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DEGREE SEQUENCES ON LINE GRAPH OF R-CORONA GRAPHS

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Abstract: A graph $G = (V, E)$ is a set of vertices, which are connected by edges. In this paper, we study the line graph of R-corona operations of complete, cycle and r-regular graphs in terms of degree sequences(DS).

Keywords and Phrases: Line graph, R- corona operations, complete, cycle and r-regular graphs.

2020 Mathematics Subject Classification: 05C76.

1. Introduction

Let $G = (V, E)$ be a simple connected graph which does not contains loops and multiple edge. The degree of vertex u is the number of vertices are adjacent to u and it is denoted as deg_u or d_u. A graph in which every two vertices are adjacent is called as a complete graph [5]. A closed walk is finite or infinite vertices and no vertex is repeated is called cycle [11]. A graph is said to be r-regular graph in which each vertex degree is r [8].

Tyshkevich et. al., [10, 4] established a correspondence between DSs of graph and some structural properties of the graph in 1981 and Bolloas started the study on DSs on the same year. The degree sequences DSs of a graph G is obtained by degree of vertices x_i of G in ascending or descending order and it is defined as $DS(G) = \{\aleph_1^{\ell_1}, \aleph_2^{\ell_2}, \aleph_3^{\ell_3}, ..., \aleph_n^{\ell_n}\}$ [2, 9].

Definition 1.1. *The line graph of a graph G is another graph $L(G)$ is defined on $V(G)$, if two vertices are adjacent in $L(G)$ if and only if their corresponding edges are adjacent in G* [3].

Definition 1.2. *The semi-total point graph $R(G)$ is obtain from G by adding one vertex($I(G)$ is the set contains additional vertices) to each edge of G and joining each new vertex ($I(G)$) to the end vertices of the corresponding edge* [7].

Definition 1.3. *Let G and H be two graphs with vertices n_1 and n_2 and edges m_1 and m_2 respectively. The R- vertex corona of graphs G and H with $(n_1+m_1+n_1n_2)$ vertices and $(3m_1 + n_1(n_2 + m_2))$ edges and is obtained from one copy $R(G)$ and $|V(G)|$ copies of H, by joining the i^{th} vertex of $V(G)$ to each vertex in the i^{th} copy of H* [1, 6].

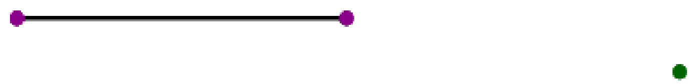

Figure 1: K_2 and K_1

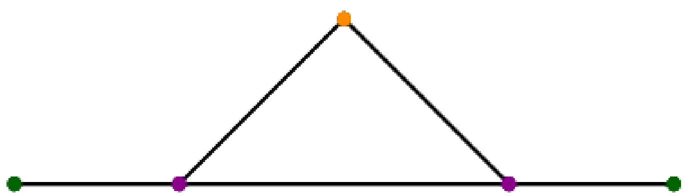

Figure 2: $K_2 \odot_R K_1$

Definition 1.4. *Let G and H be two graphs with vertices n_1 and n_2 and edges m_1 and m_2 respectively. The R- edge corona of graphs G and H with $(n_1+m_1+m_1n_2)$ vertices and $m_1(n_2+m_2+3)$ edges and is obtained from one copy $R(G)$ and $|E(G)|$ copies of H, by joining the i^{th} vertex of $I(G)$ to each vertex in the i^{th} copy of H.*

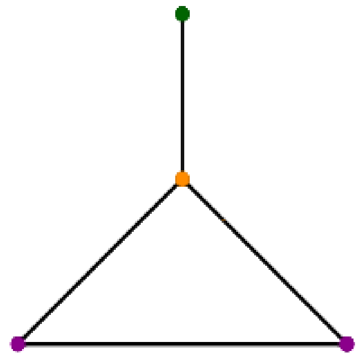

Figure 3: $K_2 \ominus_R K_1$

Definition 1.5. *Let G and H be two graphs with vertices n_1 and n_2 and edges m_1 and m_2 respectively. The R-vertex neighbourhood corona of graphs G and H with $(n_1+m_1+n_1n_2)$ vertices and $(3m_1+n_1m_2+m_1n_2)$ edges and is obtained from one copy $R(G)$ and $|V(G)|$ copies of H, by joining the neighbours of i^{th} vertex of $V(G)$ to each vertex in the i^{th} copy of H.*

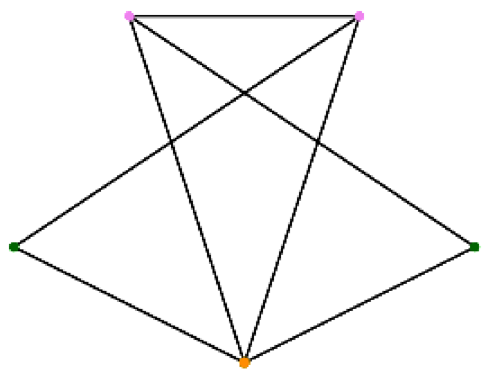

Figure 4: $K_2 \odot_{nR} K_1$

Definition 1.6. *Let G and H be two graphs with vertices n_1 and n_2 and edges m_1 and m_2 respectively. The R-edge neighbourhood corona of graphs G and H with*

$(n_1 + m_1 + m_1n_2)$ vertices and $m_1(2n_2 + m_2 + 3)$ edges and is obtained from one copy $R(G)$ and $|E(G)|$ copies of H, by joining the neighbours of i^{th} vertex of $I(G)$ to each vertex in the i^{th} copy of H.

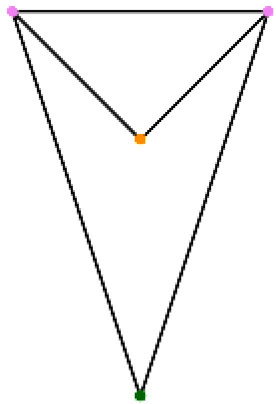

Figure 5: $K_2 \ominus_{nR} K_1$

2. Main Results

In this section, we derive the DSs of line graph of R-corona operations on K_m, C_m and r-regular graphs.

Theorem 2.1. *The DSs of line graph of R-vertex corona of complete, cycle and r-regular graphs.*

Proof. Let G and H be two simple connected graphs with n_1, m_1 and n_2, m_2 are vertex set and edge set respectively. Using the definitions 1.1 and 1.3, we obtain the line graph of R-vertex corona of G and H $[L(G \odot_R H)]$ with $(3m_1 + n_1(n_2 + m_2))$ vertices. There are four type of vertices, in which m_1 vertices having degree $(4d_G + 2n_2 - 2)$, $2m_1$ vertices having degree $(2d_G + n_2)$, n_1m_2 vertices having degree $(2d_H)$ and n_1n_2 vertices having degree $(d_H + 2d_G + n_2 - 1)$.
Therefore,

$$\begin{aligned} DS[L(G \odot_R H)] &= \{(4d_G + 2n_2 - 2)^{m_1}, (2d_G + n_2)^{2m_1}, (2d_H)^{n_1m_2}, \\ &\qquad (d_H + 2d_G + n_2 - 1)^{n_1n_2}\} \end{aligned}$$

Table 1. Degree sequences of line graph R-vertex corona for complete, cycle and r-regular graphs.

G	H	$DS[L(G \odot_R H)]$
K_n	K_m	$\{(4n+2m-6)^{\frac{n(n-1)}{2}}, (2n+m-2)^{n(n-1)}, (2m-2)^{\frac{nm(m-1)}{2}}, (2m+2n-4)^{nm}\}$
K_n	C_m	$\{(4n+2m-6)^{\frac{n(n-1)}{2}}, (2n+m-2)^{n(n-1)}, 4^{mn}, (m+2n-1)^{nm}\}$
K_n	r-regular graph with m-vertices	$\{(4n+2m-6)^{\frac{n(n-1)}{2}}, (2n+m-2)^{n(n-1)}, (2r)^{\frac{nmr}{2}}, (r+2n+m-3)^{nm}\}$
C_n	K_m	$\{(2m+6)^{n}, (m+4)^{2n}, (2n-2)^{\frac{nm(m-1)}{2}}, (2m+2n-4)^{nm}\}$
C_n	C_m	$\{(2m+6)^{n}, (m+4)^{2n}, 4^{mn}, (m+5)^{nm}\}$
C_n	r-regular graph with m-vertices	$\left\{(2m+6)^{n}, (m+4)^{2n}, (2r)^{\frac{nmr}{2}}, (r+m+3)^{nm}\right\}$
r-regular graph with n-vertices	K_m	$\{(4r+2m-2)^{\frac{nr}{2}}, (2r+m)^{nr}, (2m-2)^{\frac{nm(m-1)}{2}}, (2m+2r-2)^{nm}\}$
r-regular graph with n-vertices	C_m	$\{(4r+2m-2)^{\frac{nr}{2}}, (2r+m)^{nr}, 4^{nm}, (2r+m-1)^{nm}\}$
r_1-regular graph with n-vertices	r_2-regular graph with m-vertices	$\{(4r_1+2m-2)^{\frac{nr_1}{2}}, (2r_1+m)^{nr_1}, (2r_2)^{\frac{nmr_2}{2}}, (r_2+2r_1+m-1)^{nm}\}$

Theorem 2.2. *The DSs of line graph of R-edge corona of complete, cycle and r-regular graphs.*

Proof. Let G and H be two simple connected graphs with n_1, m_1 and n_2, m_2 are vertex set and edge set respectively. Using the definitions 1.1 and 1.4, we obtain the line graph of R-edge corona of G and H $[L(G \ominus_R H)]$ with $m_1(n_2+m_2+3)$ vertices. There are four type of vertices, in which m_1 vertices having degree $(4d_G-2)$, $2m_1$ vertices having degree $(2d_G+n_2)$, n_2m_1 vertices having degree (d_H+n_2+1) and m_1m_2 vertices having degree $(2d_H)$.

Therefore,

$$DS[L(G \ominus_R H)] = \{(4d_G-2)^{m_1}, (2d_G+n_2)^{2m_1}, (d_H+n_2+1)^{n_2m_1}, (2d_H)^{m_1m_2}\}$$

Table 2. Degree sequences of line graph R-edge corona for complete, cycle and r-regular graphs.

G	H	$DS[L(G \ominus_R H)]$
K_n	K_m	$\Big\{(4n-6)^{\frac{n(n-1)}{2}}, (2n+m-2)^{n(n-1)}, (2m)^{\frac{nm(n-1)}{2}}, (2m-2)^{\frac{nm(n-1)(m-1)}{4}}\Big\}$
K_n	C_m	$\Big\{(4n-6)^{\frac{n(n-1)}{2}}, (2n+m-2)^{n(n-1)}, (m+3)^{\frac{nm(n-1)}{2}}, 4^{\frac{mn(n-1)}{2}}\Big\}$
K_n	r-regular graph with m-vertices	$\Big\{(4n-6)^{\frac{n(n-1)}{2}}, (2n+m-2)^{n(n-1)}, (r+m+1)^{\frac{nm(n-1)}{2}}, (2r)^{\frac{mnr(n-1)}{4}}\Big\}$
C_n	K_m	$\Big\{6^n, (m+4)^{2n}, (2m)^{mn}, (2m-2)^{\frac{nm(n-1)}{2}}\Big\}$
C_n	C_m	$\{6^n, (m+4)^{2n}, (m+3)^{mn}, 4^{mn}\}$
C_n	r-regular graph with m-vertices	$\Big\{6^n, (m+4)^{2n}, (r+m+1)^{mn}, (2r)^{\frac{mnr}{2}}\Big\}$
r-regular graph with n-vertices	K_m	$\Big\{(4r-2)^{\frac{nr}{2}}, (2r+m)^{nr}, (2m)^{\frac{mnr}{2}}, (2m-2)^{\frac{mnr(m-1)}{4}}\Big\}$
r-regular graph with n-vertices	C_m	$\Big\{(4r-2)^{\frac{nr}{2}}, (2r+m)^{nr}, (m+3)^{\frac{mnr}{2}}, 4^{\frac{mnr}{2}}\Big\}$
r_1-regular graph with n-vertices	r_2-regular graph with m-vertices	$\Big\{(4r_1-2)^{\frac{nr_1}{2}}, (2r_1+m)^{nr_1}, (r_2+m+1)^{\frac{mnr_2}{2}}, (2r_2)^{\frac{mnr_1r_2}{4}}\Big\}$

Theorem 2.3. *The DSs of line graph of R-vertex neighbourhood corona of complete, cycle and r-regular graphs.*

Proof. Let G and H be two simple connected graphs with n_1, m_1 and n_2, m_2 are vertex set and edge set respectively. Using the definitions 1.1 and 1.5, we obtain the line graph of R-vertex neighbourhood corona of G and H $[L(G \odot_{nR} H)]$ with $m_1(2n_2+3)+n_1(n_2+m_2)$ vertices. There are five type of vertices, in which m_1 vertices having degree $(2d_G(2+n_2)-2)$, $2m_1$ vertices having degree $(d_G(2+n_2)+2n_2)$, $2n_2m_1$ vertices having degree $(d_H + 2d_G + 2n_2)$, n_1m_2 vertices having degree $(2(d_H + 2d_G - 1))$ and n_1n_2 vertices having degree $(4d_G + d_H + d_G n_2 - 2)$. Therefore,

$$DS[L(G \odot_{nR} H)] = \{(2d_G(2+n_2)-2)^{m_1}, (d_G(2+n_2)+2n_2)^{2m_1},$$
$$(d_H + 2d_G + 2n_2)^{2n_2m_1}, (2(d_H + 2d_G - 1))^{n_1m_2}, (4d_G + d_H + d_G n_2 - 2)^{n_1n_2}\}$$

Table 3. Degree sequences of line graph R-vertex neighbourhood corona for complete, cycle and r-regular graphs.

G	H	$DS[L(G \odot_{nR} H)]$
K_n	K_m	$\Big\{(2(n-1)(m+2)-2)^{\frac{n(n-1)}{2}}, (mn+2n+m-2)^{n(n-1)}, (2n+3m-3)^{mn(n-1)}, (mn+4n-7)^{mn}, (2[m+2n-4])^{\frac{mn(m-1)}{2}}\Big\}$
K_n	C_m	$\Big\{(2(n-1)(m+2)-2)^{\frac{n(n-1)}{2}}, (mn+2n+m-2)^{n(n-1)}, (2(m+n)^{mn(n-1)}, (mn+4n-m-4)^{mn}, (2[2n-1])^{mn}\}$
K_n	r-regular graph with m-vertices	$\Big\{(2(n-1)(m+2)-2)^{\frac{n(n-1)}{2}}, (mn+2n+m-2)^{n(n-1)}, (r+2(n+m-1))^{mn(n-1)}, (r+4n+mn-m-6)^{mn}, (2[r+2n-3])^{\frac{mnr}{2}}\Big\}$
C_n	K_m	$\{(4m+6)^n, (4m+4)^{2n}, (3m-3)^{2mn}, (2m+4)^{\frac{mn(m-1)}{2}}, (3m-5)^{mn}\Big\}$
C_n	C_m	$\{(4m+6)^n, (4m+4)^{2n}, (2m+6)^{2mn}, (10)^{mn}, (2m+8)^{mn}\}$
C_n	r-regular graph with m-vertices	$\{(4m+6)^n, (4m+4)^{2n}, (2m+r+4)^{2mn}, (2r-6)^{\frac{mnr}{2}}, (r+2m-6)^{mn}\Big\}$
r-regular graph with n-vertices	K_m	$\Big\{(2r(m+2)-2)^{\frac{nr}{2}}, (r(m+2)+2m)^{nr}, (2r+3m-1)^{mnr}, (2m+4r-4)^{\frac{mn(m-1)}{2}}, (4r+m(r+1)-3)^{mn}\}$
r-regular graph with n-vertices	C_m	$\Big\{(2r(m+2)-2)^{\frac{nr}{2}}, (r(m+2)+2m)^{nr}, (2m+2r+2)^{mnr}, (4r+2)^{mn}, (rm+4r)^{mn}\}$
r_1-regular graph with n-vertices	r_2-regular graph with m-vertices	$\Big\{(2r_1(m+2)-2)^{\frac{nr_1}{2}}, (r_1(m+2)+2m)^{nr_1}, (2m+2r_1+r_2)^{mnr_1}, (4r_1+2r_2-2)^{\frac{mnr_2}{2}}, (4r_1+r_2+r_1m-2)^{mn}\}$

Theorem 2.4. *The DSs of line graph of R-edge neighbourhood corona of complete, cycle and r-regular graphs.*

Proof. Let G and H be two simple connected graphs with n_1, m_1 and n_2, m_2 are vertex set and edge set respectively. Using the definitions 1.1 and 1.6, we obtain the line graph of R-edge neighbourhood corona of G and H $[L(G \ominus_{nR} H)]$ with $m_1(2n_2+m_2+3)$ vertices. There are four type of vertices, in which m_1 vertices having degree $(2d_G(2+n_2)-2)$, $2m_1$ vertices having degree $(d_G(2+n_2))$, m_1m_2 vertices having degree $(2d_H+2)$ and $2n_2m_1$ vertices having degree $(2d_G+d_H+n_2)$. Therefore,

$$\begin{aligned} DS[L(G \ominus_{nR} H)] &= \{(2d_G(2+n_2)-2)^{m_1}, (d_G(2+n_2))^{2m_1}, (2d_H+2)^{m_1m_2}, \\ &\qquad (2d_G+d_H+n_2)^{2n_2m_1}\} \end{aligned}$$

Table 4. Degree sequences of line graph R-edge neighbourhood corona for complete, cycle and r-regular graphs.

G	H	$DS[L(G \ominus_{nR} H)]$
K_n	K_m	$\Big\{(2(n-1)(m+2)-2)^{\frac{n(n-1)}{2}}, ((n-1)(m+2))^{n(n-1)}, (2m)^{\frac{mn(n-1)(m-1)}{4}}, (2m+2n-3)^{mn(n-1)}\Big\}$
K_n	C_m	$\Big\{(2(n-1)(m+2)-2)^{\frac{n(n-1)}{2}}, ((n-1)(m+2))^{n(n-1)}, (6)^{\frac{mn(n-1)}{2}}, (2n+m)^{mn(n-1)}\Big\}$
K_n	r-regular graph with m-vertices	$\Big\{(2(n-1)(m+2)-2)^{\frac{n(n-1)}{2}}, ((n-1)(m+2))^{n(n-1)}, (2r+2)^{\frac{mnr(n-1)}{4}}, (2(n-1)+r+m)^{mn(n-1)}\Big\}$
C_n	K_m	$\Big\{(4m+6)^n, (2m+4)^{2n}, (2m)^{\frac{mn(m-1)}{2}}, (2m+3)^{2mn}\Big\}$
C_n	C_m	$\{(4m+6)^n, (2m+4)^{2n}, (6)^{mn}, (m+6)^{2mn}\}$
C_n	r-regular graph with m-vertices	$\Big\{(4m+6)^n, (2m+4)^{2n}, (2r+2)^{\frac{mnr}{2}}, (m+r+4)^{2mn}\Big\}$
r-regular graph with n-vertices	K_m	$\Big\{(2r(m+2)-2)^{\frac{nr}{2}}, (r(m+2))^{nr}, (2m)^{\frac{mnr(m-1)}{4}}, (2r+2m-1)^{mnr}\Big\}$
r-regular graph with n-vertices	C_m	$\Big\{(2r(m+2)-2)^{\frac{nr}{2}}, (r(m+2))^{nr}, (6)^{\frac{mnr}{2}}, (2r+m+2)^{mnr}\Big\}$
r_1-regular graph with n-vertices	r_2-regular graph with m-vertices	$\Big\{(2r_1(m+2)-2)^{\frac{nr_1}{2}}, (r_1(m+2))^{nr_1}, (2r_2+2)^{\frac{mnr_1r_2}{4}}, (2r_1+r_2+m)^{mnr_1}\Big\}$

3. Conclusion

In this article, we have established the degree sequences for line graph of R-vertex(edge), R-vertex(edge) neighbourhood corona of complete, cycle and r-regular graphs.

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