Nordhaus-Gaddum type relations for the chromatic detour number of a graph.

**Theorem 3.9.** *Let $G$ and $\overline{G}$ be connected graphs of order $n \geq 4$. Then $4 \leq \chi_{dn}(G) + \chi_{dn}(\overline{G}) \leq 2n - 3$. Moreover the upper bound is sharp if and only if $G$ is the graph given in Figure 3.3.*

**Proof.** Since $G$ and $\overline{G}$ are connected graphs $\chi_{dn}(G) \geq 2$ and $\chi_{dn}(\overline{G}) \geq 2$. Therefore $\chi_{dn}(G) + \chi_{dn}(\overline{G}) \geq 4$. Since $\overline{G}$ is connected, by Theorems 3.4 and 3.5, $\chi_{dn}(G) \leq n - 1$ and $\chi_{dn}(\overline{G}) \leq n - 2$. Hence $\chi_{dn}(G) + \chi_{dn}(\overline{G}) \leq 2n - 3$. Next we prove that $\chi_{dn}(G) + \chi_{dn}(\overline{G}) = 2n - 3$ if and only if $G$ is the graph given in the Figure 3.3. From Theorem 3.5, the only graph satisfying $\chi_{dn}(G) + \chi_{dn}(\overline{G}) = 2n - 3$ is the graph given in the Figure 3.3.

**Remark 3.10.** *The lower bound in Theorem 3.9 is sharp. For the graph $G = P_4$, $\chi_{dn}(G) = 2$. Since $\overline{G} = P_4$, $\chi_{dn}(\overline{G}) = 2$. Therefore $\chi_{dn}(G) + \chi_{dn}(\overline{G}) = 4$.*

**Acknowledgment**

The first name author R. Suganya, Register Number 18223232092002, Research Scholar, Department of Mathematics, Holy Cross College (Autonomous), Nagercoil - 629004, India and second name author V. Sujin Flower, Assistant Professor, Department of Mathematics, Holy Cross College (Autonomous), Nagercoil - 629 004, India, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli - 627012 thanks to reviewer and editor for valuable guidance to improve the quality of paper.

## References

[1] Buckley F. and Harary F., Distance in graph, Addition-Wesley-wood city, CA, 1990.

[2] Chartrand G., Johns G. L. and Zhang P., On the detour number and geodetic number of a graph, Ars combinatoria, 72 (2004), 3-15.

[3] Chartrand G., Johns G. L. and Zhang P., The detour number of a graph, Utilitas Mathematica, 64 (2008), 97-113.

[4] Geir A., Greenlaw R., Graph Theory, Modeling, Application and Algorithms, Pearson, 2007.

[5] John J. and Arianayagam N., The total detour number of a graph, Journal of Discrete Mathematical Sciences and Cryptography, 17(4) (2014), 337-350.

[6] Khayoom M. M. A. and Sudhahar P. Arul P., Monophonic chromatic parameter in a graph, Int. J. Math. Analy, 11(19) (2017), 911-920.

[7] Medha I. H., Divya B., Geodetic Number and Geo-chromatic number of 2-cartesian product of some graphs, Open Journal of Discrete Mathematics, 12(1), (2022), 1-16.

[8] Piotr F. and Krzysztof T., A survey of graph coloring - its types, methods and applications, Foundations of Computing and Decision Sciences, 37 (3), (2012).

[9] Robinson S. Chellathurai and Samli S. Beulah, The Geo Dominator Chromatic Number for Corona Graphs, Infokara Research, 8(8), (2019), 304-308.

[10] Santhakumaran A. P. and Athisayanathan S., On the connected detour number of a graph, Journal of Prime Research in Mathematics, 5 (2009), 149-170.

[11] Samli S. B. and Chellathurai S. R., Geo chromatic number of a graph, International Journal of Scientific Research in Mathematical and Statistical Sciences, 5(6), (2018), 259-264.

[12] Stanis A. Arul Mary, Geo chromatic number for certain cartesian product of graphs, International Journal of Mathematics Trends and Technology (IJMTT), 66(1), (2020), 40-43.